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A robust algorithm for the tracking control of robot manipulators

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***Abstract*—For many robot manipulators tasks, the main objective consists in following accurately position and velocity time varying trajectories. Sliding Mode Control (SMC) techniques allow exact position tracking despite a poor knowledge of the robot model and in the presence of external perturbations. In the last years fixed time stability has become an area of great interest, where one of the main problems to be solved is the possible appearance of singularities. The current contribution introduces a singularity free fixed time SMC scheme for position and velocity tracking of robot manipulators in the presence of external perturbations. The control scheme avoids singularities by using linear sliding mode (LSM) surfaces, which are yielded to zero in a predefined fixed time. Although the employment of LSM allows to avoid singularities, the tracking errors tend to zero only asymptotically once the sliding surface is reached. Simulation results show that this does not represent a big drawback since tracking errors tend to zero even in the presence of external perturbations as foreseen in theory.**

***Index Terms*—Finite time, robot manipulator, robustness, position tracking.**

I. Introduction

Sliding Mode Control (SMC) theory is a well suited technique for the control of robot manipulators. In the recent years, there has been a tendency to achieve fixed time stability. Conventional SMC systems adopt linear sliding-mode (LSM) surface based controllers. Another class of SMC, the terminal sliding-mode (TSM), offers some superior properties such as fast and finite-time convergence, and high steady-state tracking precision. However, the TSM controller has a singularity problem, that is, in some areas of the state space, the TSM control may require to be infinitely large in order to maintain the ideal TSM motion. In [1] a global non-singular TSM controller for rigid manipulators is presented. The time taken to reach the equilibrium point from any initial state is guaranteed to be finite time. The singularity phenomena and the dynamic behaviors of the TSM control systems are studied in [2] and a systematic method to overcome the singularity problem for global TSM control of general dynamical systems is proposed. In [3] a fixed-time sliding mode control for the global fixed-time trajectory tracking of robot manipulators subject to uncertain dynamics and bounded external disturbances is introduced. A fixed-time sliding surface is proposed and a singularity-free fixed-time sliding mode control (SFSMC) is constructed. Lyapunov stability theory is employed to prove stability. In [4], the problem of finite-time trajectory tracking of robot manipulators with uncertain dynamics, external bounded disturbances, and bounded torque inputs is studied. In order to achieve finite-time convergence a nonlinear control algorithm based on a second-order sliding mode controller in combination with nonsingular terminal sliding mode is proposed. The controller structure is simple since it does not require the knowledge of the robot dynamic model. In [5], new global robust fixed-time stability results for scalar systems by using constant and variable exponent coefficients are proposed. Then, they are applied to global robust fixed-time stabilization of a class of uncertain nonlinear second-order systems by using SMC. In [6], it is proposed a time-varying nonsingular TSM control for a class of uncertain second order nonlinear system and its application to n-links rigid robotic manipulators. The singularity problem is avoided and the reaching phase existing in conventional TSM control is totally suppressed, which ensures the global robustness of the system against uncertainties and disturbances. However, a nominal robot model is required for implementation. In [7], a robust adaptive fixed-time SMC method for robotic systems with parameter uncertainties and input saturation is proposed. In [8], the authors explore the prescribed-time stabilization problem of strict-feedback uncertain systems, both in the presence of unknown control gains and mismatched non-parametric uncertainties. It is shown that the settling time is user-assignable a priori regardless of the initial condition. In [9], a robust controller for first-order uncertain nonlinear systems is proposed by introducing a novel hybrid sliding surface. The convergence of the system state in bounded time, independent of the initial conditions of the system, is guaranteed without singularity problems.

In this paper, a simple robust sliding mode controller is designed for robot manipulators to achieve exact tracking in the presence of external bounded perturbations. The main characteristic is that the LSM surface is reached in finite time, thus allowing the tracking errors to tend to zero exponentially while singularities are avoided. Simulation

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results are in good accordance with the developed theory. The paper is organized as follows. Section II provides some basic preliminaries on robot model properties and on the standard DREM procedure. Section III introduces the new adaptive law and the stability analysis, while simulation results are given in Section IV. The paper concludes in Section V.

II. PRELIMINARES

Consider a n-degrees of freedom rigid robot whose dynamics can be described by [10]:

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{p}} \,, \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of generalized joint coordinates, $\boldsymbol{H}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \mathbb{R}^n$ is the vector of Coriolis and centrifugal torques, $\boldsymbol{D} \in \mathbb{R}^{n\times n}$ is a symmetric positive semidefinite matrix of joint viscous friction coefficients, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^n$ is the vector of gravitational torques, $\boldsymbol{\tau}_{\mathrm{p}} \in \mathbb{R}^n$ represents external perturbations, and $\boldsymbol{\tau} \in \mathbb{R}^n$ is the vector of input torques acting at the joints. Assume for simplicity's sake that the robot has revolute joints only and that velocity measurements are available. Some useful model properties are listed below:

Property 2.1: It holds $\lambda_{\mathrm{h}}\|\boldsymbol{x}\|^2 \le \boldsymbol{x}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{x} \le \lambda_{\mathrm{H}}\|\boldsymbol{x}\|^2$ $\forall\ \boldsymbol{q} \in \mathbb{R}^n$, $\boldsymbol{x} \in \mathbb{R}^n$, and $0 < \lambda_{\mathrm{h}} \le \lambda_{\mathrm{H}} < \infty$, with $\lambda_{\mathrm{h}} = \min_{\boldsymbol{q}\in\mathbb{R}^n} \lambda_{\min}(\boldsymbol{H}(\boldsymbol{q}))$ and $\lambda_{\mathrm{H}} = \max_{\boldsymbol{q}\in\mathbb{R}^n} \lambda_{\max}(\boldsymbol{H}(\boldsymbol{q}))$. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a symmetric matrix, respectively. △

Property 2.2: By using the Christoffel symbols of the first kind to compute $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$, the matrix $\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew-symmetric. △

Property 2.3: There exists a positive constant $0 < k_{\mathrm{c}} < \infty$ such that $\|\boldsymbol{C}(\boldsymbol{q},\boldsymbol{x})\| \le k_{\mathrm{c}}\|\boldsymbol{x}\|$ holds $\forall$ $\boldsymbol{x},\boldsymbol{q} \in \mathbb{R}^n$. △

Property 2.4: The vector of the generalized gravitational torques $\boldsymbol{g}(\boldsymbol{q})$ satisfies $\|\boldsymbol{g}(\boldsymbol{q})\| \le k_{\mathrm{g}}$, $k_{\mathrm{g}} > 0$. △

III. A ROBUST CONTROL LAW FOR ROBOT MANIPULATORS

In this section, a control law for position tracking of robot manipulators will be developed. Given a bounded desired trajectory $\boldsymbol{q}_{\mathrm{d}}$ with at least bounded first and second derivatives, the control problem consists in achieving that the tracking error

$$\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_{\mathrm{d}} \,, \tag{2}$$

tends to zero asymptotically in the presence of external perturbations. Define first

$$\dot{\boldsymbol{q}}_{\mathrm{r}} = \dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\boldsymbol{e} \tag{3}$$

$$\boldsymbol{s} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\mathrm{r}} = \dot{\boldsymbol{e}} + \boldsymbol{\Lambda}\boldsymbol{e} \,, \tag{4}$$

where $\boldsymbol{\Lambda} \in \mathbb{R}^{n\times n}$ is a diagonal positive definite matrix. Definition (4) is equivalent to the stable linear filter

$$\dot{\boldsymbol{e}} = -\boldsymbol{\Lambda}\boldsymbol{e} + \boldsymbol{s} \,, \tag{5}$$

so that if $\boldsymbol{s}$ is bounded and tends to zero, so do $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$. Consider now the following control law based on [5]

$$\boldsymbol{\tau} = -\boldsymbol{K}_{\mathrm{v}} \underbrace{\mathrm{sign}(\boldsymbol{s})\,|\boldsymbol{s}|^{\frac{\boldsymbol{\lambda}_{\mathrm{s}}\boldsymbol{s}^2}{1+\boldsymbol{\mu}_{\mathrm{s}}\boldsymbol{s}^2}}}_{\boldsymbol{\tau}_{\mathrm{s}}} - \boldsymbol{K}_{\mathrm{p}}\|\boldsymbol{s}\|\boldsymbol{s} \,, \tag{6}$$

where $\boldsymbol{K}_{\mathrm{v}}, \boldsymbol{K}_{\mathrm{p}}, \boldsymbol{\lambda}_{\mathrm{s}}, \boldsymbol{\mu}_{\mathrm{s}} \in \mathbb{R}^{n\times n}$ are diagonal positive definite matrices, and for $i = 1,\ldots,n$ the i-th element $\tau_{\mathrm{s}i}$ of $\boldsymbol{\tau}_{\mathrm{s}} \in \mathbb{R}^n$ is defined as

$$\tau_{\mathrm{s}i} = \mathrm{sign}(s_i)\,|s_i|^{\frac{\lambda_{\mathrm{s}i}s_i^2}{1+\mu_{\mathrm{s}i}s_i^2}} \equiv \mathrm{sign}(s_i) f_{\mathrm{s}i}(s_i) \,, \tag{7}$$

with $\lambda_{\mathrm{s}i}$ and $\mu_{\mathrm{s}i}$ the i-th element of $\boldsymbol{\lambda}_{\mathrm{s}}$ and $\boldsymbol{\mu}_{\mathrm{s}}$, respectively, which satisfy

$$\frac{\lambda_{\mathrm{s}i}}{1+\mu_{\mathrm{s}i}} = \beta_{\mathrm{s}i} > 1 \,. \tag{8}$$

The following error dynamics for $\boldsymbol{s}$ can be computed by substituting (6) into (1)

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}_{\mathrm{p}} - \boldsymbol{K}_{\mathrm{v}}\boldsymbol{\tau}_{\mathrm{s}} - \boldsymbol{K}_{\mathrm{p}}\|\boldsymbol{s}\|\boldsymbol{s} \,. \tag{9}$$

Define

$$\boldsymbol{\tau}_{\mathrm{a}} = -\left(\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{D}\dot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{g}(\boldsymbol{q})\right) \,, \tag{10}$$

to rewrite (9) as

$$\boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{s}} = -\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{s} - \boldsymbol{D}\boldsymbol{s} + \boldsymbol{\tau}_{\mathrm{p}} + \boldsymbol{\tau}_{\mathrm{a}} - \boldsymbol{K}_{\mathrm{v}}\boldsymbol{\tau}_{\mathrm{s}} - \boldsymbol{K}_{\mathrm{p}}\|\boldsymbol{s}\|\boldsymbol{s} \,. \tag{11}$$

Assumption 3.1: The external perturbation vector is bounded, *i. e.* $\|\boldsymbol{\tau}_{\mathrm{p}}\| \le p_{\max}$, $\forall\ t \ge t_0$ and some $p_{\max} > 0$.

Theorem 3.1: Consider the closed loop error dynamics (11) generated by substituting the control law (6) into the robot dynamics (1). If Assumption 3.1 is fulfilled and

$$k_{\mathrm{v}} > \alpha_{\max} \cdot e^{\frac{\lambda_{\max}(\boldsymbol{\lambda}_{\mathrm{s}})}{2e}} \tag{12}$$

holds with $\alpha_{\max}$ defined in (23) and with $k_{\mathrm{v}} = \lambda_{\min}(\boldsymbol{K}_{\mathrm{v}})$, then

a) $\boldsymbol{s}$, $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$ remain bounded for all $t \ge t_0$.

b) $\boldsymbol{s}$ becomes zero in a time no larger than

$$\begin{aligned} T_{\max} &= T_{\mathrm{s}1} + T_{\mathrm{s}2} && (13) \\ &= \frac{\lambda_{\mathrm{H}}^{\frac{3}{2}}}{\lambda_{\mathrm{h}}^{\frac{1}{2}} \lambda_{\min}(\boldsymbol{K}_{\mathrm{p}})} \\ &+ \frac{\sqrt{\lambda_{\mathrm{h}} \lambda_{\mathrm{H}}}}{\left(k_{\mathrm{v}} e^{-\frac{\lambda_{\max}(\boldsymbol{\lambda}_{\mathrm{s}})}{2e}} - \alpha_{\max} \right)}, \end{aligned}$$

where $T_{\mathrm{s}1}$ and $T_{\mathrm{s}2}$ are defined in (35) and (41), respectively. λ_{h} and λ_{H} are defined in Property 2.1.

c) $\boldsymbol{e}$ and $\dot{\boldsymbol{e}} \to \mathbf{0}$ as $t \to \infty$.

Proof:

The proof closely follows that of Theorem 2 in [5]. First of all, consider $f_{\mathrm{s}i}$ in (7) rewritten here for simplicity as

$$f_{\mathrm{s}i}(x) = |x|^{\frac{\lambda_{\mathrm{s}i} x^2}{1+\mu_{\mathrm{s}i} x^2}} = e^{\left(\frac{\lambda_{\mathrm{s}i} x^2}{1+\mu_{\mathrm{s}i} x^2} \ln(|x|) \right)} \quad (14)$$

for $x \in \mathbb{R}$. Note that it can be shown that $f_{\mathrm{s}i}(x) = f_{\mathrm{s}i}(s_i)$ is continuous at $x = s_i = 0$ with $f_{\mathrm{s}i}(0) = 1$, so that the right-hand side of (7) is locally bounded [5]. The steps of the proof are:

a) Consider the following positive definite function

$$V_{\mathrm{s}}(t) = \frac{1}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{H}(\boldsymbol{q}) \boldsymbol{s}, \quad (15)$$

which satisfies

$$\|\boldsymbol{s}\|^2 \le V_{\mathrm{s}}(t) \le \frac{\lambda_{\mathrm{H}}}{\lambda_{\mathrm{h}}} \|\boldsymbol{s}\|^2, \quad (16)$$

with λ_{h} and λ_{H} defined in Property 2.1.

By using Property 2.2, the derivative of $V_{\mathrm{s}} = V_{\mathrm{s}}(t)$ along (11) can be shown to satisfy

$$\begin{aligned} \dot{V}_{\mathrm{s}} &= \frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{H}(\boldsymbol{q}) \dot{\boldsymbol{s}} + \frac{1}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \dot{\boldsymbol{H}}(\boldsymbol{q}) \boldsymbol{s} && (17) \\ &= -\frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{D} \boldsymbol{s} - \frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{v}} \boldsymbol{\tau}_{\mathrm{s}} \\ &\quad -\frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{p}} \|\boldsymbol{s}\| \boldsymbol{s} + \frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} (\boldsymbol{\tau}_{\mathrm{a}} + \boldsymbol{\tau}_{\mathrm{p}}). \end{aligned}$$

It is necessary to find a bound for $\boldsymbol{\tau}_{\mathrm{a}}$, but this can only be made by carrying out a local stability analysis. In fact, if $\dot{V}_{\mathrm{s}} \le 0$ then from (16) one has

$$\begin{aligned} \|\boldsymbol{s}\|^2 \le V_{\mathrm{s}}(t) &\le \frac{\lambda_{\mathrm{H}}}{\lambda_{\mathrm{h}}} \|\boldsymbol{s}(t_0)\|^2 && (18) \\ \Rightarrow \|\boldsymbol{s}\| &\le \sqrt{\frac{\lambda_{\mathrm{H}}}{\lambda_{\mathrm{h}}}} \|\boldsymbol{s}(t_0)\| = s_{\max}. \end{aligned}$$

Since $|s_i| \le \|\boldsymbol{s}\| \le s_{\max}$, then it is not difficult to show that [11]

$$|e_i(t)| \le |e_i(t_0)| + \frac{1}{\lambda_i} s_{\max}, \quad (19)$$

where e_i is the i-th element of $\boldsymbol{e}$ and λ_i is the i-th element of $\boldsymbol{\Lambda}$ in (4). This means that as long as $\dot{V}_{\mathrm{s}} \le 0$ a bound for $\boldsymbol{e}$ can be found as

$$\|\boldsymbol{e}\| \le e_{\max}. \quad (20)$$

Note also that after (5) it holds

$$\begin{aligned} \|\dot{\boldsymbol{e}}\| &\le \lambda_{\max}(\boldsymbol{\Lambda}) \|\boldsymbol{e}\| + \|\boldsymbol{s}\| && (21) \\ &\le \lambda_{\max}(\boldsymbol{\Lambda}) e_{\max} + s_{\max} = \dot{e}_{\max}. \end{aligned}$$

Next it will be shown that it is possible to set gains to satisfy $\dot{V}_{\mathrm{s}} \le 0$ by taking into account (18)-(21). Note that $\boldsymbol{s}(t_0)$ and $\boldsymbol{e}(t_0)$ are not only known, but they can even be set to zero if desired. Keeping in mind the previous discussion, recall that by assumption one has $\|\dot{\boldsymbol{q}}_{\mathrm{d}}\| \le v_{\max}$ and $\|\ddot{\boldsymbol{q}}_{\mathrm{d}}\| \le a_{\max}$ for some positive constants $v_{\max}$ and $a_{\max}$, and consider that in view of Properties 2.1, 2.3 and 2.4 it is possible to get the following bounds

$$\begin{aligned} \|\boldsymbol{\tau}_{\mathrm{a}}\| &\le \lambda_{\mathrm{H}} \|\ddot{\boldsymbol{q}}_{\mathrm{r}}\| + k_{\mathrm{c}} \|\dot{\boldsymbol{q}}\| \|\dot{\boldsymbol{q}}_{\mathrm{r}}\| + \lambda_{\max}(\boldsymbol{D}) \|\dot{\boldsymbol{q}}_{\mathrm{r}}\| + k_{\mathrm{g}} \\ &\le \lambda_{\mathrm{H}} (a_{\max} + \lambda_{\max}(\boldsymbol{\Lambda}) \dot{e}_{\max}) + k_{\mathrm{g}} && (22) \\ &\quad + (k_{\mathrm{c}} (v_{\max} + \dot{e}_{\max}) + \lambda_{\max}(\boldsymbol{D})) \cdot \\ &\quad \cdot (v_{\max} + \lambda_{\max}(\boldsymbol{\Lambda}) e_{\max}) = \tau_{a \max}, \end{aligned}$$

where (2) and (3) have been used. Then, by taking Assumption 3.1 into account, one has

$$\|\boldsymbol{\tau}_{\mathrm{a}}\| + \|\boldsymbol{\tau}_{\mathrm{p}}\| \le \tau_{a \max} + p_{\max} = \alpha_{\max}. \quad (23)$$

Now it is possible to rewrite (17) as

$$\begin{aligned} \dot{V}_{\mathrm{s}} &\le -\frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{v}} \boldsymbol{\tau}_{\mathrm{s}} - \frac{2\lambda_{\min}(\boldsymbol{K}_{\mathrm{p}})}{\lambda_{\mathrm{h}}} \|\boldsymbol{s}\|^3 \\ &\quad + \frac{2}{\lambda_{\mathrm{h}}} \alpha_{\max} \|\boldsymbol{s}\| \\ &\le -\frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{v}} \boldsymbol{\tau}_{\mathrm{s}} - \frac{2\lambda_{\min}(\boldsymbol{K}_{\mathrm{p}})}{\lambda_{\mathrm{h}}} \|\boldsymbol{s}\|^3 && (24) \\ &\quad + \frac{2}{\lambda_{\mathrm{h}}} \alpha_{\max} (|s_1| + \cdots + |s_n|). \end{aligned}$$

According to (7) one has

$$\begin{aligned} &-\frac{2}{\lambda_{\mathrm{h}}} \boldsymbol{s}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{v}} \boldsymbol{\tau}_{\mathrm{s}} + \frac{2}{\lambda_{\mathrm{h}}} \alpha_{\max} \sum_{i=1}^{n} |s_i| = && (25) \\ &-\sum_{i=1}^{n} \frac{2k_{\mathrm{v}i}}{\lambda_{\mathrm{h}}} s_i \mathrm{sign}(s_i) \, |s_i|^{\frac{\lambda_{\mathrm{s}i} s_i^2}{1+\mu_{\mathrm{s}i} s_i^2}} \\ &+ \frac{2}{\lambda_{\mathrm{h}}} \alpha_{\max} \sum_{i=1}^{n} |s_i|, \end{aligned}$$

so that for $i = 1, \ldots, n$ one can analyze separately the terms

$$-\frac{2k_{\mathrm{v}i}}{\lambda_{\mathrm{h}}} s_i \mathrm{sign}(s_i) \, |s_i|^{\frac{\lambda_{\mathrm{s}i} s_i^2}{1+\mu_{\mathrm{s}i} s_i^2}} + \frac{2}{\lambda_{\mathrm{h}}} \alpha_{\max} |s_i|. \quad (26)$$

For each and one of these terms, if $|s_i| > 1$ one has

$$-\frac{2k_{vi}}{\lambda_h}|s_i|^{\frac{\lambda_{si}s_i^2}{1+\mu_{si}s_i^2}+1}+\frac{2}{\lambda_h}\alpha_{max}|s_i| \leq -\frac{2(k_{vi}-\alpha_{max})}{\lambda_h}|s_i|^{\beta_{si}+1} \leq 0\,, \tag{27}$$

where it has been taken advantage of the fact that $|s_i| \leq |s_i|^{\frac{\lambda_{si}}{1+\mu_{si}}+1}$ whenever $|s_i| > 1$, while from (8) it holds

$$\frac{\lambda_{si}s_i^2}{1+\mu_{si}s_i^2}+1 \geq \frac{\lambda_{si}}{1+\mu_{si}}+1 = \beta_{si}+1 > 2 \tag{28}$$

also for $|s_i| > 1$. Note that in view of (12) one has $(k_{vi}-\alpha_{max}) > 0$ because $\alpha_{max}\cdot e^{\frac{\lambda_{max}(\boldsymbol{\lambda}_s)}{2e}} > \alpha_{max}$ is always true.

On the other hand, if $|s_i| \leq 1$ use for simplicity the notation $x = s_i$ to show that since $|x| \leq 1$ and $(1+\mu_{si}x^2) \geq 1$, then

$$|x|^{\frac{\lambda_{si}x^2}{1+\mu_{si}x^2}} \geq |x|^{\lambda_{si}x^2} \tag{29}$$

holds and therefore (26) satisfies

$$-\frac{2k_{vi}}{\lambda_h}x\,\mathrm{sign}(x)\,|x|^{\frac{\lambda_{si}x^2}{1+\mu_{si}x^2}}+\frac{2}{\lambda_h}\alpha_{max}|x| \leq -\frac{2k_{vi}}{\lambda_h}|x|\,|x|^{\lambda_{si}x^2}+\frac{2}{\lambda_h}\alpha_{max}|x|\,. \tag{30}$$

It can be shown that the minimum value of $|x|^{\lambda_{si}x^2} = e^{\lambda_{si}x^2\ln(|x|)}$ can take is given by $e^{-\frac{\lambda_{si}}{2e}}$ [5]. By taking this into account, (30) satisfies

$$\begin{aligned}-\frac{2k_{vi}}{\lambda_h}|s_i|\,|s_i|^{\frac{\lambda_{si}s_i^2}{1+\mu_{si}s_i^2}}+\frac{2}{\lambda_h}\alpha_{max}|s_i| &\leq \\ -\frac{2}{\lambda_h}\left(k_{vi}e^{-\frac{\lambda_{si}}{2e}}-\alpha_{max}\right)|s_i| &\leq 0\,,\end{aligned} \tag{31}$$

where once again the last inequality holds in view of (12). This means from (24) that

$$\dot{V}_s \leq \underbrace{-\frac{2}{\lambda_h}\left(\boldsymbol{s}^T\boldsymbol{K}_v\boldsymbol{\tau}_s-\alpha_{max}(|s_1|+\cdots+|s_n|)\right)}_{\leq 0} -\frac{2\lambda_{min}(\boldsymbol{K}_p)}{\lambda_h}\|\boldsymbol{s}\|^3 \leq 0 \tag{32}$$

as long as (12) is satisfied. This concludes the first part of the proof because $\boldsymbol{s}\in\mathcal{L}_\infty \Rightarrow \boldsymbol{e},\dot{\boldsymbol{e}}\in\mathcal{L}_\infty$. Note that the result can be considered semi-global since $\boldsymbol{K}_v$ can be made arbitrarily large.

b) To show that $\boldsymbol{s}$ becomes zero in a finite time, assume first that $V_s(t_0) > 1$, which from (16) implies by raising to the power $\frac{3}{2}$ that

$$1 < V_s^{\frac{3}{2}} \leq \frac{\lambda_H^{\frac{3}{2}}}{\lambda_h^{\frac{3}{2}}}\|\boldsymbol{s}\|^3 \quad\Rightarrow\quad -\|\boldsymbol{s}\|^3 \leq -\frac{\lambda_h^{\frac{3}{2}}}{\lambda_H^{\frac{3}{2}}}V_s^{\frac{3}{2}}\,, \tag{33}$$

so that (32) satisfies

$$\dot{V}_s \leq -2\lambda_{min}(\boldsymbol{K}_p)\frac{\lambda_h^{\frac{1}{2}}}{\lambda_H^{\frac{3}{2}}}V_s^{\frac{3}{2}} < 0\,. \tag{34}$$

According to the proof of Lemma 1 in [12], $V_s(t_0) > 1$ becomes $V_s(t) \leq 1$ from a fixed time

$$t \geq t_0 + \frac{1}{2\lambda_{min}(\boldsymbol{K}_p)\frac{\lambda_h^{\frac{1}{2}}}{\lambda_H^{\frac{3}{2}}}\left(\frac{3}{2}-1\right)} = t_0 + T_{s1}\,. \tag{35}$$

Once $V_s(t) \leq 1$ the following can be deduced from (16)

$$s_i^2 \leq \|\boldsymbol{s}\|^2 \leq V_s(t) \leq 1 \quad\Rightarrow\quad |s_i| \leq 1 \tag{36}$$

for $i = 1,\ldots,n$, so that from (31) one can get

$$\begin{aligned}-\frac{2}{\lambda_h}\sum_{i=1}^{n}\left(k_{vi}e^{-\frac{\lambda_{si}}{2e}}-\alpha_{max}\right)|s_i| &\leq \\ -\frac{2}{\lambda_h}\left(k_ve^{-\frac{\lambda_{max}(\boldsymbol{\lambda}_s)}{2e}}-\alpha_{max}\right)\sum_{i=1}^{n}|s_i| &\leq 0\,.\end{aligned} \tag{37}$$

Since $\sum_{i=1}^{n}|s_i| = |s_1|+\cdots+|s_n| \geq \|\boldsymbol{s}\|$, then one has from (32) and (37)

$$\dot{V}_s \leq -\frac{2}{\lambda_h}\left(k_ve^{-\frac{\lambda_{max}(\boldsymbol{\lambda}_s)}{2e}}-\alpha_{max}\right)\|\boldsymbol{s}\| \leq 0\,. \tag{38}$$

Now, from (16) one has

$$V_s^{\frac{1}{2}} \leq \sqrt{\frac{\lambda_H}{\lambda_h}}\|\boldsymbol{s}\| \quad\Rightarrow\quad -\|\boldsymbol{s}\| \leq -\sqrt{\frac{\lambda_h}{\lambda_H}}V_s^{\frac{1}{2}}\,, \tag{39}$$

which means that (38) satisfies

$$\begin{aligned}\dot{V}_s &\leq -2\frac{1}{\sqrt{\lambda_h\lambda_H}}\left(k_ve^{-\frac{\lambda_{max}(\boldsymbol{\lambda}_s)}{2e}}-\alpha_{max}\right)V_s^{\frac{1}{2}} \\ &\leq 0\,.\end{aligned} \tag{40}$$

The proof of Lemma 1 in [12] can be used again to show that once $V_s(t_0+T_{s1}) \leq 1$ then $V_s(t) \equiv 0$ in a time T_{s2} no larger than

$$T_{s2} = \frac{\sqrt{\lambda_h\lambda_H}}{\left(k_ve^{-\frac{\lambda_{max}(\boldsymbol{\lambda}_s)}{2e}}-\alpha_{max}\right)}\,. \tag{41}$$

T_{max} in (13) is gotten by considering (35) and (41).

c) Since it has been proven that $\boldsymbol{s}$ is bounded and tends to zero, then from (5) it can trivially be shown that $\boldsymbol{e},\dot{\boldsymbol{e}} \to \boldsymbol{0}$. This concludes the proof. △

IV. Simulation Results

To test the control law (6) proposed in the previous section, the simplified version of the robot model *CRS-*A465 described in [13], [14] is employed. This manipulator has three degrees of freedom and the interested reader can see the references for details. The gains for the controller given by equations (3)-(4) and (6)-(7) have been chosen as $\boldsymbol{\Lambda} = 10\boldsymbol{I}$, $\boldsymbol{K}_{\rm v} = \text{diag}\{700 \quad 200 \quad 200\}$, $\boldsymbol{K}_{\rm p} = \boldsymbol{I}$, $\lambda_{\rm s1} = \lambda_{\rm s2} = \lambda_{\rm s3} = 2$ and $\mu_{\rm s1} = \mu_{\rm s2} = \mu_{\rm s3} = 0.1$. To have a point of comparison, the algorithm introduced in [3] has also been implemented since it owns the same properties as the current proposal. The interested reader is referred to this work for details, while the parameters employed for simulation are the following[1]: $\delta = 0.3$, $\gamma_1 = 0.0873$, $\gamma_2 = 4.6129$, $\alpha = 0.7$, $r = 1.7$, $\beta = 1.9$, $\boldsymbol{C}_1 = 3\boldsymbol{I}$, $\boldsymbol{C}_2 = 3\boldsymbol{I}$, $\boldsymbol{K}_1 = 5\boldsymbol{I}$, $\boldsymbol{K}_2 = 5\boldsymbol{I}$, $\nu_1 = 2.5$, $\nu_2 = 0.5$, $a_0 = 12$, $a_1 = 2.2$, $\boldsymbol{H}_0 = \dfrac{2}{\gamma_1 + \gamma_2}\boldsymbol{I}$, $\boldsymbol{C}_0(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, $\boldsymbol{D}_0 = \boldsymbol{D}$, $\boldsymbol{g}_0(\boldsymbol{q}) = \boldsymbol{g}(\boldsymbol{q})$.

One simulation has been carried out, where the desired trajectories have been chosen to be the sum of six different sinusoidal signals for each joint shown in Figure 1, while the external perturbations shown in Figure 3. From Figure 1 it can be appreciated that both schemes have a similar performance. However, to have a better insight, Figure 2 shows the tracking errors. Here it can be recognized that the proposed scheme can deal better with the external perturbation despite it does not employ any knowledge of the robot model, on the contrary to the scheme in [3]. This better performance is confirmed in Table I, where the Position tracking RMSE's of each joint are shown. Finally, Figure 4 shows the controller outputs where it can be appreciated that the proposed scheme delivers lower values, which represents another advantage. Furthermore, no chattering arises in neither case. Note that only a representative window is shown to be able to appreciate the behavior of the input torques.

TABLE I
Position tracking RMSE.

$\boldsymbol{e}[°]$	Control (6)	Control in [3]
e_1	0.7582	1.0513
e_2	0.6337	0.9398
e_3	0.7118	0.9436

V. Conclusions

Some of the main tasks for robot manipulators can be described in a general framework as position and velocity trajectories tracking. In particular, Sliding Mode Control (SMC) techniques allow to achieve (theoretically) exact position tracking despite a poor knowledge of the robot model and in the presence of external perturbations. SMC techniques can yield to zero the tracking errors in finite (but not necessarily tunable) time. In the last years fixed time

[1]The gains were chosen as given in the reference.

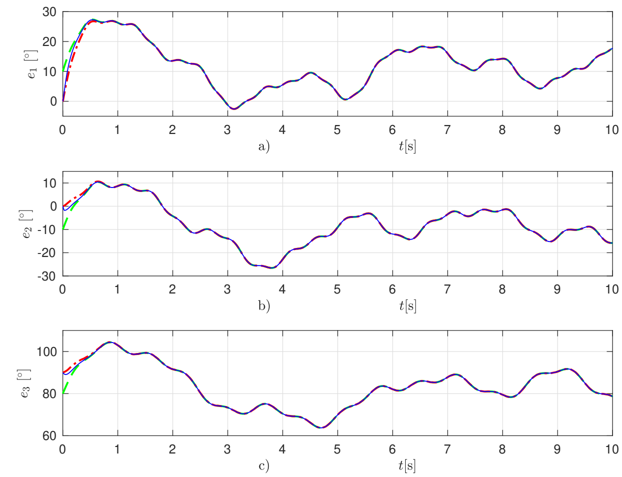

Fig. 1. Joints positions tracking. Proposed algorithm (——), algorithm in [3] (— · — · —) and desired trajectories (· — — —).

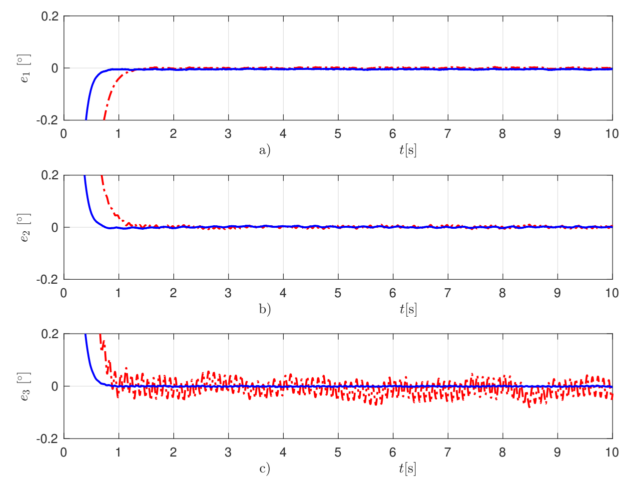

Fig. 2. Tracking errors. Proposed algorithm (——) and algorithm in [3] (— · — · —).

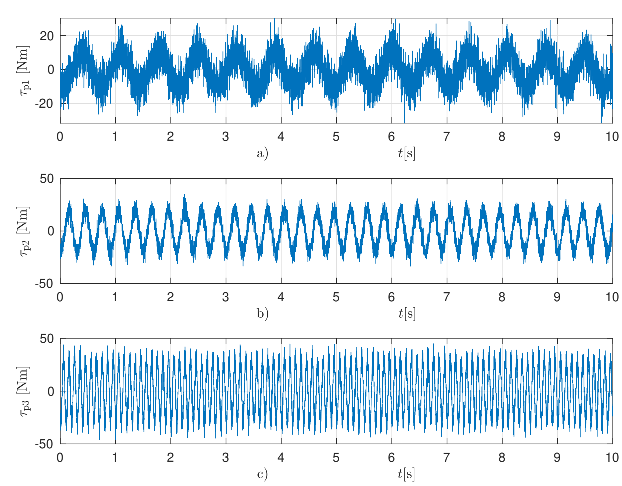

Fig. 3. Perturbation vector $\boldsymbol{\tau}_{\rm p}$ added to the second simulation.

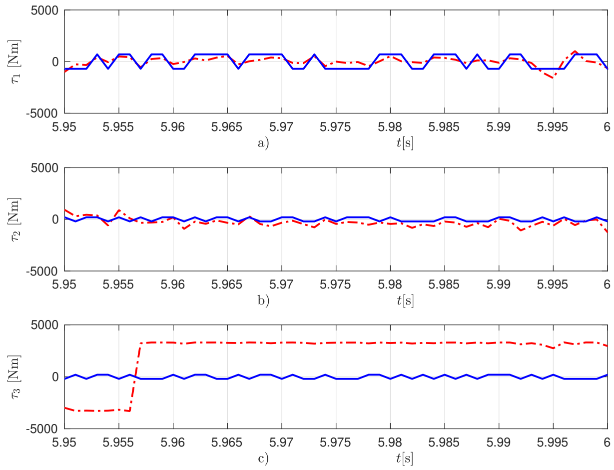

Fig. 4. Input torques $\boldsymbol{\tau}$.Proposed algorithm (——) and algorithm in [3] (— · — · —).

stability has become an area of great interest, where one of the main problems to be solved is the possible appearance of singularities. The current contribution introduces a singularity free fixed time SMC scheme for position and velocity tracking of robot manipulators in the presence of external perturbations. The control scheme avoids singularities by using linear sliding mode (LSM) surfaces, which are yielded to zero in a predefined fixed time. Although the employment of LSM allows to avoid singularities, the tracking errors tend to zero only asymptotically once the sliding surface is reached and it remains as future work to design a controller which guarantees that tracking errors do become zero in a finite time as well. Simulation results show that this does not represent a big drawback since tracking errors tend to zero even in the presence of external perturbations as foreseen in theory. A comparison with a well-known scheme is carried out to show some of the advantages of the new approach.

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