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Structured Singular Value Control with Two-Degree-of-Freedom Feedback Loop Factorization for Oscillating Plant with Uncertain Time Delay and Astatism*

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Abstract—**Application of Robust Control Toolbox for Time Delay Systems implemented in the Matlab system to the oscillating plant with uncertain time delay and astatism using the D-K iteration and algebraic approach. The algebraic approach combines the structured singular value, algebraic theory and algorithm of global optimization solving remaining issues in structured singular value framework. The algorithm for global optimization can be alternated with direct search methods such as Nelder-Mead simplex method giving solutions for problems with one local extreme. As a global optimization method, Differential Migration is used proving to be reliable in solving this type of problems. The D-K iteration represents a standard method in the structured singular value theory. The results obtained from the D-K iteration are compared with the algebraic approach.**

I. Introduction

Time delay systems are a constant issue present in control theory. In this paper, the problem of uncertain time delay in the oscillating plant with astatism is solved using Robust Control Toolbox for Time Delay Systems implemented in the Matlab system. The essential tool is the structured singular value denoted μ (see [13]) giving a measure of robust performance and stability. The algebraic approach (see [3], [4] and [5]) and evolutionary algorithm Differential Migration (see [2]) are used treating the problem of multimodality of the cost function and impossibility of deriving controller for performance weights with poles on the imaginary axis. This implies that the final controller provides zero steady-state error being impossible in the scope of the standard tools using DGKF formulae for obtaining $\mathbf{H}_\infty$ (sub)optimal controllers or other methods such as linear matrix inequality (LMI) approach leading to numerical problems in most of real world cases (see [9], [10] and [11]). The algebraic approach overcomes some difficulties connected with the *D-K* iteration, namely the fact that it does not guarantee convergence to a global or even local minimum (see [15]). Controllers obtained via the algebraic approach can have simpler structure due to the fact that there is no need of scaling matrices absorbance into generalized plant, hence, there is no need of further simplification causing deterioration of the frequency properties of the resulting controller. Moreover, the controller structure can be chosen in advance being not possible in the scope of currently used methods.

Optimization is performed via evolutionary algorithm. Evolutionary algorithms belong to the new branches of engineering (see [1], [12] and [14]) providing solution to the problems being not solvable using traditional optimization tools. In this paper, a new evolutionary algorithm – Differential Migration is used having some favourable properties compared to the existing ones, namely the fact that lower computational time is needed for obtaining a suitable solution.

In the proposed method, pole placement is performed via solving the Diophantine equation in the ring of Hurwitz-stable and proper rational functions ($\mathbf{R}_{PS}$). The structured singular value assesses the robust stability and performance of the controller.

For comparison reasons, the results obtained from the *D-K* iteration (see [8]) demonstrate the differences between the standard and proposed method. The overall performance is verified by simulations of step response for maximum values of time delays with simple feedback loop and two-degree-of-freedom structure with factorization of simple feedback controller to feed-forward, feedback and compensator part applicable to two-degree-of-freedom feedback interconnection (1DOF and 2DOF, see [6]).

The following notation is used: $\|\cdot\|_\infty$ denotes $\mathbf{H}_\infty$ norm, $\bar{\sigma}(\cdot)$ is maximum singular value, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{PS}$ denotes the ring of Hurwitz-stable and proper rational functions.

II. Preliminaries

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\mathbf{\Delta} \equiv \{\mathrm{diag}[\delta_1 I_{r_1},\ldots,\delta_S I_{r_S},\delta_1 I_{c_1},\ldots,\delta_T I_{c_T},\Delta_1,\ldots,\Delta_F,\Delta_1,\ldots,\Delta_K]: \\ \delta_s \in \mathbf{C}, s=1\ldots S, \delta_t \in \mathbf{R}, t=1\ldots T, \Delta_f \in \mathbf{C}^{m_f^1\times m_f^2}, f=1\ldots F, \Delta_k \in \mathbf{R}^{n_k^1\times n_k^2}, k=1\ldots K\} \qquad (1)$$

where S, T is the number of repeated scalar complex and real blocks,
F, K is the number of full complex and real blocks,
$r_1,\ldots, r_S, r_1,\ldots, r_T$, $m_1^i,\ldots, m_F^i$, $n_1^i,\ldots, n_K^i$, for $i = 1, 2$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{s=1}^{S} r_s + \sum_{t=1}^{T} m_t^1 + \sum_{f=1}^{F} t_f + \sum_{k=1}^{K} n_k^1 = n, \quad \sum_{s=1}^{S} r_s + \sum_{t=1}^{T} m_t^2 + \sum_{f=1}^{F} t_f + \sum_{k=1}^{K} n_k^2 = m \qquad (2)$$

Definition 1: For $\mathbf{M} \in \mathbf{C}^{n\times m}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\mathbf{\Delta}}(\mathbf{M}) \equiv \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \qquad (3)$$

If no such $\Delta \in \mathbf{\Delta}$ exists making $\mathbf{I} - \mathbf{M}\Delta$ singular then $\mu_{\mathbf{\Delta}}(\mathbf{M}) = 0$.

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Consider a complex matrix **M** partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \quad (4)$$

and suppose there is a defined block structure Δ_2 which is compatible in size with $\mathbf{M}_{22}$ (for any $\Delta_2 \in \mathbf{\Delta}_2$, $\mathbf{M}_{22}\Delta_2$ is square). For $\Delta_2 \in \mathbf{\Delta}_2$, consider the following loop equations

$$\begin{aligned} e &= \mathbf{M}_{11}d + \mathbf{M}_{12}w \\ z &= \mathbf{M}_{21}d + \mathbf{M}_{22}w \\ w &= \Delta_2 z \end{aligned} \quad (5)$$

If the inverse to $\mathbf{I} - \mathbf{M}_{22}\Delta_2$ exists, then e and d must satisfy $e = \mathbf{F}_L(\mathbf{M}, \Delta_2)d$, where

$$\mathbf{F}_L(\mathbf{M}, \Delta_2) = \mathbf{M}_{11} + \mathbf{M}_{12}\Delta_2(I - \mathbf{M}_{22}\Delta_2)^{-1}\mathbf{M}_{21} \quad (6)$$

is a linear fractional transformation on **M** by Δ_2, and in a feedback diagram appears as the loop in Fig. 1.

The subscript L on $\mathbf{F}_L$ pertains to the *lower* loop of **M** and is closed by Δ_2. An analogous formula describes $\mathbf{F}_U(\mathbf{M}, \Delta_1)$, which is the resulting matrix obtained by closing the *upper* loop of **M** with a matrix $\Delta_1 \in \mathbf{\Delta}$.

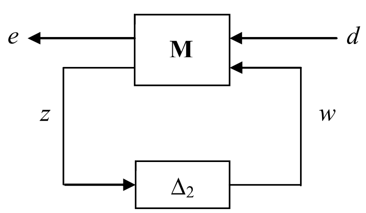


Figure 1. LFT interconnection

Theorem 1: Let $\beta > 0$. For all $\Delta_2 \in \mathbf{\Delta}_2$ with $\left|\overline{\sigma}(\Delta_2)\right| < \dfrac{1}{\beta}$, the loop shown in Fig. 1 is well-posed, internally stable, and $\left\|\mathbf{F}_L(\mathbf{M}, \Delta_2)\right\|_\infty \le \beta$ if and only if

$$\sup_{\omega \in \Re} \mu_\Delta[\mathbf{M}(j\omega)] \le \beta \quad (7)$$

with $\Delta \equiv \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}, \Delta_1 \in \mathbf{\Delta}_1, \Delta_2 \in \mathbf{\Delta}_2$.

Proof: Proof is the same as in [7] and [13] except for the fact that perturbations are complex matrices, which simplifies the proof and complies with the definition of μ-function.

III. Algebraic μ-Synthesis

The algebraic μ-synthesis can be applied to any control problem that can be transformed to the loop in Fig. 2, where **G** denotes the generalized plant, **K** is the controller, Δ_{del} is the perturbation matrix, r is the reference and e is the output.

For the purposes of the algebraic μ-synthesis, the MIMO system with l inputs and l outputs has to be decoupled into l identical SISO plants. The nominal model is defined in terms of transfer functions:

$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & \cdots & P_{1l}(s) \\ \vdots & \ddots & \vdots \\ P_{l1}(s) & \cdots & P_{ll}(s) \end{bmatrix} \quad (8)$$

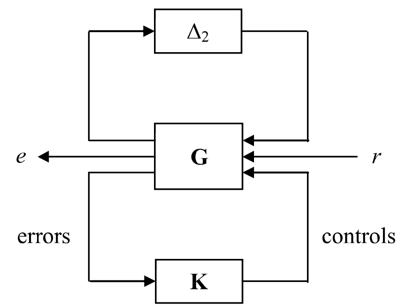


Figure 2. Closed loop interconnection

For decoupling the nominal plant $\mathbf{P}_{nom}$ ($\mathbf{P}_{nom}$ invertible) it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\mathbf{I}_l \frac{1}{P_{xy}(s)} adj[\mathbf{P}_{nom}(s)] \quad (9)$$

where P_{xy} is an element of $adj[\mathbf{P}_{nom}(s)] = \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}$ with the highest degree of numerator $\{adj[\mathbf{P}_{nom}(s)]$ denotes adjugate matrix of $\mathbf{P}_{nom}\}$. The choice of the decoupling matrix prevents the controller from cancelling any poles or zeros from the right half-plane so that internal stability of the nominal feedback loop is held. The MIMO problem is reduced to finding a controller $K(s)$ which is tuned via setting the poles of the nominal feedback loop with the plant

$$\begin{aligned} \mathbf{P}_{dec}(s) &= \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)]\mathbf{I}_l \end{aligned} \quad (10)$$

Define

$$P_{dec} \equiv \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)] \quad (11)$$

Transfer function P_{dec} can be approximated by a system P_{dec}^* with lower order than P_{dec}

$$P_{dec}^*(s) = \frac{b(s)}{a(s)} \quad (12)$$

which can be rewritten in terms of its coefficients and transformed to the elements of $\mathbf{R}_{\mathrm{PS}}$

$$P_{dec}^*(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1}s^{n-1}}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}} = \frac{B}{A},\ A, B \in \mathbf{R}_{\mathrm{PS}} \quad (13)$$

The controller $K = N_K/D_K$ is obtained by solving the Diophantine equation

$$AD_K + BN_K = 1 \quad (14)$$

with A, B, D_K, $N_K \in \mathbf{R}_{\mathrm{PS}}$. Equation (14) is often called the Bezout identity. All feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT}, \qquad N_{K_0}, D_{K_0} \in \mathbf{R}_{\mathrm{PS}} \quad (15)$$

where $N_{K_0}, D_{K_0} \in \mathbf{R}_{\mathrm{PS}}$ are particular solutions of (14) and T is an arbitrary element of $\mathbf{R}_{\mathrm{PS}}$.

The controller K satisfying equation (14) guarantees the BIBO (bounded input bounded output) stability of the feedback loop in Fig. 3. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G},\mathbf{K})$ making possible usage of performance weights with integration property implying non-existence of state space solutions using DGKF formulae (see [9]) due to zero eigenvalues of appropriate Hamiltonian matrices. Such procedure, however, results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (14). This technique is neither possible in the scope of the standard μ-synthesis using DGKF formulae, nor using LMI approach (see [10]) leading to numerical problems in most of real-world applications.

The aim of synthesis is to design a controller which satisfies the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},\mathbf{K})(j\omega,\alpha_1,\ldots,\alpha_{n+n_1+n_2},t_1,\ldots,t_{n_2})] \le 1,\ \omega \in [0,\infty) \qquad (16)$$

where ω is angular velocity in Fourier transform, $n+n_1+$ $+n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution K_0, t_i are arbitrary parameters in $T = \dfrac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1}+s)\cdot\ldots\cdot(\alpha_{n_1+n_2}+s)}$ and $\mu_{\mathbf{\Delta}}$ denotes the structured singular value of LFT on generalized plant $\mathbf{G}$ and controller $\mathbf{K}$.

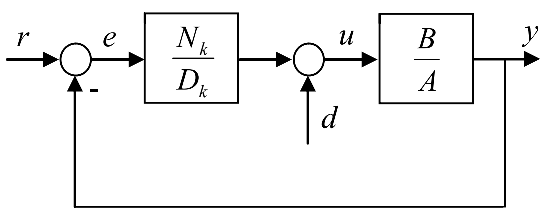


Figure 3. Nominal feedback loop

Tuning parameters are positive and constrained to the real axis since parameters of the transfer function have to be real and due to the fact that non-real poles cause oscillations of the nominal feedback loop.

A crucial problem of the cost function in (16) is the fact that many local extremes are present. Hence, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary optimization solving the task very efficiently.

IV. Problem Formulation

The problem to solve is general 3rd order system with uncertain time delay:

$$\mathbf{P} \equiv \left\{ \frac{b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0} e^{-\tau s} : 0 \le \tau \le T_0 \right\} \qquad (17)$$

The control objective is to find a controller that guarantees the robust stability and performance for every plant from the set P. The time delay is treated by multiplicative uncertainty

$$\{P(1+W_2\Delta) : \|\Delta_{del}\|_\infty \le 1\} \qquad (18)$$

Define the nominal plant

$$P(s) = \frac{b}{a} = \frac{b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0} \qquad (19)$$

then for the weighting function W_2 the following inequality must be held

$$\left| \frac{P'(j\omega)}{P(j\omega)} - 1 \right| < |W_2(j\omega)|,\ \forall \omega \in \overline{\Re}_+,\ \forall P' \in \mathbf{P} \qquad (20)$$

equivalent with

$$\left| e^{-\tau j\omega} - 1 \right| < |W_2(j\omega)|,\ \forall \omega \in \overline{\Re}_+,\ \tau \in [0 \quad T_0] \qquad (21)$$

The weight W_2 is defined as an envelope curve of $\left| e^{-\tau j\omega} - 1 \right|$.

V. Problem Solution

A. Structured Singular Value Framework

The problem defined in previous section can be solved using interconnection in Fig. 4. Here, $\mathbf{G}$ denotes the generalized plant partitioned to

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & G_{22} \end{bmatrix} \qquad (22)$$

where the block structure of $\mathbf{G}$ corresponds with the input and output variables in Fig. 4:

$$\begin{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ v \end{bmatrix} = \mathbf{G} \cdot \begin{bmatrix} \begin{bmatrix} w \\ d \end{bmatrix} \\ u \end{bmatrix} \qquad (23)$$

The design objective is to find a stabilizing controller K such that

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_l(\mathbf{G},K)] \qquad (24)$$

is minimal, where

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G},K) = \mathbf{G}_{11} + \mathbf{G}_{12} K (1 - G_{22} K)^{-1} \mathbf{G}_{21} \qquad (25)$$

is the lower linear fractional transformation on generalized plant $\mathbf{G}$ and controller K (see Fig. 4).

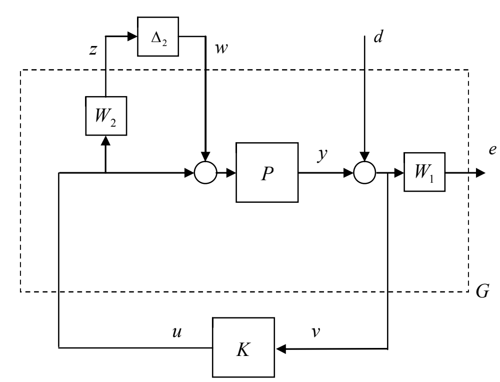


Figure 4. Closed-loop interconnection for μ-synthesis

B. Algebraic Approach

The controller $K = \frac{N_K}{D_K}$ is obtained by solving the Diophantine equation (14).

By the analysis of the polynomial degrees of a and b, the transfer functions A, B, D_K and N_K were chosen so that the number of closed loop poles is minimal and the asymptotic tracking is achieved:

$$A = \frac{a}{\prod_{i=1}^{n}(s+\alpha_i)}, \; B = \frac{b}{\prod_{i=1}^{n}(s+\alpha_i)} \tag{26}$$

$$D_K = \frac{s d_k}{\prod_{i=n+1}^{2n}(s+\alpha_i)}, \; N_K = \frac{n_k}{\prod_{i=n+1}^{2n}(s+\alpha_i)} \tag{27}$$

where n is the actual degree of polynomial a obtained by omitting zero parameters a_i.

The resulting controller has the general PID structure:

$$K(s) = \frac{n_{k,n}s^n + \ldots + n_{k,1}s + n_{k,0}}{s(s^{n-1} + s^{n-2}d_{k,n-2} + \ldots + d_{k,0})} \tag{28}$$

VI. Example of Time Delay System Control

The plant family is defined as 3rd order oscillating system with uncertain time delay and first order astatism:

$$\mathbf{P} \equiv \left\{ \frac{e^{-\tau s}}{s^3 + s} : 0 \le \tau \le 0.5 \right\} \tag{29}$$

The control objective is to find a controller that will guarantee the robust stability and performance for every plant from the set **P**. The time delay is treated by multiplicative uncertainty

$$\{P(1 + W_2\Delta) : \|\Delta\|_\infty \le 1\} \tag{30}$$

Define the nominal plant

$$P(s) = \frac{b}{a} = \frac{1}{s^3 + s} \tag{31}$$

For the weighting function W_2, the following inequality must be held

$$\left| \frac{P'(j\omega)}{P(j\omega)} - 1 \right| \le |W_2(j\omega)|, \; \forall \omega \in \overline{\Re}_+, \; \forall P' \in \mathbf{P} \tag{32}$$

equivalent with

$$\left| e^{-\tau j\omega} - 1 \right| \le |W_2(j\omega)|, \; \forall \omega \in \overline{\Re}_+, \; \tau \in [0 \quad 0.5] \tag{33}$$

The weight W_2 can be defined as envelope curve of $\left| e^{-T_0 j\omega} - 1 \right|$ for $T_0 = 0.5$ (see Fig. 5):

$$W_2(s) = 2.3\frac{2s}{2s+5} \tag{34}$$

The performance condition is of the form:

$$\|W_1 S\|_\infty < 1 \tag{35}$$

where S is the sensitivity function and weight W_1 is designed so that the asymptotic tracking is achieved:

$$W_1^A(s) = \frac{2}{10s^3 + 100s^2 + s} \tag{36}$$

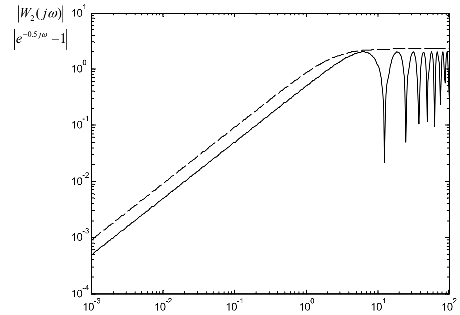


Figure 5. Bode plot of W_2 (dashed) and $\left| e^{-0.5j\omega} - 1 \right|$ (full)

A. Algebraic Approach

The controller $K = \frac{N_K}{D_K}$ is obtained by solving the Diophantine equation (14). By the analysis of the polynomial degrees of a and b, the transfer functions A, B, D_K and N_K were chosen so that the number of closed loop poles is minimal and the asymptotic tracking is achieved:

$$A = \frac{a}{\prod_{i=1}^{3}(s+\alpha_i)}, \; B = \frac{b}{\prod_{i=1}^{3}(s+\alpha_i)} \tag{37}$$

$$D_K = \frac{s d_k}{\prod_{i=4}^{6}(s+\alpha_i)}, \; N_K = \frac{n_k}{\prod_{i=4}^{6}(s+\alpha_i)} \tag{38}$$

and degrees of polynomials d_k, n_k are:

$$\partial d_k = 2, \partial n_k = 3 \tag{39}$$

The resulting controller has general PID structure:

$$K(s) = \frac{n_{k,3}s^3 + n_{k,2}s^2 + n_{k,1}s + n_{k,0}}{s(s^2 + s d_{k,1} + d_{k,0})} \tag{40}$$

By the optimization of the poles α_i via the Differential Migration, resulting poles were obtained:

$$\alpha_1 = 2.768, \alpha_2 = 0.997, \alpha_3 = 0.705, \alpha_4 = 0.608, \alpha_5 = 0.561, \alpha_5 = 0.485 \tag{41}$$

yielding the controller

$$K_A(s) = \frac{9.014s^3 + 3.838s^2 + 2.667s + 0.3225}{s^3 + 6.126s^2 + 12.72s} \tag{42}$$

B. D-K Iteration

In order to satisfy state-space formulae assumptions for $\mathbf{H}_\infty$ suboptimal controller the performance weight W_1 has to be modified so that it does not have integrating behaviour:

$$W_1^{D-K}(s) = \frac{4}{10s^3 + 100s^2 + s + 10^{-5}} \tag{43}$$

The controller obtained from the *D-K* iteration was approximated by 5th order transfer function:

$$K_{D-K}(s)=\frac{-0.0056s^5+2.612s^4-0.149s^3+2.312s^2+0.3617s+1.477\cdot10^{-5}}{s^5+3.186s^4+6.269s^3+6.366s^2+0.0621s+6.959\cdot10^{-7}} \quad (44)$$

The μ-plot in Fig. 6 shows that both controllers have the supremum of μ below one and the robust stability and performance condition is satisfied with maximum values:

$$\sup_{\omega}\mu_{\Delta}[\mathbf{F}_l(\mathbf{G},K_A)]=0.86\ ,\ \sup_{\omega}\mu_{\Delta}[\mathbf{F}_l(\mathbf{G},K_{D-K})]=0.89\ .$$

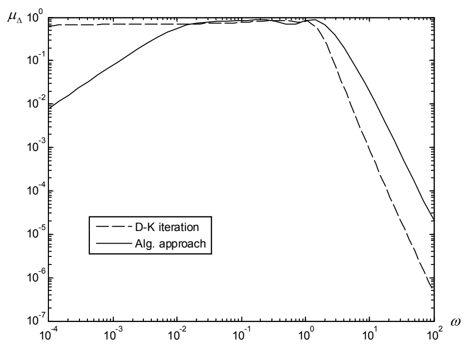


Figure 6. μ-plot for the controllers obtained by the *D-K* iteration and algebraic approach

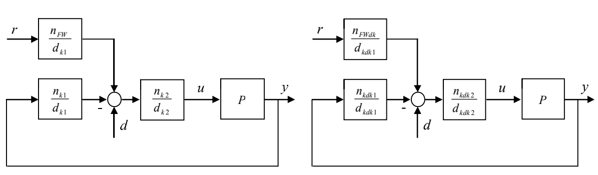
C. Factorization for 2DOF feedback loop

The controllers for 2DOF feedback loop (Fig. 7a, 7b - algebraic approach and *D-K* iteration, respectively) have the compensator (n_{k2}, d_{k2}, n_{kdk2}, d_{kdk2}) defined as fraction of the factors corresponding with most stable zero and least stable pole of K_A and K_{D-K} and feedback (n_{k1}, d_{k1}, n_{kdk1}, d_{kdk1}) and feed-forward part (n_{FW}, d_{k1}, n_{FWdk}, d_{kdk1}) defined by the fraction of the factors corresponding with remaining zeros and poles of K_A and K_{D-K} with $n_{FW}=n_{k1,0}$ and $n_{FWdk}=n_{kdk1,0}$ ($n_{k1,0}$, $n_{kdk1,0}$ being the coefficients of n_{k1} and n_{kdk1} of zero exponent of s):

$$\frac{n_{k1}}{d_{k1}}=\frac{9.014s^2+4.76s+3.154}{s^2+6.126s+12.72}\ ,\ \frac{n_{FW}}{d_{k1}}=\frac{3.154}{s^2+6.126s+12.72}\ ,\ \frac{n_{k2}}{d_{k2}}=\frac{s+0.1023}{s} \quad (45)$$

$$\frac{n_{kdk1}}{d_{kdk1}}=\frac{-0.00561s^4+2.613s^3-0.5441s^2+2.394s+9.776\cdot10^{-5}}{s^4+3.186s^3+6.269s^2+6.366s+0.062}\ ,$$

$$\frac{n_{FWdk}}{d_{kdk1}}=\frac{9.776\cdot10^{-5}}{s^4+3.186s^3+6.269s^2+6.366s+0.062}\ ,\ \frac{n_{kdk2}}{d_{kdk2}}=\frac{s+0.151}{s+1.122\cdot10^{-5}} \quad (46)$$



(a) (b)

Figure 7. 2DOF feedback loop

D. Comparison Study

Simulations for the maximum time delay 2DOF and simple feedback loop in Fig. 8, 9, 10 and 11 show that both the algebraic approach and *D-K* iteration controllers yield stable response. The algebraic approach gives similar results with *D-K* iteration for simple feedback loop with 100% overshoot compared to 75% for the reference method but lower number of oscillations. The *D-K* iteration has no overshoot for 2DOF feedback loop but 1000 times longer time needed for reaching steady state with non-zero steady state tracking error compared to algebraic approach with zero steady state tracking error but 80% overshoot.

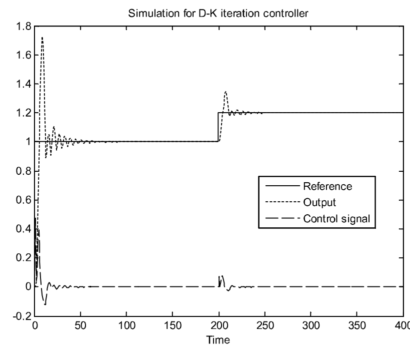


Figure 8. Simulation for simple feedback loop – *D-K* iteration

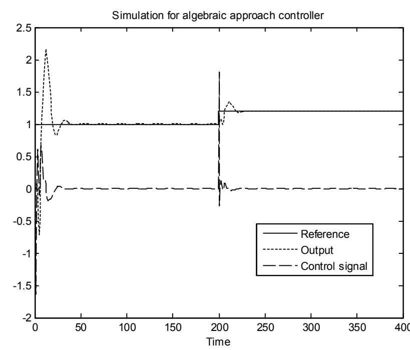


Figure 9. Simulation for simple feedback loop algebraic approach

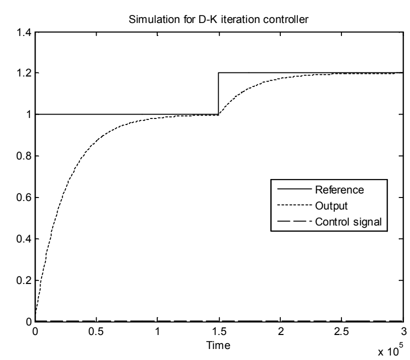


Figure 10. Simulation for 2DOF feedback loop – *D-K* iteration

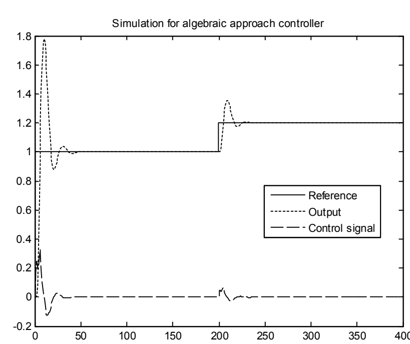


Figure 11. Simulation for 2DOF feedback loop – algebraic approach

VII. Download

The Robust Control Toolbox for Time Delay Systems toolbox can be downloaded from [16].

VIII. Conclusion

The paper showed usage of the Robust Control Toolbox for Time Delay Systems for the Matlab system. An outline of the algebraic approach was given with application to time delay plant with oscillating poles and first order astatism in nominal model. The plots and simulations of control using the presented Matlab toolbox showed the benefits of the algebraic approach in comparison with the *D-K* iteration as the reference procedure for robust control design using structured singular value.

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