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Integrated evacuation planning and spatial field estimation using level-set based guidance with trajectory reconfiguration

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Abstract— **This work proposes mid-course trajectory reconfigurations for humans escaping hazardous environments in indoor surroundings as part of disaster management in civil infrastructures. Hazardous environments are interpreted as spatial fields such as carbon monoxide concentrations, and have accumulated effects on escaping humans within indoor environments. When the spatial field is known and available to an evacuee then a level-set based guidance can provide an optimal trajectory to an escape exit that corresponds to the smallest accumulated amount of hazardous material in the evacuee's lungs. However, when the spatial field is unknown to an evacuee, an integrated estimation and trajectory planning scheme is warranted. This paper combines the asymptotic embedding approach for state estimation of spatially distributed processes via mobile sensor with a modified level-set trajectory generation scheme. Incorporating realism due to computing and planning time, a trajectory cycle is decomposed into a planning stage in which a mobile agent (human) is immobile and uses the most recent state estimate to generate viable escape trajectories, and the travel stage in which the mobile agent is executing the trajectory computed during the planning stage. As new process state information is updated, the escape trajectories are recalculated thereby leading to continuous escape trajectory reconfiguration. In both stages of a given cycle the mobile agent is continuously estimating the spatially distributed process but only using the most recent snapshot of the spatial process estimate for trajectory recalculation. Extensive numerical studies are included to shed light on the detrimental effects of accumulated amounts of hazardous environments on the escape trajectories of humans during indoor evacuation.**

I. Introduction

The problem of trajectory generation in indoor environments is significantly more difficult when one has to take into account the accumulated effects of a hazardous environment, such as carbon monoxide, on an evacuee's ability to move and complete a path to safety. Safety is interpreted as reaching an emergency exit and hence access to breathable air free of carbon monoxide.

Following the earlier work [1], [2] in which the accumulated and instantaneous effects of a hazardous field inhaled by an evacuee significantly influenced the escape trajectory from the interior of an indoor environment such as music/concert halls, stations, and terminals, this work adds another level of complexity in the generation of viable escape trajectories. The earlier works presented a level-set escape trajectory as a means to reach the optimal escape exit that also ensured the levels of the accumulated amount of carbon monoxide present in the human lungs were as far below as possible from a critical threshold. Exceeding such a threshold in the lungs means the human is incapacitated and unable to complete the trajectory to safety. Such a level-set based human guidance required the full knowledge of the species concentration (carbon monoxide concentration) throughout the indoor environment. This meant that the human must have complete knowledge of the concentration for each spatial point in the indoor environment. The evacuation guidance proposed in the earlier work [1], [2] was also assumed to be executed over a single cycle meaning that an evacuee had to find the optimal trajectory for each of the available escape exits at the beginning of the escape flight and select the escape trajectory that yielded the smallest accumulated amount of the hazardous substance inhaled. It should be noted that the level-set guidance used in [1], [2] was an extension of earlier works on level-sets for human navigation [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], having the added element of minimizing the accumulated amount present in the lungs over a selected trajectory.

In this work, the assumption of the knowledge of the spatial field representing the concentration of the hazardous substance is removed with the evacuee having to estimate the concentration in real-time in order to continuously re-evaluate the escape trajectory. This means that as an updated estimate of the spatial field is accessed, the escape trajectory is recalculated on-the-fly. This knowledge of the environment, via a state estimate, is available to the agent at discrete time instances (snapshots). The unknown spatial field is assumed to be described by a Poisson-type PDE (elliptic PDE) that is time invariant [15]; that is, constant-in-time, but varying-in-space. To provide an estimate of the state using the measured concentration at the current location of an evacuee (i.e., mobile sensor), we consider asymptotic embedding methods to set-up a state observer with mobile sensors [16]. It should be emphasized that while the true environment is spatially varying, its on-line estimate is spatiotemporally varying. The state estimate is made available to the escaping agent at discrete time instances.

The mathematical formulation of the various components of a hazardous environment in an indoor setting is presented in Section II. A level-set guidance modified for on-line estimated snapshots of a spatially varying field is presented in Section III. Extensive numerical results are discussed in Section IV with conclusions following in Section V.

II. Mathematical Formulation

The various modeling aspects of a hazardous environment in which an evacuee is traveling through are detailed.

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A. Hazardous field over rectangular indoor domain

The spatial field that is assumed to have accumulated effects on an evacuating agent, is modelled by a Poisson-type PDE over a rectangular domain $\Omega = [0, L_\xi] \times [0, L_\zeta]$. This of course represents the steady-state equation of an unsteady advection-diffusion PDE. The solution to the elliptic PDE is denoted by $c(\xi,\zeta)$ and is the solution to

$$0 = \nabla \cdot (D\nabla c) - \nabla \cdot (uc) + f, \tag{1}$$

where $f = f(\xi,\zeta)$ denotes the (negative of the) source terms, and is furnished with the appropriate boundary conditions. For part of the boundary $\partial\Omega$, Dirichlet conditions are imposed and for the remainder, Neumann conditions are used with $\Gamma_D \cup \Gamma_N = \partial\Omega$.

The spatial field with concentration $c(\xi,\zeta)$ is not available to the evacuee for each spatial coordinate $(\xi,\zeta) \in \Omega$. Instead, it is assumed that the amount of the hazardous field inhaled by the evacuee at a given coordinate position $(x_1(t), x_2(t))$ within the domain Ω is given as a function of the "measured" quantity. This is given by

$$y(t) = \int_\Omega \delta(\xi - x_1(t))\delta(\zeta - x_2(t))\, c(\xi,\zeta)\, \mathrm{d}\zeta \mathrm{d}\xi,$$

where $\delta(\cdot)$ denotes the Dirac delta function. The simplified expression for the "measured" field, as inhaled by the evacuee is given by

$$y(t) = c(x_1(t), x_2(t)), \tag{2}$$

and which assigns to the measured output the value of the unknown concentration evaluated at the current evacuee position $(x_1(t), x_2(t))$. It is noted that while the concentration $c(\xi,\zeta)$ is time-invariant, the measured quantity $y(t)$ is time varying because of the motion of the evacuee.

B. Evacuee equations of motion

A simple kinematic model is assumed to model the motion of the evacuee in the spatial domain Ω. This simplified version was proposed in the earlier work [1], [2], and is described by the kinematic equations of a mobile robot

$$\begin{aligned} \dot{x}_1(t) &= \upsilon(t)\cos(\theta), \\ \dot{x}_2(t) &= \upsilon(t)\sin(\theta), \end{aligned} \quad (x_1(0), x_2(0)) = (x_{10}, x_{20}), \tag{3}$$

where $(x_1(t), x_2(t)) \in \Omega$ are the evacuee's coordinates in Ω, $\upsilon(t)$ is the evacuee (linear) speed and θ is the angle between the direction of motion and the horizontal axis ξ. While elaborate models include both the above kinematics and the dynamics (angular velocity as the control signal plus Newton's translational motion and Euler's rotational motion, [17]), we have opted for the simplified equations of motion due to the low values of human speeds. In the above model, the angle $\theta(t)$ is taken to be the control variable.

C. Inhalation model

The accumulated amount of the hazardous substance up to the current time t in the evacuee's lungs is obtained from the line integral of the concentration $c(\xi,\zeta)$ along the path towards an escape exit. Using the derivation in [1], [2], we summarize the trajectory-dependent amount due to the inhale-exhale cycle via the cost

$$J(\theta) = \frac{1}{2}\int_{\theta(t)} c(\mathbf{r})\, \mathrm{d}s,$$

where $\mathbf{r}(t) = (x_1(t), x_2(t))$. The factor $1/2$ in front of the integral represents the ratio between the time of inhalation and the total time for a breath cycle, see [1].

Using the fact that the concentration at each spatial coordinate is denoted by $c(x_1(t), x_2(t))$, then use of the line integral equation [18] yields $\mathrm{d}s = \upsilon(t)\mathrm{d}t$ which then produces

$$J(0,t,\theta(t)) = \frac{1}{2}\int_0^t \upsilon(\tau)c(x_1(\tau), x_2(\tau))\, \mathrm{d}\tau. \tag{4}$$

When the speed υ is constant and the evacuee moves along a level curve (isoline) of the concentration $L_m(c) = \{(\xi,\zeta) : c(\xi,\zeta) = m\}$, then the accumulated amount (4) simplifies to $J(0,t,\theta(t)) = \upsilon m t/2$.

The accumulated amount can be explicitly expressed in terms of the measured signal $y(t)$ for a constant speed via

$$J(0,t,\theta(t)) = \frac{\upsilon}{2}\int_0^t y(\tau)\, \mathrm{d}\tau.$$

D. Observer based estimation of time-invariant fields

Since the concentration field is unknown, then it must be estimated using the available measured signal $y(t)$ given by (2). One way to find the time-varying estimate of $c(\xi,\zeta)$ is to use asymptotic embedding methods presented in [19], [16]. The idea behind this is to express (1) as the steady-state equation of the evolution equation corresponding to the unsteady advection-diffusion PDE. Using $x = c(\cdot,\cdot)$ as the state, we have that (1) is abstractly given as

$$0 = \mathcal{A}x + f, \tag{5}$$

where $\mathcal{A}$ is the elliptic operator $\mathcal{A}\varphi = \nabla \cdot (D\nabla\varphi) - \nabla \cdot (u\varphi)$, for $\varphi \in H_0^1(\Omega)$. The measurement (2) can be written in terms of the output operator

$$y(t) = \mathcal{C}(t)x, \tag{6}$$

where, for $\varphi \in H_0^1(\Omega)$, $\mathcal{C}(\cdot) \in H^{-1}(\Omega)$ is given by

$$\mathcal{C}(t)\varphi = \int_\Omega \delta(\xi - x_1(t))\delta(\zeta - x_2(t))\, \varphi(\xi,\zeta)\, \mathrm{d}\zeta \mathrm{d}\xi.$$

Using (5), (6), the associated filter is given by

$$\dot{\widehat{x}}(t) = \mathcal{A}\widehat{x}(t) - \mathcal{L}(t)\big(y(t) - \mathcal{C}\widehat{x}(t)\big), \tag{7}$$

where $\widehat{x}(t)$ denotes the state estimate of x, in other words, it is the estimate of $c(\xi,\zeta)$. In terms of the PDE setting, the estimator (7) is

$$\begin{aligned} \frac{\partial\widehat{c}(t,\xi,\zeta)}{\partial t} &= \nabla \cdot (D\nabla\widehat{c}(t,\xi,\zeta)) - \nabla \cdot (u\widehat{c}(t,\xi,\zeta)) \\ &\quad - \ell(t,\xi,\zeta)\big(c(x_1(t),x_2(t)) - \widehat{c}(t,x_1(t),x_2(t))\big), \end{aligned}$$

where $\ell(t,\xi,\zeta)$ is the kernel of the adjoint of the filter operator $\mathcal{L}(t)$ in (7). As presented in [16], when the filter operator is decoupled to the sensor guidance and is selected as the adjoint of the output operator, one has that (7) simplifies to a Luenberger observer

$$\dot{\widehat{x}}(t) = \mathcal{A}\widehat{x}(t) - \gamma\mathcal{C}^*(t)\left(y(t) - \mathcal{C}\widehat{x}(t)\right), \tag{8}$$

where $\gamma > 0$ is a user-defined Luenberger observer gain. In terms of the PDE setting, the Luenberger observer is

$$\frac{\partial \widehat{c}(t,\xi,\zeta)}{\partial t} = \nabla \cdot (D\nabla \widehat{c}(t,\xi,\zeta)) - \nabla \cdot (u\,\widehat{c}(t,\xi,\zeta)) - \gamma\delta(\xi - x_1(t))\delta(\zeta - x_2(t)) \times \big(c(x_1(t),x_2(t)) - \widehat{c}(t,x_1(t),x_2(t))\big). \tag{9}$$

Summarizing, the state estimator for the hazardous field concentration $c(\xi,\zeta)$ using the mobile measurement (2) is given by (8). The guidance of the mobile sensor (i.e., the evacuee) is given by the appropriate selection of the control angle $\theta(t)$ in the equation of motion (3) that minimizes (4).

E. The planning and travelling stages in a guidance cycle

The assumption here is that the evacuee's guidance is completed over multiple cycles which contain a *planning stage* and a *travel stage*. During the planning stage the evacuee is stationary (does not move) and simply uses this time interval to compute a viable escape trajectory. The evacuee in this stage uses the estimate of the concentration field evaluated at the beginning of the planning stage (the snapshot of the estimated field) $\widehat{c}(t_k,\xi,\zeta)$, $\forall(\xi,\zeta) \in \Omega$ to generate the escape trajectory that predicts the smallest accumulated amount of the hazardous substance in the lungs. The state observer (8) provides the estimate $\widehat{c}(t,\xi,\zeta)$ for all time *but* the agent is using the snapshot $\widehat{c}(t_k,\xi,\zeta)$ to find the optimal escape trajectory. The snapshots of the estimated concentration field are available only at the discrete time instances t_k, $k = 1,2,\ldots$. These instances t_k signal the beginning of the guidance cycles.

During the travel stage of a given cycle, the agent executes the escape guidance that was computed during the planning stage. At the completion of a given travel stage, the agent halts and a new cycle is initiated. The duration of a single guidance cycle is denoted by τ_{cycle} with τ_{plan} and τ_{travel} denoting the planning and traveling stages, respectively. Thus, we have

$$\tau_{cycle} = \tau_{plan} + \tau_{travel}. \tag{10}$$

Since the time instance t_{esc} wherein the agent reaches any of the escape exits is unknown, one cannot a priori define the interval $[0,t_{esc}]$ or its decompositions that yield the duration of a given cycle. Instead, one defines the duration τ_{cycle} of a cycle and the duration of either the planning stage or the travel stage. When the length of the subintervals is a priori defined, then the time instances t_k corresponding to the beginnings of a new guidance cycle are easily defined as

$$t_k = (k-1)\tau_{cycle}, \qquad k = 1,2,\ldots,N. \tag{11}$$

One observes that the planning stage occurs for $t \in [t_k, t_k + \tau_{plan})$ and the learning stage in $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$.

In summary, one has the stages of a given guidance cycle

i) Planning stage: For each $t \in [t_k, t_k + \tau_{plan})$, the evacuee is stationary but is searching for the optimal escape trajectory for each escape exit. When the optimal trajectories corresponding to the smallest predicted accumulated amount inhaled are calculated, it selects the trajectory for the travel stage. In this case, it generates the trajectory $\theta(t;t_k,\infty)$ for $[t_k + \tau_{plan}, t_k + \tau_{cycle})$.

ii) Travel stage: For each $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$, the evacuee implements the trajectory $\theta(t;t_k,\infty)$ obtained during the planning stage, *but* terminates it at $t = t_k + \tau_{cycle}$, in other words it executes $\theta(t;t_k,t_k + \tau_{cycle})$.

III. Level-set guidance using snapshots $\widehat{c}(t_k,\xi,\zeta)$ of filter estimates of a steady field $c(\xi,\zeta)$

The spatial field $c(\xi,\zeta)$ is unknown and a state observer is employed to provide an estimate $\widehat{c}(t,\xi,\zeta)$. The model of the state observer is given in (8). While the estimate is generated continuously, only a snapshot is available to the agent during a given planning stage in order to compute the escape trajectory needed for the corresponding travel stage.

The level-set guidance presented in [1], [2] is modified in order to account for the use of an estimated spatial field and also the intermittent availability of such an estimate. The modifications to the level-set guidance are as follows:

i) Throughout the planning stage $t \in [t_k, t_k + \tau_{plan})$, the agent is not moving and is using the spatially varying function $\widehat{c}(t_k,\xi,\zeta)$ to compute the escape trajectory that yields the smallest amount of accumulated substance. While the guidance is only using $\widehat{c}(t_k,\xi,\zeta)$, the state observer (8) continues to generate the state estimate using the arrested measurements

$$\frac{\partial \widehat{c}(t,\xi,\zeta)}{\partial t} = \nabla \cdot (D\nabla \widehat{c}(t,\xi,\zeta)) - \nabla \cdot (u\widehat{c}(t,\xi,\zeta)) - \gamma\delta(\xi - x_1(t_k))\delta(\zeta - x_2(t_k)) \times \big(c(x_1(t_k),x_2(t_k)) - \widehat{c}(t,x_1(t_k),x_2(t_k))\big), \tag{12}$$

for all $t \in [t_k, t_k + \tau_{plan})$. The state observer in (12) uses an arrested learning that was first presented [20], since it uses the frozen-in-time concentration measurement $c(x_1(t_k),x_2(t_k))$ which is the sensor measurement at the fixed location $(x_1(t_k),x_2(t_k))$. Since the actual process is time invariant and the sensor is immobile, then this measurement is constant for $t \in [t_k, t_k + \tau_{plan})$. However, the estimated concentration at the sensor location is $\widehat{c}(t,x_1(t_k),x_2(t_k))$ which is time-varying for all $t \in [t_k, t_k + \tau_{plan})$. The observer kernel $\delta(\xi - x_1(t_k))\delta(\zeta - x_2(t_k))$ is constant in time since the delta functions are evaluated at a fixed position within the spatial domain Ω. Thus, the estimated state in (12) will continue to vary with time for all $t \in [t_k, t_k + \tau_{plan})$.

The associated optimal trajectory $\theta^{opt}(t;t_k,\infty)$ corresponds to the optimization of the accumulated cost J for the interval $t \in [t_k + \tau_{plan}, \infty)$, which means that the agent is attempting to find the optimal trajectory $\theta^{opt}(t;t_k,\infty)$ that corresponds to an escape exit with the smallest accumulated J. Additionally, the level-set guidance proposed in [1], [2] must be modified since the maximum value of the true field needed for the derivation of the level-set guidance is not available. In its place, one uses the estimated value $\widehat{c}(t_k,\xi,\zeta)$ to generate the level-set guidance.

ii) For all times $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$ in the travel stage, the evacuee executes the guidance associated with the optimal trajectory $\theta^{opt}(t; t_k, \infty)$ but terminates it at the time instance $t = t_k + \tau_{cycle}$ which designates the end of the current guidance cycle. At the next cycle the next updated information is $\widehat{c}(t_{k+1}, \xi, \zeta)$. During the current travel stage the agent is using the following state observer

$$
\begin{aligned}
&\frac{\partial \widehat{c}(t,\xi,\zeta)}{\partial t} = \nabla \cdot (D \nabla \widehat{c}(t,\xi,\zeta)) - \nabla \cdot (u \widehat{c}(t,\xi,\zeta)) \\
&-\gamma \delta(\xi - x_1(t)) \delta(\zeta - x_2(t)) \times \\
&\big(c(x_1(t), x_2(t)) - \widehat{c}(t, x_1(t), x_2(t))\big),
\end{aligned} \tag{13}
$$

for all $t \in [t_k + t_{plan}, t_k + \tau_{cycle})$.

The details of the level-set guidance using the snapshots of the estimated field are presented in Algorithm 1.

IV. Numerical Studies

We consider a rectangular domain in 2D, given by $\Omega = [0, L_\xi] \times [0, L_\zeta] = [0, 100] \times [0, 30]$m having three escape exits at its boundary with coordinates $(\xi_1^d, \zeta_1^d) = (100, 10)$m, $(\xi_2^d, \zeta_2^d) = (100, 20)$m and $(\xi_3^d, \zeta_3^d) = (90, 30)$m.

The selected cycle duration is $\tau_{cycle} = 6$s with $\tau_{plan} = 2$s, meaning that the escapee agent does not move for 2 seconds while planning and moves at the remaining 4 seconds till completion of the current cycle. While the state estimator is running continuously even when the agent is not moving, the trajectory planning scheme receives the state estimate every 6 seconds in the form of $\widehat{c}(t_k, \xi, \zeta)$.

The true spatial field is governed by the elliptic PDE

$$0 = \nabla^2 c(\xi, \zeta) + f(\xi, \zeta)$$

having Dirichlet boundary conditions with $\nabla^2 = \Delta$ denoting the Laplacian operator in 2D

$$\nabla^2 c(\xi, \zeta) = \frac{\partial^2 c}{\partial \xi^2} + \frac{\partial^2 c}{\partial \zeta^2},$$

and $f(\xi, \zeta)$ denotes the negative of the unknown source. The solution to the above PDE is

$$c(\xi, \zeta) = 4100 \exp\Big(\frac{-(\xi - \mu_\xi)^2}{2\sigma_\xi^2}\Big) \exp\Big(\frac{-(\zeta - \mu_\zeta)^2}{2\sigma_\zeta^2}\Big)$$

where $\mu_\xi = 0.7 L_\xi$, $\sigma_\xi = 0.1 L_\xi$, $\mu_\zeta = 0.75 L_\zeta$ and $\sigma_\zeta = \sqrt{L_\zeta}$. The associated state operator $\mathcal{A}$ in (7) is

$$\langle \mathcal{A}\varphi, \psi \rangle = \int_\Omega \Big(\frac{\partial^2 \varphi(\xi, \zeta)}{\partial \xi^2} + \frac{\partial^2 \varphi(\xi, \zeta)}{\partial \zeta^2}\Big) \psi(\xi, \zeta)\, d\zeta d\xi,$$

for all $\varphi, \psi \in H_0^1(\Omega)$ and the source term given by the negative of the Laplacian of the selected solution. The measurement model is given by

$$
\begin{aligned}
y(t) &= \int_0^{L_\xi} \int_0^{L_\zeta} \delta(\xi - x_1(t)) \delta(\zeta - x_2(t)) c(\xi, \zeta)\, d\zeta\, d\xi \\
&= c(x_1(t), x_2(t)).
\end{aligned}
$$

The agent starts at the position $x_1(0) = L_\xi/3$, $x_2(0) = L_\zeta/3$, and travels inside Ω with a constant speed of $\upsilon = 7 m/s$.

The agent assumes the unknown spatial field is given by the initial guess

$$\widehat{c}(0, \xi, \zeta) = 4100 \exp\Big(\frac{-(\xi - \mu_\xi)^2}{2\sigma_\xi^2}\Big) \exp\Big(\frac{-(\zeta - \mu_\zeta)^2}{2\sigma_\zeta^2}\Big)$$

with $\mu_\xi = L_\xi/2$, $\sigma_\xi = L_\xi/5$, $\mu_\zeta = L_\zeta/3$ and $\sigma_\zeta = L_\zeta/4$.

When the agent has full access to the spatial field and implements the level-set guidance presented in [1], [2], the

Algorithm 1 Arrested estimation-based evacuation guidance over $[t_k, t_k + \tau_{cycle})$ using discrete time estimated field information $\widehat{c}(t_k, \xi, \zeta)$ with on-the-fly trajectory recalculation

1: **initialize:** Using sampling constraints, define the instances t_k in (11) and define the cycle duration $\tau_{cycle} = t_{k+1} - t_k$. Using individual agent capacity, select the planning stage τ_{plan} and travel stage τ_{travel} durations in $\tau_{cycle} = \tau_{plan} + \tau_{travel}$. Use an initial state estimate $\widehat{x}(0)$ for the observer in (12). Using initial $(x_1(t_0), x_2(t_0))$ and desired locations (escape exits) (ξ_j^d, ζ_j^d), $j = 1, \ldots, n_{exits}$, determine the first trajectory planning $\theta(t, t_1)$ for $t \in [\tau_{plan}, \infty)$, but implement in $t \in [\tau_{plan}, \tau_{plan} + \tau_{travel})$. Obtain sensor measurements in both $t \in [t_0, t_0 + \tau_{plan})$ and $t \in [t_0 + \tau_{plan}, t_0 + \tau_{plan} + \tau_{travel})$ and use them to implement the observer (12), (13).
2: **iterate:** $k = 2$
3: **loop**
4: Define next cycle $[t_k, t_k + \tau_{cycle})$ with $t_k = (k-1)\tau_{cycle}$. For each time $t \in [t_k, t_k + \tau_{cycle})$ continue to obtain sensor measurements and implement the arrested estimation (12) regardless of the agent motion.
5: In the k^{th} *planning stage* of duration τ_{plan} with $t \in [t_k, t_k + \tau_{plan})$, use the most recent arrested estimate of the field $\widehat{c}(t_k, \xi, \zeta)$ to plan the trajectory for $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$ using the level-set based guidance in [1], [2]. Continue the nominal estimation (13).
6: **for** $j = 1$ to n_{exits} **do**
7: from current position $(x_1(t_k), x_2(t_k))$, compute the level-set trajectory to each j^{th} exit in $[t_k + \tau_{plan}, \infty)$
8: compute anticipated accumulated amount $J_j(t_k, \infty, \theta(t, t_k))$ for each exit and truncate to $J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k))$
9: select optimal trajectory $\theta_k^{opt}(t)$ using

$$\theta_k^{opt}(t) = \arg\min_j J_j(t_k, t_k + \tau_{cycle}, \theta(t, t_k)) \tag{14}$$

10: **end for**
11: In the k^{th} *travel stage* of duration τ_{travel} with $t \in [t_k + \tau_{plan}, t_k + \tau_{cycle})$, implement the level-set based path planning developed at the most recent planning stage.
12: At the end of the k^{th} *cycle* $t_{k+1} = t_k + \tau_{cycle}$, propagate the state estimate of the spatial field using (13).
13: **if** $\sqrt{(x_1(t_{k+1}) - \xi_j^d)^2 + (x_2(t_{k+1}) - \zeta_j^d)^2} > 0$ **then**
14: $k \leftarrow k + 1$
15: **goto** 2
16: **else**
17: terminate-success: reached a safety exit!
18: **end if**
19: **end loop**

trajectory selected is for exit #1 and provides a minimum accumulated exposure to the hazardous field. Figure 1 depicts the optimal trajectory and the contours of the hazardous field. As expected, the agent essentially moves in a slightly curved path to minimize the accumulated exposure and marches towards the nearest exit that would predict the smallest accumulated amount. Exits #2 and #3 would have a path passing through the higher concentrations of the spatial field and thus are deemed unsuitable trajectories.

However, when the spatial field is not known to the agent, the selected path may go through the highest values of the hazardous field. The escapee trajectory is presented in Figure 2 with the intermediate escape exit selections in each cycle. In all three cycles, exit #3 was selected as the viable exit with the smallest accumulated amount predicted. While exit #3 was always selected, the path generated was not given by a straight line from $(x_1(0), x_2(0))$ to (ξ_3^d, ζ_3^d). This of course ensured that the accumulated amount at the instance of reaching exit #3 was the smallest possible. Table I summarizes the results for three different cases: the first one uses the proposed guidance that uses the on-line estimate of the spatial field to implement the level-set guidance with trajectory reconfiguration. The second case is that of a field-agnostic agent who is not aware of the presence of the hazardous field and selects a path to escape using line-of-sight to escape to the exit with the shortest distance. The third case is that of an agent that is aware of the presence and effects of the hazardous field and essentially implements the level-set guidance proposed in [1], [2]. As expected, the field-agnostic agent traverses the distance to exit #3 in the shortest time possible of 8.6 seconds. The agent using the proposed level-set guidance with trajectory reconfiguration based on the estimate of the spatial field takes 20.2 seconds to reach safety. The agent that uses the knowledge of the spatial field without any trajectory reconfiguration requires 14.8 seconds to reach safety. However, when the accumulated amount of the harmful substance inhaled are taken into account, the field-agnostic agent has $J = 45,260$ppm while the field-aware agent has a lower level at $J = 9,533$ppm. When the agent has full knowledge of the spatial field and does not implement a trajectory reconfiguration, it reaches safety with a lower value at $J = 254$ppm, see also Figure 1. As expected, when the agent has knowledge of the field and completes the trajectory without any reconfiguration, it has a better performance, both in terms of time-to-escape and accumulated amount, than the agent that has to estimate the spatial field in order to generate the trajectories with reconfiguration. The case of a field-agnostic agent demonstrates the detrimental effects on an evacuee's health when the effects of the accumulated amount are not taken into account in the trajectory planning.

The viable trajectories of the field-agnostic agent based on simple time-to-escape (line of sight) paths are depicted in Figure 3. Purely using time (or distance with constant speed) as an escape selection is not the correct criterion when hazardous substances in the indoor domain are present.

The effects of the field on trajectory selection are seen in Figure 4. While the true and unknown field is shown in

case	time to exit (sec)	J at exit
field-aware	20.19	9,533
field-agnostic	8.58	45,260
nominal case	10.50	254

TABLE I: Escape times and accumulated J for the trajectory-reconfiguration guidance and the field-agnostic agent.

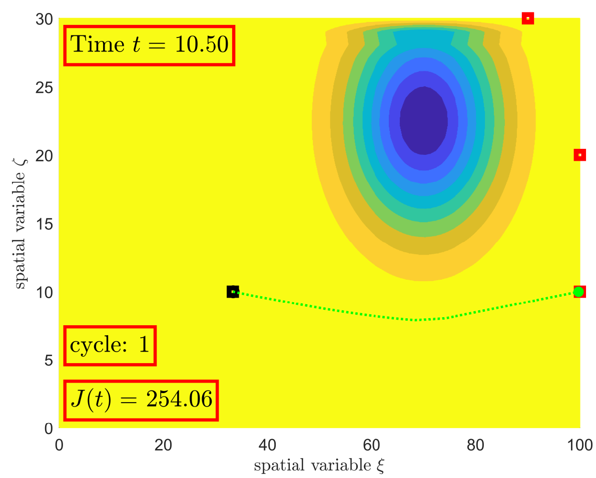


Fig. 1: Trajectory using full access to spatial field.

Figure 4b, the agent unaware of the true field, is using its own estimate $\widehat{c}(t, \xi, \zeta)$ in Figure 4a for trajectory selection.

V. Conclusions

This paper described a modification to level-set based evacuation guidance over hazardous indoor environments. The modification introduced a planning period, in which the agent was computing viable escape trajectories using snapshots of the estimated spatial field , along with a travel period in which the agent was implementing the optimal trajectory over a given time cycle. Earlier works on asymptotic embedding methods for the state estimation of spatial

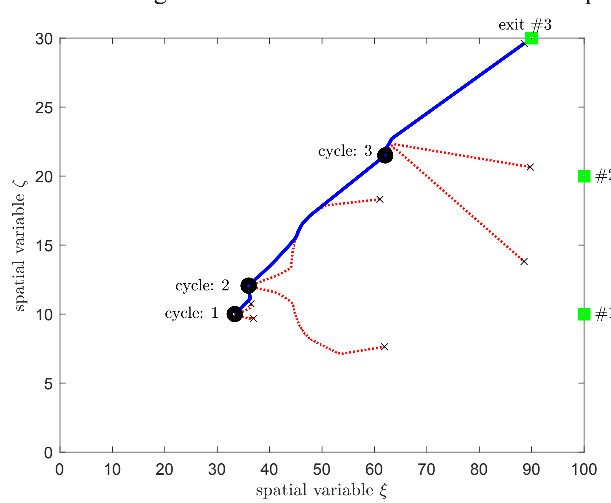


Fig. 2: Trajectory defined over different cycles with intermediate trajectory recalculations; dotted lines represent trajectories computed in each planning stage that were not selected in the travel stage.

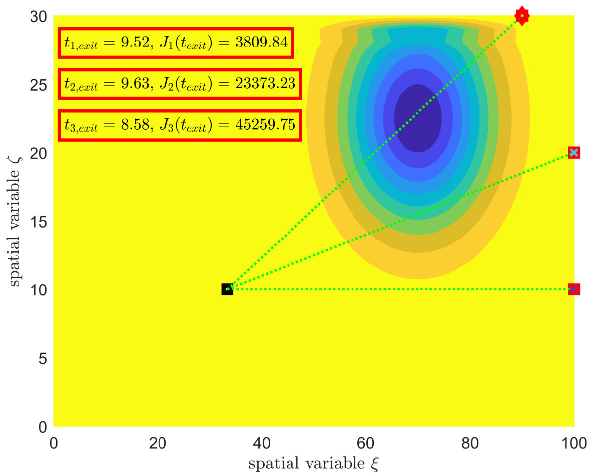
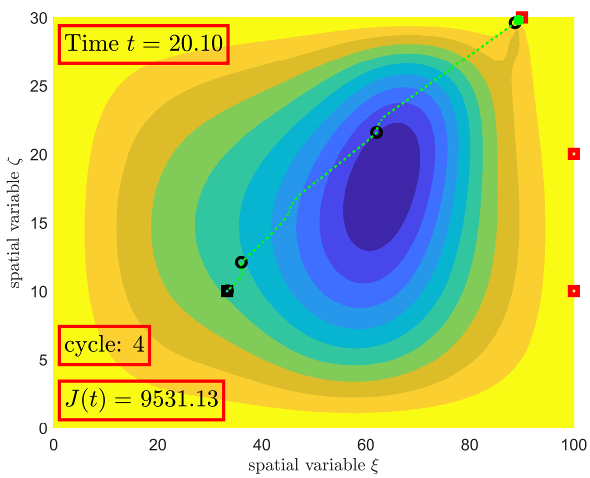
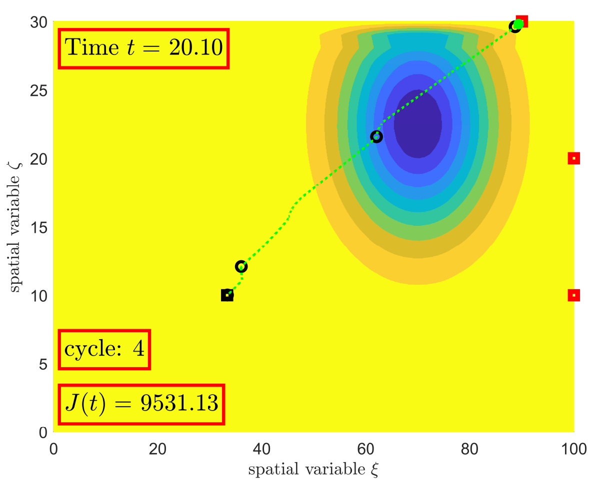


Fig. 3: Viable trajectories of field-agnostic agent using line-of-sight path planning. Shortest escape path leads to exit #3 in 8.6 seconds with an accumulated amount of 45,260ppm.



(a) On-line estimated spatial field.



(b) True spatial field.

Fig. 4: Evacuation guidance based on snapshots of on-line estimated spatial field; (a) estimated field, (b) true field.

fields with mobile sensor were incorporated in the level-set trajectory generation in order to generate viable escape trajectories with guaranteed accumulated amounts below life-threatening thresholds. Numerical studies presented the modifications using snapshots of the estimated spatial field and pointed to the improvements of the modified level-set guidance with partial field information.

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