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Distributed Event-Triggered Observer-based Control for Linear Networked Multi-Agent Systems

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Abstract— **This paper studies the distributed event-triggered observer-based control problem for linear multi-agent systems with heterogeneous dynamics and external disturbances. In particular, the multi-agent system and its observers communicate via multiple independent and asynchronous networks, the observers can communicate with each other, and each controller is designed via the corresponding observer. In such a framework, we first investigate the information transmission in the closed-loop system, then apply local information to develop distributed event-triggered mechanisms, and finally derive sufficient conditions for the co-design strategy to ensure the desired performance. Further discussions are presented to show the generality of the proposed framework. A numerical example from power systems is presented to illustrate the derived results.**

I. Introduction

Distributed cooperative control of multi-agent systems (MASs) has gained increasing attention in the past decades due to its potential applications in many fields including intelligent microgrids, multirobot systems, and intelligent transportation systems [1], [2]. A key challenge in distributed cooperative control is how to design control schemes to achieve certain agreement for multiple agents by exploiting information from each agent and its neighbors. In order to deal with this challenge, many results can be found in the literature; see [1]–[3] and references therein. In addition, with rapid advancements in the field of computer science and technology, digital networks are involved in MASs to improve system efficiency and flexibility, and to reduce installation/maintenance time and costs [4]–[6]. For the MAS structures where different components are connected via communication networks, the information transmission over the networks plays an essential role in the overall performance [7]. However, how to address the effects of the communication networks and how to minimize the frequency of the information transmission deserve further study. In this paper we consider linear MASs where multiple networks are involved for the information transmission.

For networked MASs, both observer and controller design are two fundamental problems, which have attracted numerous attention from diverse fields [8]–[10]. The observer design is to estimate the agent states, while the controller design is to apply the estimation information to generate control inputs such that the desired performance is achieved. Hence, the observers and controllers are usually separated into two different layers that interact with each other. Regarding the observer and controller design, different approaches have been developed in the literature; see [1]–[3] and references therein. However, we note that each agent only has the local information from its neighbor agents and is able to take actions independently. This distributed setting imposes more difficulties in the observer and controller design. In addition, the information transmission via multiple networks results in that only local/partial information is transmitted to the agents due to the limited capacity of communication networks, which can be a reason for the performance deterioration [23]. In these respects, how to implement partial and local information to design the observers and controllers deserve further study and thus is the topic of this work.

In this paper, we investigate the distributed event-triggered observer-based control problem of linear multi-agent systems, whose dynamics are heterogeneous and are perturbed by external disturbances. A general framework is proposed in this work: multiple independent and asynchronous networks are applied to ensure the communication between the agents and observers; the distributed event-triggered mechanisms (DETMs) are derived to reduce the communication burden of multiple networks; and the agent states are estimated by the observers which are further involved in the controller design. This framework recovers many existing architectures in [11]–[13] for NCS and in [8]–[10] for MAS as special cases. With this framework, we aim to co-design the DETMs, the observers and the controllers to guarantee the stabilization of MASs. To this end, we start with the analysis of the information transmission via each ETM and communication network to reveal their effects on the system dynamics, then apply the transmitted information to design the DETMs and to compute the maximally allowable sampling periods explicitly, and finally establish sufficient conditions for both the observer and controller design to guarantee the system stability. In addition, we further discuss the generality and potential extensions/variants of the proposed framework in several directions. In contrast to our previous work [10, Section VI] on the observer design only, here we make a further step to the co-design of the ETMs, the observers and the controllers in a distributed and networked way.

This paper is organized as follows. Section II formulates the problem. The detailed co-design strategy is derived in Section III. A numerical example is given in Section IV followed by the conclusions in Section V.

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II. Preliminaries and Problem Formulation

Let $\mathbb{R} := (-\infty, +\infty)$; $\mathbb{R}_{\geq 0} := [0, +\infty)$; $\mathbb{R}_{>0} := (0, +\infty)$; $\mathbb{N} := \{0, 1, 2, \ldots\}$; $\mathbb{N}_+ := \{1, 2, \ldots\}$. $\|\cdot\|$ denotes the Euclidean norm. Given two vectors $x, y \in \mathbb{R}^n$, $(x, y) := (x^\top, y^\top)^\top$ for simplicity of notation, and $\langle x, y\rangle$ denotes the usual inner product. I denotes the identity matrix of appropriate dimension, and $\operatorname{diag}\{A, B\}$ denotes the block diagonal matrix made of the matrices A and B. Given a function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $f(t^+) := \limsup_{s\to 0^+} f(t+s)$ and the $\mathcal{L}_\infty$ norm is $\|f\|_\infty := \operatorname{ess.sup}_{t\in\mathbb{R}_{\geq 0}} \|f(t)\|$.

A. Agent Dynamics

Consider the MAS, where the dynamics of each agent is of the following linear and heterogenous form:

$$\begin{aligned} \dot{x}_{\mathrm{p}}^i &= A_{\mathrm{p}}^{ii} x_{\mathrm{p}}^i + \sum\nolimits_{j\in\mathcal{N}_{\mathrm{p}}^i} A_{\mathrm{p}}^{ij}(x_{\mathrm{p}}^j - x_{\mathrm{p}}^i) + B_{\mathrm{p}}^i u_i + F_i w_1, \\ y_{\mathrm{p}}^i &= C_{\mathrm{p}}^i x_{\mathrm{p}}^i + G_i w_2, \end{aligned} \tag{1}$$

where $i \in \mathcal{N} := \{1, \ldots, N\}$, $x_{\mathrm{p}}^i \in \mathbb{R}^{n_{\mathrm{p}}^i}$ is the agent state, $u_i \in \mathbb{R}^{n_u^i}$ is the control input, $y_{\mathrm{p}}^i \in \mathbb{R}^{n_y^i}$ is the agent output, and $A_{\mathrm{p}}^{ii}, A_{\mathrm{p}}^{ij}, C_{\mathrm{p}}^i, B_{\mathrm{p}}^i, F_i, G_i$ are the matrices with appropriate dimensions. $w_1 \in \mathbb{R}^{n_1}$ and $w_2 \in \mathbb{R}^{n_2}$ are unknown but bounded disturbances imposed on the agent dynamics and the agent output, respectively. Assume that w_2 and its time derivative have finite $\mathcal{L}_\infty$ norms.

In (1), $\mathcal{N}_{\mathrm{p}}^i \subseteq \mathcal{N}$ denotes the set of all neighbor agents that can communicate with the i-th agent, and thus the item $\sum_{j\in\mathcal{N}_{\mathrm{p}}^i} A_{\mathrm{p}}^{ij}(x_{\mathrm{p}}^j - x_{\mathrm{p}}^i)$ is to show the coupling among all agent. The physical coupling among all agents is denoted by an undirected and connected graph $\mathcal{G}_{\mathrm{p}} := (\mathcal{N}, \mathcal{E}_{\mathrm{p}})$ with $\mathcal{E}_{\mathrm{p}} \subseteq \mathcal{N}\times\mathcal{N}$. Let $x_{\mathrm{p}} := (x_{\mathrm{p}}^1, \ldots, x_{\mathrm{p}}^N) \in \mathbb{R}^{n_{\mathrm{p}}}$ with $n_{\mathrm{p}} := \sum_{i=1}^N n_{\mathrm{p}}^i$. Hence, we can rewrite (1) as

$$\dot{x}_{\mathrm{p}} = \mathbf{A}_{\mathrm{p}} x_{\mathrm{p}} + \mathbf{B}_{\mathrm{p}} u + \mathbf{F} w_1, \quad y_{\mathrm{p}} = \mathbf{C}_{\mathrm{p}} x_{\mathrm{p}} + \mathbf{G} w_2, \tag{2}$$

where $u := (u_1, \ldots, u_N) \in \mathbb{R}^{n_u}$ and $y_{\mathrm{p}} := (y_{\mathrm{p}}^1, \ldots, y_{\mathrm{p}}^N) \in \mathbb{R}^{n_y}$ with $n_u := \sum_{i=1}^N n_u^i$ and $n_y := \sum_{i=1}^N n_y^i$. $\mathbf{A}_{\mathrm{p}} := [\mathbf{A}_{\mathrm{p}}^{ij}]_{N\times N}$ with $\mathbf{A}_{\mathrm{p}}^{ij} := A_{\mathrm{p}}^{ii} - \sum_{j\in\mathcal{N}_{\mathrm{p}}^i} A_{\mathrm{p}}^{ij}$ for $i = j$ and $\mathbf{A}_{\mathrm{p}}^{ij} := A_{\mathrm{p}}^{ij}$ for $i \neq j$. Note that $A_{\mathrm{p}}^{ij} = 0$ if $j \notin \mathcal{N}_{\mathrm{p}}^i$. $\mathbf{B}_{\mathrm{p}} := \operatorname{diag}\{B_{\mathrm{p}}^1, \ldots, B_{\mathrm{p}}^N\}$, $\mathbf{C}_{\mathrm{p}} := \operatorname{diag}\{C_{\mathrm{p}}^1, \ldots, C_{\mathrm{p}}^N\}$, $\mathbf{F} := (F_1, \ldots, F_N)$ and $\mathbf{G} := (G_1, \ldots, G_N)$.

B. Problem Formulation

Our *objective* is to design the distributed observer-based controllers via the event-triggered network communication for each agent such that all agents converge to a region around the origin. The proposed control framework is illustrated in Fig. 1, which consists of the following three parts.

1) Event-Triggered Network Communication: All agents and their observers/controllers communicate via multiple networks, which are assumed to be asynchronous and independent of each other. In this way, all agents do not need to transmit their information simultaneously. The event-triggered mechanisms (ETMs) are to decide whether the information needs to be transmitted, thereby aiming to reduce the transmission cost. Hence, the key of this part is how to design the distributed ETMs to ensure the information transmission and the desired performance simultaneously.

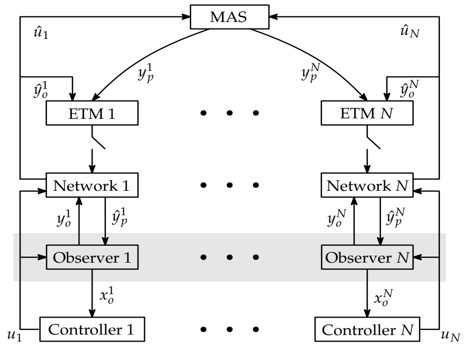

Fig. 1. The distributed event-triggered observer-based control structure.

2) Distributed Observer Design: In the network-free case, the extended-state observer for each agent is designed as

$$\begin{aligned} \dot{x}_{\mathrm{o}}^i &= A_{\mathrm{o}}^{ii} x_{\mathrm{o}}^i + \sum\nolimits_{j\in\mathcal{N}_{\mathrm{o}}^i} A_{\mathrm{o}}^{ij}(x_{\mathrm{o}}^i - x_{\mathrm{o}}^j) + L_i(y_{\mathrm{p}}^i - y_{\mathrm{o}}^i) \\ &\quad + B_{\mathrm{o}}^i u_i + D_i \vartheta_i, \\ \dot{\vartheta}_i &= E_i(y_{\mathrm{p}}^i - y_{\mathrm{o}}^i) + S_i \vartheta_i, \quad y_{\mathrm{o}}^i = C_{\mathrm{p}}^i x_{\mathrm{o}}^i, \end{aligned} \tag{3}$$

where $x_{\mathrm{o}}^i \in \mathbb{R}^{n_{\mathrm{p}}^i}$ is the estimate of the agent state, $\vartheta_i \in \mathbb{R}^{m_i}$ is the additional variable, and $y_{\mathrm{o}}^i \in \mathbb{R}^{n_y^i}$ is the observer output. In (3), $A_{\mathrm{o}}^{ii}, A_{\mathrm{o}}^{ij}, B_{\mathrm{o}}^i, L_i, D_i, E_i, S_i$ are the matrices with appropriate dimensions.

The physical coupling among all observers is denoted by an undirected and connected graph $\mathcal{G}_{\mathrm{o}} := (\mathcal{N}, \mathcal{E}_{\mathrm{o}})$ with $\mathcal{E}_{\mathrm{o}} \subseteq \mathcal{N}\times\mathcal{N}$. Let $x_{\mathrm{o}} := (x_{\mathrm{o}}^1, \ldots, x_{\mathrm{o}}^N) \in \mathbb{R}^{n_{\mathrm{o}}}$ with $n_{\mathrm{o}} := \sum_{i=1}^N n_{\mathrm{o}}^i$. Hence, we can rewrite the state-space equation in (3) as

$$\dot{x}_{\mathrm{o}}^i = \mathbf{A}_{\mathrm{o}} x_{\mathrm{o}} + \mathbf{L}(y_{\mathrm{p}} - y_{\mathrm{o}}) + \mathbf{B}_{\mathrm{o}} u + \mathbf{D}\vartheta, \tag{4}$$

where $\vartheta := (\vartheta_1, \ldots, \vartheta_N) \in \mathbb{R}^m$ with $m := \sum_{i=1}^N m_i$ and $y_{\mathrm{o}} := (y_{\mathrm{o}}^1, \ldots, y_{\mathrm{o}}^N) \in \mathbb{R}^{n_y}$. $\mathbf{A}_{\mathrm{o}} := [\mathbf{A}_{\mathrm{o}}^{ij}]_{N\times N}$ with $\mathbf{A}_{\mathrm{o}}^{ij} := A_{\mathrm{o}}^{ii} - \sum_{j\in\mathcal{N}_{\mathrm{o}}^i} A_{\mathrm{o}}^{ij}$ for $i = j$ and $\mathbf{A}_{\mathrm{o}}^{ij} := A_{\mathrm{o}}^{ij}$ for $i \neq j$. Note that $A_{\mathrm{o}}^{ij} = 0$ if $j \notin \mathcal{N}_{\mathrm{o}}^i$. $\mathbf{B}_{\mathrm{o}} := \operatorname{diag}\{B_{\mathrm{o}}^1, \ldots, B_{\mathrm{o}}^N\}$, $\mathbf{D} := \operatorname{diag}\{D_1, \ldots, D_N\}$ and $\mathbf{L} := \operatorname{diag}\{L_1, \ldots, L_N\}$.

3) Observer-based Controller Design: With the observer (3), the controller of each agent is designed below:

$$\dot{x}_{\mathrm{c}}^i = R_i x_{\mathrm{c}}^i + T_i x_{\mathrm{o}}^i, \quad u_i = K_i x_{\mathrm{c}}^i + H_i x_{\mathrm{o}}^i, \tag{5}$$

where $x_{\mathrm{c}}^i \in \mathbb{R}^{n_{\mathrm{c}}^i}$ is the controller state, u_i is the control input, and R_i, T_i, K_i, H_i are the matrices with appropriate dimensions. Similar to (2) and (4), we have

$$\dot{x}_{\mathrm{c}} = \mathbf{R} x_{\mathrm{c}} + \mathbf{T} x_{\mathrm{o}}, \quad u = \mathbf{K} x_{\mathrm{c}} + \mathbf{H} x_{\mathrm{o}}, \tag{6}$$

where $x_{\mathrm{c}} := (x_{\mathrm{c}}^1, \ldots, x_{\mathrm{c}}^N) \in \mathbb{R}^{n_{\mathrm{c}}}$ with $n_{\mathrm{c}} := \sum_{i=1}^N n_{\mathrm{c}}^i$, $\mathbf{R} := \operatorname{diag}\{R_1, \ldots, R_N\}$, $\mathbf{T} := \operatorname{diag}\{T_1, \ldots, T_N\}$, $\mathbf{K} := \operatorname{diag}\{K_1, \ldots, K_N\}$ and $\mathbf{H} := \operatorname{diag}\{H_1, \ldots, H_N\}$.

From the above three parts, the objective of this paper is the co-design of the ETMs, the observers and the controllers in a distributed and networked way. Hence, all agents are assumed to be observable while all observers are assumed to be controllable. The ETM design is to minimize the frequencies of the information transmission over multiple networks. The observer design is to guarantee the estimation error $\eta_i := x_{\mathrm{p}}^i - x_{\mathrm{o}}^i$ to converge asymptotically to a region around the origin. The observer-based controller design is to ensure the asymptotic convergence of all agents to a region around the origin. The sizes of these regions are determined by the external disturbances and the networks, which will be shown clearly in Section III-B.

III. Co-design Strategy

In this section we present the co-design strategy in detail. The information transmission via multiple networks is studied in Section III-A, the distributed ETMs are designed in Section III-B, and the main result is stated in Section III-C.

A. Information Transmission

For each network, the information to be transmitted is denoted as $z_i := (y^i_{\rm p}, y^i_{\rm o}, u_i) \in \mathbb{R}^{n^i_z}$ with $n^i_z := 2n^i_y + n^i_u$. That is, the outputs of the agent, the observer and the controller are transmitted via the network. For each agent, the sampling time sequence is given by $\{t^i_j : i \in \mathcal{N}, j \in \mathbb{N}_+\}$, which is assumed to be strictly increasing. To reduce the transmission cost, each sampled data is evaluated via the ETM at each t^i_j. Only when certain event-triggered condition in the ETM is satisfied can the sampled data be transmitted via the corresponding network. For each network, the event-triggered condition is defined as $\Lambda_i \geq 0$, where the function $\Lambda_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ will be designed in Subsection III-B.

Due to the band-limited capacity of the communication networks, each network is assumed to have $\ell_i \in \mathbb{N}_+$ nodes. At each t^i_j, one and only one node is granted to access to the i-th network, while which node to be chosen is determined by the time-scheduling protocol [14]. To correspond to all nodes of each network, z_i is partitioned into ℓ_i parts. For each $i \in \mathcal{N}$ and all $j \in \mathbb{N}_+$, the sampling intervals are defined as $h^i_j := t^i_{j+1} - t^i_j$, which is assumed to be bounded.

Assumption 1: For each $i \in \mathcal{N}$ and all $j \in \mathbb{N}_+$, there exist $\tau^i_{\rm masp} \geq 0$ and $\tau^i_{\rm miati} \in (0, \tau^i_{\rm masp})$ such that $h^i_j \in [\tau^i_{\rm miati}, \tau^i_{\rm masp}]$.

In Assumption 1, $\tau^i_{\rm masp} > 0$ is called the *maximally allowable sampling period (MASP)* and $\tau^i_{\rm miati} > 0$ is the *minimally achievable transmission interval (MIATI)*. The constant $\tau^i_{\rm miati}$ is determined by the hardware constraints [5] and ensures the exclusion of Zeno phenomena. However, the MASP is to be determined later to balance the system performance and the transmission cost.

After the information transmission via networks, the received measurement is denoted as $\hat{z}_i := (\hat{y}^i_{\rm p}, \hat{y}^i_{\rm o}, \hat{u}_i)$. The network-induced error is defined as $e_i := (e^i_{\rm p}, e^i_{\rm o}, e^i_u)$ with $e^i_{\rm p} := \hat{y}^i_{\rm p} - y^i_{\rm p}$, $e^i_{\rm o} := \hat{y}^i_{\rm o} - y^i_{\rm o}$ and $e^i_u := \hat{u}_i - u_i$. In $[t^i_j, t^i_{j+1}]$, the received measurement $\hat{z}_i$ is assumed to be implemented via the zero-order hold (ZOH) mechanism, that is,

$$\dot{\hat{z}}_i(t) = 0, \quad \forall t \in [t^i_j, t^i_{j+1}]. \tag{7}$$

At each t^i_j, $\hat{z}_i$ is updated based on the ETM and the latest received measurement. If $\Lambda_i \geq 0$, then $\hat{z}_i$ is updated with the latest received measurement; otherwise, $\hat{z}_i$ is kept constant.

$$\hat{z}_i(t^{i+}_j) = \begin{cases} z_i(t^i_j) + h_i(\kappa_i(t^i_j), e_i(t^i_j)), & \Lambda_i(t^i_j) \geq 0, \\ \hat{z}_i(t^i_j), & \Lambda_i(t^i_j) < 0, \end{cases} \tag{8}$$

where $\kappa_i : \mathbb{R}_{\geq 0} \to \mathbb{N}$ is a counter to record the number of the transmission successes. That is, $\kappa_i(t^{i+}_j) = \kappa_i(t^i_j) + 1$ if $\Lambda_i(t^i_j) \geq 0$, and $\kappa_i(t^{i+}_j) = \kappa_i(t^i_j)$ otherwise. $h_i \in \mathbb{R}^{n_z}$ is called the *update function* and depends on the time-scheduling protocol. Let $h_i := (h^i_{\rm p}, h^i_{\rm o}, h^i_u)$ and

$$\begin{aligned} \hat{z}_i(t^{i+}_j) &= (1 - \mathrm{sgn}(\Lambda_i(t^i_j)))\hat{z}_i(t^i_j) \\ &\quad + \mathrm{sgn}(\Lambda_i(t^i_j))[z_i(t^i_j) + h_i(\kappa_i(t^i_j), e_i(t^i_j))], \end{aligned} \tag{9}$$

where $\mathrm{sgn} : \mathbb{R} \to \{0, 1\}$ is defined as $\mathrm{sgn}(\Lambda_i) = 1$ if $\Lambda_i \geq 0$ and $\mathrm{sgn}(\Lambda_i) = 0$ otherwise. From (9), e_i is updated by

$$\begin{aligned} e_i(t^{i+}_j) &= \hat{z}_i(t^{i+}_j) - z_i(t^{i+}_j) \\ &= (1 - \mathrm{sgn}(\Lambda_i(t^i_j)))e_i(t^i_j) \\ &\quad + \mathrm{sgn}(\Lambda_i(t^i_j))h_i(\kappa_i(t^i_j), e_i(t^i_j)). \end{aligned} \tag{10}$$

B. Distributed Event-Triggered Mechanisms

With the analysis of the information transmission in the previous subsection, in this subsection the distributed ETMs are established via the information to be transmitted and the network-induced errors. For this purpose, we start with the formulation of the closed-loop system.

1) The Closed Loop over Each Network: Let $x_i := (x^i_{\rm p}, \eta_i, x^i_{\rm c}, \vartheta_i)$ be the augmented state and $\mathbf{w} := (w_1, w_2, \dot{w}_2)$ be the augmented disturbance. From (1)-(6), we derive the following dynamics:

$$\dot{x}_i = \hat{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{B}}_i e_i + \hat{\mathbf{C}}_i\mathbf{w} \tag{11}$$

with $\hat{\mathbf{A}} := [\hat{\mathbf{A}}_{ij}]_{N\times N}$ and

$$\hat{\mathbf{A}}_{ii} = \begin{bmatrix} \mathbf{A}^{ii}_{\rm p} + B^i_{\rm p}H_i & -B^i_{\rm p}H_i & B^i_{\rm p}K_i & \mathbf{0} \\ \mathbf{A}^{ii}_{\rm p} - \mathsf{A}_i & \mathsf{A}_i - L_iC^i_{\rm p} & B^i_{\rm o}K_i & \mathbf{0} \\ T_i & -T_i & R_i & \mathbf{0} \\ \mathbf{0} & E_iL_iC^i_{\rm p} & \mathbf{0} & S_i \end{bmatrix},$$

$$\hat{\mathbf{A}}_{ij} = \begin{bmatrix} \mathbf{A}^{ij}_{\rm p} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}^{ij}_{\rm p} - \mathbf{A}^{ij}_{\rm o} & \mathbf{A}^{ij}_{\rm o} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\hat{\mathbf{B}}_i = \begin{bmatrix} \mathbf{0} & \mathbf{0} & B^i_{\rm p} \\ -L_i & L_i & B^i_{\rm p} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ E_iL_i & -E_iL_i & \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{C}}_i = \begin{bmatrix} F_i & \mathbf{0} & \mathbf{0} \\ F_i & -L_iG_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & E_iL_iG_i & \mathbf{0} \end{bmatrix},$$

where $\mathsf{A}_i := \mathbf{A}^{ii}_{\rm o} - (B^i_{\rm p} - B^i_{\rm o})H_i$.

Define a Lyapunov function candidate $V_i(x_i) := x_i^\top P_i x_i$ with a positive definite symmetric matrix P_i.

$$\lambda_{\min}(P_i)\|x_i\|^2 \leq V_i(x_i) \leq \lambda_{\max}(P_i)\|x_i\|^2, \tag{12}$$

where $\lambda_{\min}(P_i)$ and $\lambda_{\max}(P_i)$ are respectively the smallest and largest eigenvalues of P_i. With (11), the time derivative of $V_i(x_i)$ is given as follows.

$$\begin{aligned} &\left\langle \nabla V_i(x_i), \hat{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{B}}_i e_i + \hat{\mathbf{C}}_i\mathbf{w} \right\rangle \\ &= x_i^\top \left(P_i\hat{\mathbf{A}}_{ii} + \hat{\mathbf{A}}_{ii}^\top P_i\right) x_i + x_i^\top P_i \left(\sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{B}}_i e_i \right. \\ &\quad \left. + \hat{\mathbf{C}}_i\mathbf{w}\right) + \left(\sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{B}}_i e_i + \hat{\mathbf{C}}_i\mathbf{w}\right)^\top P_i x_i \end{aligned}$$

$$\leq x_i^\top \left(P_i\hat{\mathbf{A}}_{ii} + \hat{\mathbf{A}}_{ii}^\top P_i + \varepsilon_i^{-1}P_i\right) x_i + \varepsilon_i e_i^\top \hat{\mathbf{B}}_i^\top P_i \hat{\mathbf{B}}_i e_i + x_i^\top P_i \left(\sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{C}}_i\mathbf{w}\right)^\top + \left(\sum\nolimits_{j\in\mathcal{N}} \hat{\mathbf{A}}_{ij}x_j + \hat{\mathbf{C}}_i\mathbf{w}\right) P_i x_i, \tag{13}$$

where $\varepsilon_i > 0$ can be arbitrary and the inequality "$\leq$" holds from the triangle inequality.

2) The Dynamics of Network-induced Errors: Since the transmitted information is denoted as z_i in Section III-A, we have from (11) that

$$\begin{aligned} z_i &= \tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}, \\ \dot{z}_i &= \bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j + \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}, \end{aligned} \tag{14}$$

where $\bar{\mathbf{A}}_{ii} = \tilde{\mathbf{A}}_i\hat{\mathbf{A}}_{ii}, \bar{\mathbf{A}}_{ij} = \tilde{\mathbf{A}}_i\hat{\mathbf{A}}_{ij}, \bar{\mathbf{B}}_i = \tilde{\mathbf{A}}_i\hat{\mathbf{B}}_i$ and

$$\tilde{\mathbf{A}}_i = \begin{bmatrix} C_{\mathrm{p}}^i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ C_{\mathrm{p}}^i & -C_{\mathrm{p}}^i & \mathbf{0} & \mathbf{0} \\ H_i & -H_i & K_i & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{0} & G_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\bar{\mathbf{C}}_i = \tilde{\mathbf{A}}_i\hat{\mathbf{C}}_i + \begin{bmatrix} \mathbf{0} & \mathbf{0} & G_i \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

Let $\varphi_i(z_i) = \|z_i\|^2$, and from (14),

$$\begin{aligned} \dot{\varphi}_i(z_i) &= \left(\bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j + \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}\right)^\top \\ &\times \left(\tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}\right) + \left(\tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}\right)^\top \\ &\times \left(\bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j + \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}\right) \\ &\leq \varphi_i(z_i) + 2x_i^\top \bar{\mathbf{A}}_{ii}^\top \bar{\mathbf{A}}_{ii}x_i + \mathbf{w}^\top \tilde{\mathbf{C}}_i^\top \tilde{\mathbf{C}}_i\mathbf{w} + \Big(\sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j \\ &+ \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}\Big)^\top \left(\tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}\right) + \left(\tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}\right)^\top \\ &\times \left(\sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j + \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}\right). \end{aligned} \tag{15}$$

From (7) and (14), the dynamics for the network-induced error e_i is derived below:

$$\dot{e}_i = -\bar{\mathbf{A}}_{ii}x_i - \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j - \bar{\mathbf{B}}_i e_i - \bar{\mathbf{C}}_i\mathbf{w}. \tag{16}$$

The Lyapunov function candidate for (16) is defined as $W_i : \mathbb{N}_+ \times \mathbb{R}^{n_e^i} \to \mathbb{R}_{\geq 0}$, which is related to κ_i and e_i. The explicit form of W_i cannot be determined in a unified way, since it depends on the applied scheduling protocol for each network. For instance, for the Round-Robin (RR) protocol, $W^2(\kappa_i, e_i) = \sum_{j=1}^{\ell_i} a_j(\kappa_i)\|e_{ij}\|^2$ with $e_i = (e_{i1}, \ldots, e_{i\ell_i})$ and $a_j(\kappa_i) \in \{1, \ldots, \ell_i\}$; see also [14, Example 3]. For the try-once-discard (TOD) protocol, the function W_i is defined as $\|e_i\|$ simply. For these scheduling protocols, W_i is assumed to satisfy the following properties [14]:

$$\mathfrak{a}_i\|e_i\| \leq W_i(\kappa_i, e_i) \leq \mathfrak{b}_i\|e_i\|, \tag{17a}$$

$$\left\|\frac{\partial W_i(\kappa_i, e_i)}{\partial e_i}\right\| \leq \mathfrak{g}_i, \tag{17b}$$

$$W_i(\kappa_i + 1, h_i(\kappa_i, e_i)) \leq \lambda_i W_i(\kappa_i, e_i). \tag{17c}$$

The condition (17) is valid for many existing cases. For instance, $\mathfrak{a}_i = 1, \mathfrak{b}_i = \mathfrak{g}_i = \sqrt{\ell_i}$ for the RR protocol, whereas $\mathfrak{a}_i = \mathfrak{b}_i = \mathfrak{g}_i = 1$ for the TOD protocol. In addition,

$$\begin{aligned} &\left\langle \frac{\partial W_i(\kappa_i, e_i)}{\partial e_i}, -\bar{\mathbf{A}}_{ii}x_i - \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j - \bar{\mathbf{B}}_i e_i - \bar{\mathbf{C}}_i\mathbf{w} \right\rangle \\ &\leq \mathfrak{g}_i\|\bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j + \bar{\mathbf{B}}_i e_i + \bar{\mathbf{C}}_i\mathbf{w}\| \\ &\leq \mathfrak{g}_i\|\bar{\mathbf{B}}_i e_i\| + \mathfrak{g}_i\|\bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j\| + \mathfrak{g}_i\|\bar{\mathbf{C}}_i\|\|\mathbf{w}\| \\ &\leq \mathfrak{a}_i^{-1}\mathfrak{g}_i\|\bar{\mathbf{B}}_i\|W_i(\kappa_i, e_i) + \mathfrak{g}_i\|\bar{\mathbf{A}}_{ii}x_i + \sum\nolimits_{j\in\mathcal{N}} \bar{\mathbf{A}}_{ij}x_j\| \\ &\quad + \mathfrak{g}_i\|\bar{\mathbf{C}}_i\|\|\mathbf{w}\|. \end{aligned} \tag{18}$$

3) Distributed Event-Triggered Conditions: We next show how to design the distributed ETMs via the applied Lyapunov function candidates and the transmitted information. From (13)-(18), the function Λ_i in (8) is defined as

$$\Lambda_i(\kappa_i, z_i, e_i) := \gamma_i W_i^2(\kappa_i, e_i) - \rho_i\delta_i\|z_i\|^2, \tag{19}$$

where $\gamma_i := \varepsilon_i\lambda_{\max}(\hat{\mathbf{B}}_i^\top P_i\hat{\mathbf{B}}_i)$ is from (13), $\rho_i \in [0, (1+\gamma_i)^{-1})$ and $\delta_i := \max\{\lambda_i, (1-\rho_i)^{-1}\rho_i\gamma_i\}$. That is, the event-triggered condition is $\Lambda_i(\kappa_i, z_i, e_i) \geq 0$, which is based on the Lyapunov function candidates W_i, V_i and the transmitted information z_i. In (19), ε_i can be viewed as an adjustable variable to tune the event-triggered condition.

With the aforementioned analysis and designed ETMs, the maximally allowable transmission period (MASP) for each network can be derived explicitly as follows.

$$\tau_{\text{masp}}^i := \begin{cases} \dfrac{\mathfrak{a}_i}{\mathfrak{g}_i\mathfrak{h}_i\|\bar{\mathbf{B}}_i\|}\arctan(\theta_i), & \mathfrak{a}_i\gamma_i > \|\bar{\mathbf{B}}_i\|, \\ \dfrac{1-\delta_i}{1+\delta_i}\dfrac{\mathfrak{a}_i}{\mathfrak{g}_i\|\bar{\mathbf{B}}_i\|}, & \mathfrak{a}_i\gamma_i = \|\bar{\mathbf{B}}_i\|, \\ \dfrac{\mathfrak{a}_i}{\mathfrak{g}_i\mathfrak{h}_i\|\bar{\mathbf{B}}_i\|}\operatorname{arctanh}(\theta_i), & \mathfrak{a}_i\gamma_i < \|\bar{\mathbf{B}}_i\|, \end{cases} \tag{20}$$

where $\mathfrak{h}_i := \sqrt{|(\mathfrak{a}_i\gamma_i/\|\bar{\mathbf{B}}_i\|)^2 - 1|}$ and $\theta_i := \frac{(1-\delta_i^2)\mathfrak{h}_i}{2\delta_i(\mathfrak{a}_i\gamma_i/\|\bar{\mathbf{B}}_i\|-1)+(1+\delta_i)^2}$. From [15], [16], the MASP (20) is the solution to the following differential equation:

$$\dot{\phi}_i = -2\mathfrak{a}_i^{-1}\mathfrak{g}_i\|\bar{\mathbf{B}}_i\|\phi_i - \gamma_i(\mathfrak{g}_i^2\phi_i^2 + 1). \tag{21}$$

That is, given the initial condition $\phi_i(0) = \delta_i^{-1}$, we have $\phi_i(\tau_{\text{masp}}^i) = \delta_i$. Here we emphasize that due to the item $\mathfrak{g}_i^2$ in (21), the property (17b) affects the values of MASPs.

C. Performance Analysis

In this subsection, we show the satisfaction of the desired performance under the designed ETMs and observer-based controllers. The next theorem presents the LMI conditions for the design of the observer (3) and the controller (5).

Theorem 1: Consider the system (1)-(6) and let Assumption 1 hold. There exist $\chi, \alpha, \psi > 0$ such that for all $t \geq 0$,

$$\|(x(t), e(t))\|^2 \leq \chi e^{-\alpha t}\|(x(0), e(0))\|^2 + \psi\|\mathbf{w}\|_\infty^2, \tag{22}$$

if there exist matrices $P_i > 0$, $i \in \mathcal{N}$, such that

$$\Sigma := \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & \Sigma_{23} \\ * & * & \Sigma_{33} \end{bmatrix} < 0, \tag{23}$$

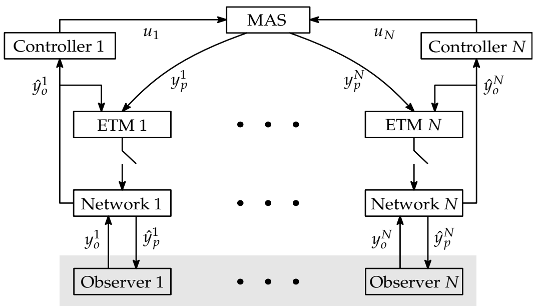

Fig. 2. Illustration of an alternative structure for the distributed event-triggered observer-based control strategy.

with $\Sigma_{11} = \hat{\mathbf{A}}^\top\mathbf{P} + \mathbf{P}\hat{\mathbf{A}} + \alpha I + 2\hat{\mathbf{A}}^\top\tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}\hat{\mathbf{A}} + \tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}$, $\Sigma_{12} = \mathbf{P}\hat{\mathbf{B}} + \hat{\mathbf{A}}^\top\tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}\hat{\mathbf{B}}$, $\Sigma_{13} = \mathbf{P}\hat{\mathbf{C}} + 2\hat{\mathbf{A}}^\top\tilde{\mathbf{A}}^\top(\tilde{\mathbf{A}}\hat{\mathbf{C}} + \bar{\mathbf{G}}) + \tilde{\mathbf{A}}\tilde{\mathbf{C}}$, $\Sigma_{22} = \hat{\mathbf{B}}^\top\tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}\hat{\mathbf{B}} + \alpha I - \Gamma$, $\Sigma_{23} = \hat{\mathbf{B}}^\top\tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}\tilde{\mathbf{C}}$, $\Sigma_{33} = 2(\tilde{\mathbf{A}}\hat{\mathbf{C}} + \bar{\mathbf{G}})^\top(\tilde{\mathbf{A}}\hat{\mathbf{C}} + \bar{\mathbf{G}}) - \psi I + \tilde{\mathbf{C}}^\top\tilde{\mathbf{A}}^\top\tilde{\mathbf{A}}\tilde{\mathbf{C}}$, where $\mathbf{P} := \mathrm{diag}\{P_1, \ldots, P_N\}$, $\Gamma := \mathrm{diag}\{\mathfrak{a}_1^2\gamma_1^2, \ldots, \mathfrak{a}_N^2\gamma_N^2\}$, and all the matrices are given below (11) and (14).

From Theorem 1, α, ψ can be set via the desired performance, and (23) is solved to establish the observers and controllers guaranteeing the desired performance. In this respect, the observers and controllers can be co-designed based on the performance requirement. In addition, if (23) is satisfied, then $\chi = (\sum_{i=1}^N(\lambda_{\min}(P_i) + \mathfrak{a}_i^2\gamma_i\phi_i(\tau_{\text{masp}}^i)))^{-1}(\sum_{i=1}^N(\lambda_{\max}(P_i) + \rho_i + \mathfrak{b}_i^2\gamma_i\phi_i(0)))$ from the detailed computation. For the proposed co-design strategy, further discussions are presented below.

1) Discussion on the MASPs: Let $\bar{\phi}_i := \mathfrak{g}_i\phi_i$ and $\mathsf{L}_i := \mathfrak{a}_i^{-1}\mathfrak{g}_i\|\bar{\mathbf{B}}_i\|$. Hence, (21) can be rewritten as

$$\dot{\bar{\phi}}_i = -2\mathsf{L}_i\bar{\phi}_i - \mathfrak{g}_i\gamma_i(\bar{\phi}_i^2 + 1), \tag{24}$$

which shows explicitly the differences from the one in existing works [14], [16] and the effects of the constant $\mathfrak{g}_i$ on the MASPs. In particular, if $\mathfrak{g}_i = 1$, then (24) is similar to the one in [10], [16], while the derived MASPs are larger than the one in [10], [16] if $\mathfrak{g}_i < 1$.

2) Single Node for Each Network: If there exists one and only one node for each network, then all information of each agent is transmitted at the transmission times, which is also the case in many works on MASs [17]–[19]. In the single-node case, the scheduling protocol does not work, the Lyapunov function candidate W_i can be defined as $\|e_i\|$ directly, and thus it is easy to verify $\mathfrak{g}_i \equiv 1$. In this respect, this work extends the results [9], [17] on the time-triggered state omniscience to the event-triggered case.

3) Alternative Co-design Structure: In the proposed co-design strategy, the control inputs are transmitted via multiple networks. A special case is that the control inputs are transmitted directly to the agents, which is shown in Fig. 2 and is an alternative structure. In this case, the information to be transmitted is reduced to $(y_{\text{p}}^i, y_{\text{o}}^i)$, and the ETMs can be derived in a similar fashion. Moreover, this alternative structure is similar to the one in [20], where only the first-order dynamics is considered and the communication network is the observer graph $\mathcal{G}_{\text{o}}$ here.

IV. Numerical Results

In order to illustrate the derived results, in this section we present a numerical example from multi-machine power systems [21]. We assume that 3 interconnected power systems are involved, each of which has the dynamics of the form (1). In particular, for each power system, the state $x_{\text{p}}^i = (x_{\text{p}}^{i1}, x_{\text{p}}^{i2}, x_{\text{p}}^{i3}, x_{\text{p}}^{i4}) \in \mathbb{R}^4$ consists of the rotor angle x_{p}^{i1} in radian, the relative speed x_{p}^{i2} in radian/s, the mechanical power x_{p}^{i3} in per unit, and the steam valve opening x_{p}^{i4} in per unit. The matrices in (1) are given below.

$$A_{\text{p}}^{ii} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-D_c^i}{2H^i} & \frac{w_0}{2H^i} & 0 \\ 0 & 0 & \frac{-1}{T_m^i} & \frac{K_m^i}{T_m^i} \\ 0 & \frac{-K_e^i}{T_e^i R^i w_0} & 0 & \frac{-1}{T_e^i} \end{bmatrix}, \quad B_{\text{p}}^i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_e^i} \end{bmatrix},$$

$$A_{\text{p}}^{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ p_{ij}\alpha_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_{\text{p}}^{i\top} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $p_{ij} \in \{0, 1\}$ is to show the connection between the i-th and j-th systems. $D_c^1 = 5, D_c^2 = D_c^3 = 3, H^1 = 4, H^2 = H^3 = 5.1, T_m^1 = T_m^2 = T_m^3 = 0.35, T_e^1 = T_e^2 = T_e^3 = 0.2, R^1 = R^2 = R^3 = 0.05, K_m^1 = K_m^2 = K_m^3 = 1, K_e^1 = K_e^2 = K_e^3 = 1, w_0 = 314.159, \alpha_{12} = \alpha_{13} = -27.49$ and $\alpha_{21} = \alpha_{23} = \alpha_{31} = \alpha_{32} = -23.10$. In addition, $w_1 = (\sin(5t), 0.5\cos(10t), \exp(-0.5t), \exp(-t))$ and $w_2 = \sin(t)\exp(-t)$.

For these three power systems, the observers and controllers are designed as follows:

$$\dot{x}_{\text{o}}^i = A_{\text{p}}^{ii}x_{\text{p}}^i + B_{\text{p}}^i u_i + L_i(y_{\text{p}}^i - y_{\text{o}}^i),$$
$$y_{\text{o}}^i = C_{\text{p}}^i x_{\text{o}}^i, \quad u_i = K_i x_{\text{o}}^i.$$

Since the communication between the power systems and the observers are via multiple networks, we assume that the protocols for each networks are different. Let the protocols of the first and third networks be the TOD, while the protocol of the second network be the RR. Hence, next we need to design the ETM, the observer gain L_i and the controller gain K_i for each system such that both the system states and the estimation errors are asymptotically convergent.

To this end, we first consider the network-free case and have the following gains satisfying the LMI condition (23): $K_1 = -\begin{bmatrix} 462.321 & 81.115 & 229.122 & 19.084 \end{bmatrix}$, $K_2 = K_3 = -\begin{bmatrix} 459.784 & 81.491 & 211.049 & 18.536 \end{bmatrix}$, $L_1 = (99.1663, 97.3051, 1.5329, -7.6471)$ and $L_2 = L_3 = (88.2933, 75.1120; 1.3593, -6.7894)$. With these gains, we next consider the networked case. $\lambda_1 = \lambda_2 = \lambda_3 = \sqrt{2/3}$ from [14]. Let $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.1$ and $\rho_i = 0.01/(1 + \gamma_i)$. We can compute $\gamma_1 = 3.8726 \cdot 10^3, \gamma_2 = \gamma_3 = 2.6971 \cdot 10^3, \rho_1 = 2.5815 \cdot 10^{-6}, \rho_2 = \rho_3 = 3.7063 \cdot 10^{-6}$ and $\delta_1 = \delta_2 = \delta_3 = 0.8165$. Hence, the event-triggered conditions are designed as $\Lambda_i(\kappa_i, z_i, e_i) = \gamma_i\|e_i\|^2 - \rho_i\bar{\lambda}_i\|\tilde{\mathbf{A}}_i x_i + \tilde{\mathbf{C}}_i\mathbf{w}\|^2 \geq 0$. Furthermore, we have $\|\bar{\mathbf{B}}_1\| = 7.6290 \cdot 10^4, \|\bar{\mathbf{B}}_2\| = \|\bar{\mathbf{B}}_3\| = 6.6295 \cdot 10^4, \mathfrak{h}_1 = 0.9987, \mathfrak{h}_2 = \mathfrak{h}_3 = 0.9992, \theta_1 = 0.1903$ and $\theta_2 = \theta_3 = 0.1922$. We follow (20) to derive all MASPs, that is, $\tau_{\text{masp}}^1 = 1.4596 \cdot 10^{-6}$ and $\tau_{\text{masp}}^2 = \tau_{\text{masp}}^3 = 1.6961 \cdot 10^{-6}$. With the designed ETMs and the MASPs,

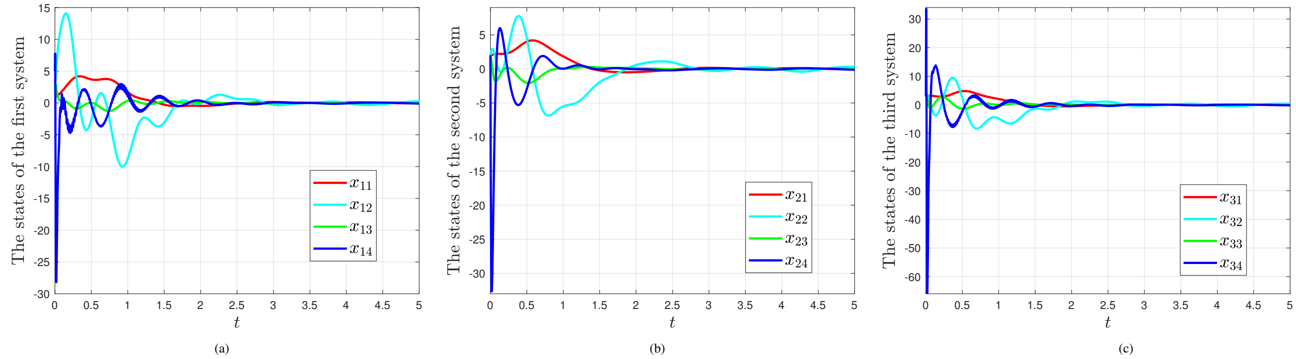

Fig. 3. Illustration of the state trajectories of the considered three systems. (a) The system state of the first system. (b) The system state of the second system. (c) The system state of the third system.

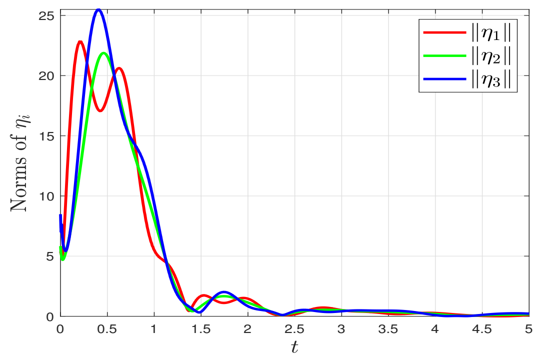

Fig. 4. Illustration of the norms of different estimation errors.

we can depict the state trajectories of the three systems, as shown in Fig. 3, and conclude the convergence of all system states. From Fig. 4, all estimation errors are convergent and thus the desired objective is achieved.

V. Conclusion

We presented a general control framework for the event-triggered observer-based control problem of linear NCS, where the information communication is via multiple asynchronous networks. We first investigated the information transmission to design distributed event-triggered mechanisms, and then showed how to design the observers and controllers to ensure the system stability. In particular, the maximally allowable transmission periods were computed explicitly and further discussions were presented to show the generality of the proposed framework. Future work will be devoted to the nonlinear and stochastic cases and the case of NCS under attacks.

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