2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Simultaneous Compensation of Actuation and Communication Delays for Heterogeneous Platoons via Predictor-Feedback CACC with Integral Action

Amirhossein Samii[1] and Nikolaos Bekiaris-Liberis[1]

***Abstract*—We construct a predictor-feedback cooperative adaptive cruise control (CACC) design with integral action, which achieves simultaneous compensation of long, actuation and communication delays, for platoons of heterogeneous vehicles whose dynamics are described by a third-order linear system with input delay. The key ingredients in our design are an underlying predictor-feedback law that achieves actuation delay compensation and an integral term of the difference between the delayed (by an amount equal to the respective communication delay) and current speed of the preceding vehicle. The latter, essentially, creates a virtual spacing variable, which can be regulated utilizing only delayed position and speed measurements from the preceding vehicle. We establish individual vehicle stability, string stability, and regulation for vehicular platoons, under the control design developed. The proofs rely on combining an input-output approach (in the frequency domain), with derivation of explicit solutions for the closed-loop systems, and they are enabled by the actuation and communication delays-compensating property of the design. We demonstrate numerically the control and model parameters' conditions of string stability, while we also present simulation results, in a realistic scenario, considering a heterogeneous platoon of ten vehicles, for validating the performance of the design.**

I. Introduction

String stability is a crucial requirement and serves as an indicator of the safety and efficiency properties of platoons consisting of vehicles equipped with Adaptive Cruise Control (ACC) and CACC capabilities, see, for example, [8], [16], [18]. This property is imperiled when delays affect actuation, sensing, or communication of vehicular systems, see, for example, [4], [6], [10], [15], [17], [22], [23], [27]. In particular, communication delay, stemming from vehicle-to-vehicle (V2V) communication, poses a significant challenge to string stability, particularly when both actuation and communication delays coexist and they are large [7], [13], [16], [20], [25], [27].

For this reason, compensation of such delays becomes an essential mechanism that could be integrated with nominal, ACC/CACC laws. This integration may lead to a substantial enhancement in string stability properties of vehicular platoons. This is already evident in works that address small actuation delays only [11], [22], or small communication delays only [1], [7], [17], [19], or both [5], [10], [13], [16], [25], [26]. To address larger actuation or communication delays a predictor-based approach is required. Predictor-based control designs addressing long actuation and communication delays can be found in [3], [4], [6], [9], [14], [15], [21], [23] and [24], respectively; while [27] presents a predictor-based design to address both long actuation and communication delays. Here we complement [27], developing an alternative, less complex predictor-feedback CACC design with integral action, which enables development of a systematic/constructive stability and string stability analysis strategy, as well as it sheds further light on the mechanisms that allow simultaneous actuation and communication delays compensation.

In the present paper, we build upon the predictor-feedback CACC law from [4], which is constructed to compensate actuation delay only. While in [20] it is established that string stability of the CACC law from [4] is robust to small communication delay, a predictor-feedback CACC design addressing simultaneously, long actuation and communication delays is not available. The main reason for this unavailability is the fact that exact predictor states (over a prediction horizon equal to the actuation delay) cannot be constructed anymore in the presence of communication delays. Nevertheless, as we establish here, to achieve string stability it is not necessarily required to construct exact predictor states, but to rather cancel the effect of communication delay by aiming at regulation of spacing and speed of the ego vehicle, essentially, to the past (rather than the current) spacing and speed of the preceding vehicle.

Towards this end, we construct a linear, predictor-feedback CACC law augmented with an integral term of the difference between the preceding vehicle's, delayed, by an amount equal to the respective communication delay, speed and its current speed. We consider platoons of vehicles with heterogeneous dynamics described by a third-order linear system with actuation delay. The control design developed achieves $\mathcal{L}_2$ string stability with respect to speed/acceleration errors propagation (and with respect to spacing errors propagation as well, in the particular case of homogeneous vehicles). String stability is achieved relying on the following two mechanisms embedded in the control law developed– an underlying predictor-feedback CACC design that aims at actuation delay compensation and the integral term of the

The research in this paper was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to Support Faculty Members & Researchers" (Project Number: 3537/ORAMA).

[1]A. Samii and N. Bekiaris-Liberis are with the Department of Electrical & Computer Engineering, Technical University of Crete, Chania, 73100, Greece. Email addresses: asamii@tuc.gr and bekiaris-liberis@ece.tuc.gr.

difference between the delayed and current speed of the preceding vehicle (which, in fact, may be viewed as a type of spacing variable). The latter, essentially, modifies the objective of the original control law to aiming at regulating the spacing (and speed) of the ego vehicle accounting for the delayed, rather than the current, position and speed of the preceding vehicle. This, in a way, aligns the regulation objectives of the controller with the available information for the preceding vehicle's state at the current time, which is beneficial for string stability. Furthermore, the control design achieves stability of individual vehicles (that is a prerequisite for string stability) and zero, steady-state speed and spacing tracking errors, for a constant leader's speed. To achieve zero, steady-state spacing tracking error it is required to reduce the original time-headway by an amount equal to the respective communication delay, which imposes a condition that the desired time-headway is larger than the respective communication delay. This, in fact, is reasonably expected since the controller reacts to past rather than current information of the preceding vehicle's state. Nevertheless, the values of actuation and communication delays themselves are not restricted.

The proof of string stability relies on an input-output approach, deriving the respective transfer functions between the speed of the ego and the preceding vehicle, together with deriving explicit conditions on control/model parameters and time-headway. The proofs of individual vehicle stability and regulation rely on deriving explicit solutions of the closed-loop systems, capitalizing on the ability of the control design developed to achieve actuation and communication delays compensation. The analytical string stability conditions are also illustrated numerically. Furthermore, we present consistent simulation results of a platoon of ten vehicles, for the practical scenario in which a vehicle cuts in the platoon (described by considering initial condition deviations from equilibrium) and it subsequently performs an acceleration/deceleration maneuver.

II. Predictor-Feedback CACC for Heterogeneous Platoons with Both Actuator and Communication Delays

A. Vehicle Model and Nominal Delay-Free Design

a) Vehicle dynamics: We consider a heterogeneous string of vehicles (see Fig. 1) each one modeled by the following third-order, linear system with actuator delay that describes vehicle dynamics (see, e.g., [1], [23], [24], [25])

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$

$$\dot{v}_i(t) = a_i(t), \tag{2}$$

$$\dot{a}_i(t) = -\frac{1}{\tau_i}a_i(t) + \frac{1}{\tau_i}u_i(t-D), \tag{3}$$

$i = 1, ..., N$, where $s_i = x_{i-1} - x_i - l$ and x_i is the position of vehicle i and l is its length, v_i is vehicle speed, a_i is vehicle acceleration, τ_i is lag, capturing, engine dynamics, u_i is the individual vehicle's control variable, $D \geq 0$ is input delay, and $t \geq 0$ is time. Note that for the leading vehicle we assume similarly that it has the same type of third-order dynamics as the rest of the vehicles. The difference is that u_{l} acts as a time-varying, exogenous input rather than as feedback control input. We adopt the convention that $v_0 = v_l$ and $a_0 = a_l$ are the speed and acceleration of the string leader, respectively.

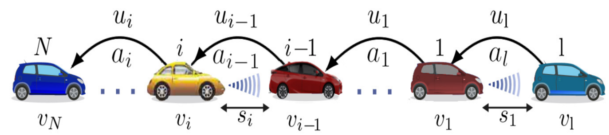

Fig. 1. Platoon of $N+1$ heterogeneous vehicles following each other in a single lane without overtaking. The dynamics of each vehicle $i = 1, ..., N$ are governed by system (1)–(3). Each vehicle can measure its own speed, the relative speed with the preceding vehicle, and the spacing with respect to the preceding vehicle. The control input and acceleration of each vehicle is communicated to the following vehicle via V2V communication.

b) Available measurements: For CACC platoons the measurements available to the ego vehicle i are its own spacing s_i, speed v_i, acceleration a_i, and control input u_i as well as the speed of the preceding vehicle v_{i-1}. It is possible to obtain this information through on-board sensors. Furthermore, the control input of the preceding vehicle, as well as its acceleration and speed are also available and are denoted by $u_{i-1,\mathrm{m}}$, $a_{i-1,\mathrm{m}}$, and $v_{i-1,\mathrm{m}}$ respectively. These measurements are transmitted from the preceding vehicle, through V2V communication. Due to the presence of communication delay these measurements are modeled by $v_{i-1,\mathrm{m}}(t) = v_{i-1}(t - D_{\mathrm{c},i-1})$, $a_{i-1,\mathrm{m}}(t) = a_{i-1}(t - D_{\mathrm{c},i-1})$ and $u_{i-1,\mathrm{m}}(\theta) = u_{i-1}(\theta - D_{\mathrm{c},i-1}), \theta \in [t-D, t]$, respectively, where $D_{\mathrm{c},i-1} \geq 0$, $i = 1, ..., N$, are communication delays[1].

c) Nominal control design: Without input delay, the following control strategy is constructed

$$u_i(t) = \tau_i \alpha_i \left(\frac{s_i(t)}{h_i} - v_i(t) \right) + \tau_i b_i (v_{i-1}(t) - v_i(t)) + \tau_i c_i a_i(t), \tag{4}$$

where $\alpha_i > 0$, $b_i > 0$, and $c_i \in \mathbb{R}$ are design parameters, and $h_i > 0$ is time-headway.

B. Communication Delay-Compensating Predictor-Feedback Control Design

The predictor-based control laws with communication delay compensation for system (1)–(3) are given by

$$u_i(t) = \frac{\tau_i \alpha_i}{h_i} q_{i,1}(t) - \tau_i(\alpha_i + b_i) q_{i,2}(t) + \tau_i b_i q_{i,3}(t) + \tau_i c_i q_{i,4}(t) + \frac{\tau_i \alpha_i}{h_i} \sigma_i(t), \tag{5}$$

$$\dot{\sigma}_i(t) = v_{i-1,\mathrm{m}}(t) - v_{i-1}(t), \tag{6}$$

$$q_i(t) = \mathrm{e}^{\Gamma_i D} \bar{x}_i(t) + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_i u_i(\theta)\, d\theta + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_{1i} u_{i-1,\mathrm{m}}(\theta)\, d\theta, \tag{7}$$

[1]The initial conditions $v_{i-1}(s) = v_{i-1_0}(s)$, $s \in [-D_{\mathrm{c},i-1}, 0]$, $a_{i-1}(s) = a_{i-1_0}(s)$, $s \in [-D_{\mathrm{c},i-1}, 0]$ and $u_{i-1}(s) = u_{i-1_0}(s)$, $s \in [-D - D_{\mathrm{c},i-1}, 0)$ are assumed to be continuous functions.

where

$$q_i = \begin{bmatrix} q_{i,1} & q_{i,2} & q_{i,3} & q_{i,4} & q_{i,5} \end{bmatrix}^{\mathrm{T}}, \tag{8}$$

$$\bar{x}_i = \begin{bmatrix} s_i & v_i & v_{i-1,\mathrm{m}} & a_i & a_{i-1,\mathrm{m}} \end{bmatrix}^{\mathrm{T}}, \tag{9}$$

$$B_i = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau_i} & 0 \end{bmatrix}^{\mathrm{T}}, B_{1i} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{\tau_{i-1}} \end{bmatrix}^{\mathrm{T}}, \tag{10}$$

$$\Gamma_i = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\tau_i} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{i-1}} \end{bmatrix}. \tag{11}$$

Implementation of control laws (5) requires measurements of the ego vehicle's spacing s_i, speed v_i, acceleration a_i, and control input u_i as well as the relative speed with the preceding vehicle. Utilizing on-board sensors, this information can be obtained. The preceding vehicle's speed $v_{i-1,\mathrm{m}}$, acceleration $a_{i-1,\mathrm{m}}$, and control input $u_{i-1,\mathrm{m}}$, which are also required, can be obtained through V2V communication that, however, they are subject to communication delay. It is important to note that we employ in our control design two different measurements for the preceding vehicle speed, one from on-board sensors v_{i-1} and one from V2V communication $v_{i-1,\mathrm{m}}$. Note that for control implementation the value of the communication delay is not needed, because $v_{i-1,\mathrm{m}}$ can be obtained directly from V2V communication. If $D_{\mathrm{c},i-1}$ is known to vehicle i, then one could, alternatively, employ $v_{i-1,\mathrm{m}}(t)$ via $v_{i-1,\mathrm{m}}(t) = v_{i-1}(t - D_{\mathrm{c},i-1})$.

III. String Stability Despite Actuation and Communication Delays

We start providing the definition of string stability employed. A platoon of vehicles indexed by $i = 1, ..., N$, following each other within one lane without overtaking, is $\mathcal{L}_2$ string stable with reference to speed errors if the following condition holds

$$\sup_{\omega} |G_i(j\omega)| \leq 1, \qquad i = 1, ..., N, \tag{12}$$

where $G_i(j\omega)$ denotes the transfer function between the i-th vehicle's speed and the speed of its preceding vehicle $i-1$ (see, e.g., [8], [10]). Note that the respective transfer functions, corresponding to acceleration states, are identical to those for speed states. We now state our main result.

Theorem 1: Consider a platoon of vehicles with heterogeneous dynamics modeled by (1)–(3), under control laws (5) with (6)–(11). Let the leading vehicle's speed be uniformly bounded and continuous. For any $D \geq 0$, $h_i > 0$, the platoon is $\mathcal{L}_2$ string stable with respect to speed errors propagation provided that the following conditions hold: $\frac{1}{\tau_i} - c_i > 0$, $\left(\frac{1}{\tau_i} - c_i\right)(\alpha_i + b_i) - \frac{\alpha_i}{h_i} > 0$, $\left(c_i - \frac{1}{\tau_i}\right)^2 - 2(\alpha_i + b_i) > 0$, and $\frac{2}{h_i}\left(c_i - \frac{1}{\tau_i}\right) + 2b_i + \alpha_i > 0$, $i = 1, ..., N$. Furthermore, all states remain bounded and, for a constant leading vehicle's speed, say v^*, regulation is achieved with $\lim_{t\to+\infty} a_i(t) = 0$, $\lim_{t\to\infty} v_i(t) = v^*$, and $\lim_{t\to+\infty} s_i(t) = h_i v^* - \lim_{t\to+\infty} \sigma_i(t)$, where $\lim_{t\to+\infty} \sigma_i(t) = \sigma_i(0) + \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds - D_{\mathrm{c},i-1} v^*$, $i = 1, ..., N$.

Proof: The proof can be found in Appendix A.

Remark 1: Communication delay is compensated by regulating the speed of the ego vehicle to match the speed of the preceding vehicle, also accounting for the respective communication delay. This regulatory action in the presence of communication delays alters the equilibrium point, resulting in the loss of zero, steady-state tracking error, as the controller aims to regulate $s_i + \sigma_i$ (rather than s_i) to $h_i v_i$ (this phenomenon also appears in, e.g., [24]). To address steady-state error when communication delay is known (e.g., as a known, average network delay), we can set $\sigma_i(0) = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$ and $h_i = h_{i,\mathrm{des}} - D_{\mathrm{c},i-1}$ (assuming $h_{i,\mathrm{des}} > D_{\mathrm{c},i-1}$, which is a reasonable requirement given that the controller reacts with $D_{\mathrm{c},i-1}$ delay and that, typically, the values of communication delay are much smaller that the desired headways), which results in a steady-state value for s_i to be $h_{i,\mathrm{des}} v^*$. Note that the choice for $\sigma_i(0)$ can be implemented at $t = 0$ using the past measurements of v_{i-1}, which are available. On the other hand, if $D_{\mathrm{c},i-1}$ is unknown, we can set $\sigma_i(0) = 0$. This results in a steady-state error for s_i of $D_{\mathrm{c},i-1} v^* - \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$. Nevertheless, it is worth noting that in practice, $D_{\mathrm{c},i-1}$ is typically much smaller than h_i, and thus, the steady-state error is expected not to be large, particularly when the initial condition for speed is close to the leader's equilibrium speed or, at least, an estimate $\hat{D}_{\mathrm{c},i-1}$ of actual communication delay $D_{\mathrm{c},i-1}$ is available.

Remark 2: The first two conditions of Theorem 1 come from the Routh-Hurwitz criterion and they are a prerequisite for string stability of the platoon. While the remaining two conditions are derived from the string stability criterion in speed error propagation. Feasibility of simultaneous satisfaction of the four conditions in Theorem 1 is explained noting, for example, that, since α_i and b_i are positive, the first three conditions can be satisfied with a proper choice of $\frac{1}{\tau_i} - c_i$ (via a proper choice of c_i); while the last condition can be satisfied, subsequently, with a proper choice of α_i and b_i.

IV. Numerical illustration of String Stability

In this section, we numerically analyze the string stability properties of the closed-loop system, according to Theorem 1. The transfer function $G_i = \frac{V_i}{V_{i-1}}$, which corresponds to the closed-loop systems described by equations (1)–(3), (5)–(11), along with choices (made for simplicity of illustration)

$$\alpha_i = -h_i p_i^3, \tag{13}$$

$$b_i = h_i p_i^3 + 3p_i^2, \tag{14}$$

$$c_i = \frac{1}{\tau_i} + 3p_i, \tag{15}$$

for some $p_i < 0$ and all i, is determined as

$$G_i(s) = \frac{V_i(s)}{V_{i-1}(s)} = \frac{-p_i^3 + p_i^2(p_i h_i + 3)s}{(s - p_i)^3} \mathrm{e}^{-sD_{\mathrm{c},i-1}}. \tag{16}$$

The numerical performance of the predictor-feedback CACC design (5) is showcased, focusing on $\mathcal{L}_2$ string stability definition in relation to (16). Fig. 2 depicts $\sup_\omega |G_i(j\omega)|$ as a function of p_i and h_i, where G_i is defined in (16). The conditions in Theorem 1, reduce to condition $h_i^2 p_i^2 + 6h_i p_i + 6 < 0$, which should hold to guarantee string stability. In Fig. 2, the region between the red curves indicates where condition $h_i^2 p_i^2 + 6h_i p_i + 6 < 0$ is satisfied.

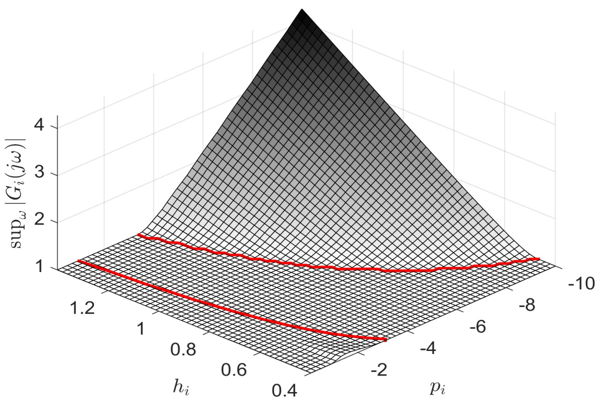

Fig. 2. The values of function $\sup_\omega |G_i(j\omega)|$ corresponding to transfer function (16) for heterogeneous vehicles, for different values of time-headway h_i and control parameter p_i.

V. Simulation Results

We demonstrate the performance of the actuation/communication delays-compensating predictor-feedback CACC law. We consider a heterogeneous platoon of ten vehicles in order to make the numerical example more practical. For a heterogeneous platoon of ten vehicles with third-order dynamics given by (1)–(3), we consider a case in which $\tau_i = 0.1s$, $i = 1, 2, 6, 9$; $\tau_i = 0.2s$, $i = 0, 3, 5$; and $\tau_i = 0.25s$, $i = 4, 7, 8$. The desired time-headways are $h_{i,\mathrm{des}} = 0.75$, $i = 3, 4, 7, 9$; $h_{i,\mathrm{des}} = 0.9$, $i = 2, 5$; $h_{i,\mathrm{des}} = 1.2$, $i = 1, 6, 8$. The actuation delay is set to $D = 0.7$ and communication delays are $D_{\mathrm{c},i-1} = 0.1$, $i = 1, 4, 6$; $D_{\mathrm{c},i-1} = 0.15$, $i = 5, 8$; $D_{\mathrm{c},i-1} = 0.2$, $i = 3$; $D_{\mathrm{c},i-1} = 0.25$, $i = 2, 9$; and $D_{\mathrm{c},i-1} = 0.35$, $i = 7$. Following Remark 1, we assume that the communication delay is known. To address steady-state error, we employ in (5) time-headways $h_i = h_{\mathrm{des},i} - D_{\mathrm{c},i-1}$ (all h_i, $i = 1, 2, ..., 9$, satisfy the conditions in Theorem 1; see Fig. 2) and choose $\sigma_{i_0} = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$ for all vehicles. Moreover, zero, steady-state spacing tracking errors are achieved as $\lim_{t\to+\infty} s_i(t) = h_{i,\mathrm{des}} v^*$, $i = 1, 2, ..., N$ (see Remark 1). We choose control gains according to (13)–(15) with $p_i = \frac{-2.5}{h_i}$, $i = 1, 2, ..., 9$ which satisfy the conditions in Theorem 1. Moreover, we consider a scenario in which $a_{i-1}(s) = 0, s \in [-D_{\mathrm{c},i-1}, 0]$ and $u_i(s) = 0$, $s \in [-D - D_{\mathrm{c},i-1}, 0)$ for each vehicle i. While we set $v_{i_0} = 15\left(\frac{m}{s}\right)$, $i = 1, 2, ..., 9$ and $v_{l_0} = \frac{4v_{i_0}}{5} = 12\left(\frac{m}{s}\right)$; $v_l(s) = 12, s \in [-D_{\mathrm{c},0}, 0]$

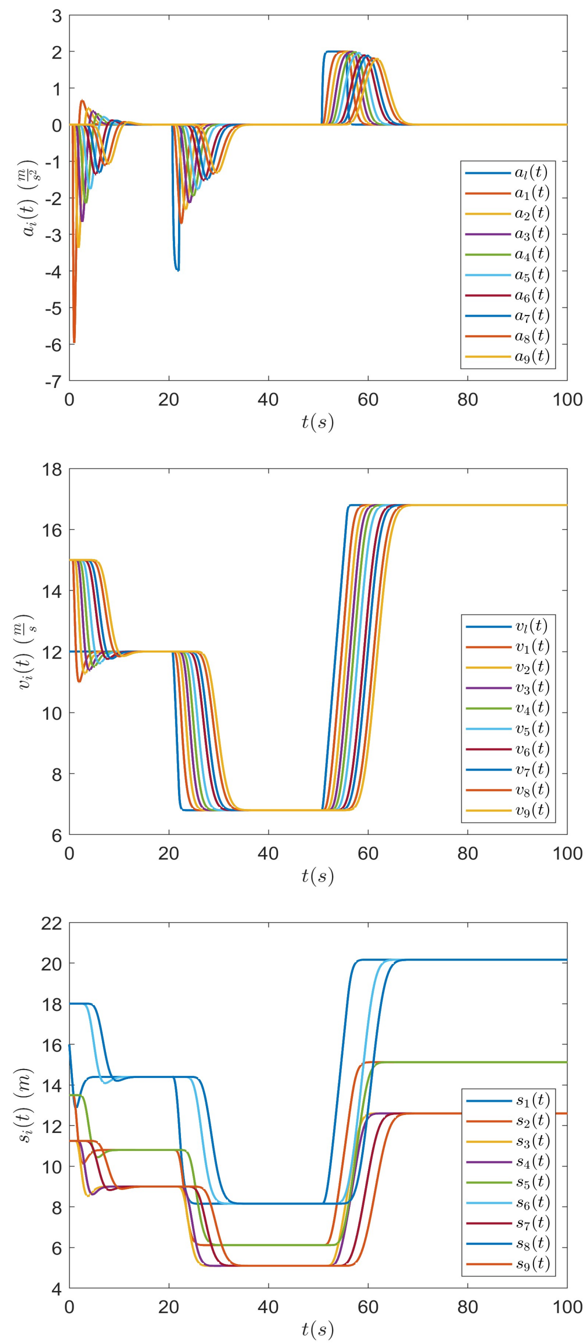

Fig. 3. Acceleration (top), speed (middle), and spacing (bottom) of ten vehicles, with dynamics described by (1)–(3), where $D = 0.7$, $\tau_i = 0.1s$, $i = 1, 2, 6, 9$; $\tau_i = 0.2s$, $i = 0, 3, 5$; and $\tau_i = 0.25s$, $i = 4, 7, 8$, following a leader that performs an acceleration/deceleration maneuver, under the CACC control laws (5), where $D_{\mathrm{c},i-1} = 0.1$, $i = 1, 4, 6$; $D_{\mathrm{c},i-1} = 0.15$, $i = 5, 8$; $D_{\mathrm{c},i-1} = 0.2$, $i = 3$; $D_{\mathrm{c},i-1} = 0.25$, $i = 2, 9$; and $D_{\mathrm{c},i-1} = 0.35$, $i = 7$. The desired time-headways are $h_{i,\mathrm{des}} = 0.75$, $i = 3, 4, 7, 9$; $h_{i,\mathrm{des}} = 0.9$, $i = 2, 5$; $h_{i,\mathrm{des}} = 1.2$, $i = 1, 6, 8$; while control parameters are chosen according to (13)–(15) with $p_i = \frac{-2.5}{h_i}$ and $h_i = h_{i,\mathrm{des}} - D_{\mathrm{c},i-1}$. Initial conditions are $v_{i_0} = 15\left(\frac{m}{s}\right)$, $i = 1, 2, ..., 9$, $v_{l_0} = \frac{4v_{i_0}}{5} = 12\left(\frac{m}{s}\right)$; $s_{i_0} = h_{i,\mathrm{des}} v_{i_0} = h_{i,\mathrm{des}} \times 15\ m$, $i = 2, 3, ..., 9$, $s_{1_0} = 16\ m$; $\sigma_{i_0} = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$ and $u_{i0} \equiv 0$, for $i = 1, 2, ..., 9$.

and $v_{i-1}(s) = 15, s \in [-D_{c,i-1}, 0]$, $i = 2, ..., 9$; $s_{i_0} = h_{\mathrm{des},i} v_{i_0} = h_{\mathrm{des},i} \times 15\ m$, $i = 2, 3, ..., 9$, $s_{1_0} = 16\ m$. Furthermore, the leading vehicle performs both deceleration and acceleration maneuvers. As depicted in Fig. 3, the speed and acceleration responses to these maneuvers by the leading vehicle exhibit characteristics devoid of oscillations and overshoot. Furthermore, it is interesting to note that all states diverged with the nominal control law (4) in the presence of actuation/communication delays.

We note that if communication delays are not known exactly then we could still employ the choices $h_i = h_{i,\mathrm{des}} - \hat{D}_{\mathrm{c},i-1}$ and $\sigma_{i_0} = -\int_{-\hat{D}_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s) ds$, with an estimate $\hat{D}_{\mathrm{c},i-1}$ of $D_{\mathrm{c},i-1}$. It is anticipated that steady-state, spacing tracking errors would remain small. The only case in which steady-state spacing errors would be large is when $D_{\mathrm{c},i-1}$ are both completely unknown and large which, in practice, may not be as realistic.

VI. Conclusions

In the present paper, we design a predictor-feedback CACC law with integral action, which achieves simultaneous actuation and communication delays compensation. We consider heterogeneous platoons with vehicles whose dynamics are described by a linear, third-order model with delayed actuation. The control design developed achieves string stability with respect to speed errors propagation, individual vehicle stability, and zero steady-state tracking errors. We provide constructive proof strategies that rely on a combination of an input-output approach and on deriving explicit solutions of the closed-loop systems. We demonstrate numerically the string stability conditions obtained and we provide simulation results for a platoon of ten vehicles, considering a realistic scenario of a vehicle cutting in the platoon and performing acceleration/deceleration maneuvers. As next step we aim at validating the performance of the design developed in simulation, using vehicles' trajectories from real traffic data.

Appendix A

Due to space limitations we provide only elements of the complete proof. In order to studying stability and string stability of speed error propagation, we first compute the transfer functions

$$G_i(s) = \frac{V_i(s)}{V_{i-1}(s)}, \quad i = 1, ..., N, \tag{A.1}$$

viewing as input the preceding vehicle's speed and as output the current vehicle's speed. Taking Laplace transform of the predictor states (7) we get

$$\begin{aligned} Q_i(s) = {} & \mathrm{e}^{\Gamma_i D} \bar{X}_i(s) + M_{1,i}(s) U_i(s) \\ & + M_{2,i}(s) U_{i-1}(s) \mathrm{e}^{-s D_{\mathrm{c},i-1}}, \end{aligned} \tag{A.2}$$

where

$$M_{1,i}(s) = (sI_{5\times5} - \Gamma_i)^{-1} \left(I_{5\times5} - \mathrm{e}^{\Gamma_i D} \mathrm{e}^{-sD}\right) B_i, \tag{A.3}$$

$$\begin{aligned} M_{2,i}(s) = {} & (sI_{5\times5} - \Gamma_i)^{-1} \\ & \times \left(I_{5\times5} - \mathrm{e}^{\Gamma_i D} \mathrm{e}^{-sD}\right) B_{1i}. \end{aligned} \tag{A.4}$$

Using the i-th vehicle's model (1)–(3), to express $\bar{X}_i(s)$ as a function of U_i and U_{i-1}, and computing $\mathrm{e}^{\Gamma_i D}$ and $(sI_{5\times5} - \Gamma_i)^{-1}$ we derive $\frac{U_i}{U_{i-1}}$, which, multiplying it by $\frac{s\tau_{i-1}+1}{s\tau_i+1}$, gives

$$G_i(s) = \frac{\left(b_i s + \frac{\alpha_i}{h_i}\right) \mathrm{e}^{-D_{c,i-1} s}}{s^3 + \left(\frac{1}{\tau_i} - c_i\right) s^2 + (\alpha_i + b_i) s + \frac{\alpha_i}{h_i}}. \tag{A.5}$$

String stability in $\mathcal{L}_2$ is guaranteed when $|G_i(j\omega)| \le 1$, for all $\omega \ge 0$. The condition is satisfied for $\omega = 0$ since $|G_i(0)| = 1$. With straightforward computations, we conclude that, under the conditions on the parameters a_i, b_i, c_i, τ_i, h_i of Theorem 1, relation $|G_i(j\omega)| \le 1$, for all $\omega \ge 0$, holds.

We next show that boundedness of all states is achieved. Using the delay-compensating property of predictor feedback (see e.g., [2]), we can write (5) as

$$\begin{aligned} u_i(t) = {} & \frac{\tau_i \alpha_i}{h_i} s_i(t+D) - \tau_i(\alpha_i + b_i) v_i(t+D) \\ & + \tau_i c_i a_i(t+D) + \frac{\tau_i \alpha_i}{h_i} \sigma_i(t+D) \\ & + \tau_i b_i v_{i-1,\mathrm{m}}(t+D). \end{aligned} \tag{A.6}$$

Thus, for $t \ge \max\left\{D, \max_i \{D_{\mathrm{c},i-1}\}\right\} = \bar{D}$ it holds

$$\begin{aligned} \begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} = {} & \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \frac{a_i}{h_i} & -(a_i + b_i) & c_i - \frac{1}{\tau_i} \end{bmatrix} \begin{bmatrix} s_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix} \\ & + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_{i-1}(t) + \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} v_{i-1,\mathrm{m}}(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{a_i}{h_i} \end{bmatrix} \sigma_i(t), \end{aligned} \tag{A.7}$$

$$\dot{\sigma}_i(t) = v_{i-1,\mathrm{m}}(t) - v_{i-1}(t). \tag{A.8}$$

The solution to (A.7), (A.8) is given as

$$\begin{aligned} \begin{bmatrix} s_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix} = {} & \mathrm{e}^{\bar{A}_i(t-\bar{D})} \begin{bmatrix} s_i(\bar{D}) \\ v_i(\bar{D}) \\ a_i(\bar{D}) \end{bmatrix} \\ & + \int_{\bar{D}}^{t} \mathrm{e}^{\bar{A}_i(t-s)} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_{i-1}(s) + \begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix} v_{i-1,\mathrm{m}}(s) \right. \\ & \left. + \begin{bmatrix} 0 \\ 0 \\ \frac{a_i}{h_i} \end{bmatrix} \sigma_i(s) \right) ds, \end{aligned} \tag{A.9}$$

$$\sigma_i(t) = \sigma_i(\bar{D}) + \int_{\bar{D}}^{t} \left(v_{i-1,\mathrm{m}}(s) - v_{i-1}(s)\right) ds, \tag{A.10}$$

where $\bar{A}_i = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \frac{a_i}{h_i} & -(a_i + b_i) & c_i - \frac{1}{\tau_i} \end{bmatrix}$. Under the conditions in Theorem 1, $\bar{A}_i$ always has eigenvalues with strictly negative real part, which means that the states s_i, v_i, a_i remain bounded, provided that σ_i and v_{i-1} are bounded. We establish next the boundedness of σ_1 under the assumption that the leader's speed, denoted as v_0, is bounded by, say, M_{v_0}, i.e., $|v_0(t)| \leq \mathrm{M}_{v0}$, for all $t \geq -D_{\mathrm{c},0}$. We derive

$$\sigma_1(t) = \sigma_1(0) + \int_{-D_{\mathrm{c},0}}^{0} v_0(s)ds - \int_{t-D_{\mathrm{c},0}}^{t} v_0(s)ds. \quad \text{(A.11)}$$

Considering the assumption on the leader's speed being bounded we can derive that

$$\int_{t-D_{\mathrm{c},0}}^{t} |v_0(s)|\, ds \leq \int_{t-D_{\mathrm{c},0}}^{t} \mathrm{M}_{v_0} ds = \mathrm{M}_{v_0} D_{\mathrm{c},0}. \quad \text{(A.12)}$$

Thus, considering (A.11), (A.12), it follows that σ_1 is uniformly bounded, with $|\sigma_1(t)| \leq \mathrm{M}_{\sigma_1}$, where $\mathrm{M}_{\sigma_1} = \sigma_1(0) + 2\mathrm{M}_{v_0} D_{\mathrm{c},0}$, $t \geq 0$. For showing boundedness of v_1 we proceed as follows. By using (A.9) for $i = 1$, with the fact that $\left|\mathrm{e}^{\bar{A}_1(t-\bar{D})}\right| \leq k_1 \mathrm{e}^{-\lambda_1(t-\bar{D})}$, for some positive constants k_1, λ_1 (because A_1 is Hurwitz, see, e.g., [12]) we get for $t \geq \bar{D}$

$$|v_1(t)| \leq \bar{v}_1(t), \quad \text{(A.13)}$$

where

$$\bar{v}_1(t) = r_{1,1}(t) + \frac{(1+b_1)\,k_1}{\lambda_1} \mathrm{M}_{v_0} + \frac{k_1 \alpha_1}{\lambda_1 h_1} \mathrm{M}_{\sigma_1}, \quad \text{(A.14)}$$

$$r_{1,1}(t) = k_1 \mathrm{e}^{-\lambda_1(t-\bar{D})} \left(\left|s_1\left(\bar{D}\right)\right| + \left|v_1\left(\bar{D}\right)\right| + \left|a_1\left(\bar{D}\right)\right| \right). \quad \text{(A.15)}$$

Relations (A.13)–(A.15) imply that v_1 is uniformly bounded with $|v_1(t)| \leq \mathrm{M}_{v_1}$, $t \geq -D_{\mathrm{c},1}$. In a similar manner given the boundedness of v_1, we conclude that σ_2 is uniformly bounded, and thus, from (A.9) that v_2 is also bounded. This pattern continues iteratively up to $i = N$. Consequently, we can deduce by induction that v_i and σ_i, $i = 1, ..., N$, are bounded. From (A.9) and the fact that the $\bar{A}_i$ matrices are Hurwitz we conclude that the system's states s_i and a_i are also bounded.

For constant leader's speed $v_0 \equiv v^*$, regulation is proved using (A.7), (A.9), and the fact that $\bar{A}_i$, $i = 1, ..., N$, are Hurwitz. In particular, we deduce that $\lim_{t\to+\infty} v_i(t) = \lim_{t\to+\infty} v_{i-1}(t) = v^*$. Moreover, we conclude that $\lim_{t\to+\infty} s_i(t) = \lim_{t\to+\infty} (h_i v_i(t) - \sigma_i(t))$, where (see (A.11) for $i = 1$)

$$\lim_{t\to+\infty} \sigma_i(t) = \sigma_i(0) + \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds - D_{\mathrm{c},i-1} v^*. \quad \text{(A.16)}$$

REFERENCES

[1] E. Abolfazli, B. Besselink, and T. Charalambous, "Minimum time headway in platooning systems under the MPF topology for different wireless communication scenario", *IEEE Transactions on Intelligent Transportation Systems*, in press, 2023.

[2] N. Bekiaris-Liberis and M. Krstic, *Nonlinear Control Under Nonconstant Delays*, SIAM, 2013.

[3] N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou, "Predictor-based adaptive cruise control design", *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, pp. 3181–3195, 2018.

[4] N. Bekiaris-Liberis, "Robust string stability and safety of CTH predictor-feedback CACC", *IEEE Transactions on Intelligent Transportation Systems*, vol. 24, pp. 8209-8221, 2023.

[5] B. Caiazzo, F. Di Rosa, D. Giuseppe Lui, A. Petrillo, and S. Santini, "Distributed resilient control for autonomous connected vehicles platoon over uncertain V2X communication links", *IEEE International Conference on Intelligent Transportation Systems*, Bilbao, Spain, 2023.

[6] B. Caiazzo, D. Giuseppe Lui, A. Mungiello, A. Petrillo, and S. Santini, "On the resilience of autonomous connected vehicles platoon under DoS attacks: a predictor-based sampled data control", *IEEE International Conference on Intelligent Transportation Systems*, Bilbao, Spain, 2023.

[7] M. di Bernardo, A. Salvi, and S. Santini, "Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays", *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, pp. 102–112, 2015.

[8] A. Bose and P. A. Ioannou, "Analysis of traffic flow with mixed manual and semiautomated vehicles", *IEEE Transactions on Intelligent Transportation Systems*, vol. 4, pp. 173–188, 2003.

[9] L. C. Davis, "Method of compensation for the mechanical response of connected adaptive cruise control vehicles", *Physica A: Statistical Mechanics and its Applications*, vol. 562, paper no. 125402, 2021.

[10] J. I. Ge and G. Orosz "Dynamics of connected vehicle systems with delayed acceleration feedback", *Transportation Research Part C: Emerging Technologies*, vol. 46, pp. 46–64, 2014.

[11] S. Huang and W. Ren, "Autonomous intelligent cruise control with actuator delays", *J. Intelligent & Robotic Syst.*, vol. 23, pp. 27-–43, 1998.

[12] H. K. Khalil, *Nonlinear control*, Pearson, 2015.

[13] F. Ma, J. Wang, S. Zhu, S. Gelbal, Y. Yang, B. Aksun-Guvenc, and L. Guvenc, "Distributed control of cooperative vehicular platoon with nonideal communication condition", *IEEE Transactions on Vehicular Technology*, vol. 69, pp. 8207–8220, 2020.

[14] T. G. Molnar, A. K. Kiss, A. D. Ames, and G. Orosz, "Safety-critical control with input delay in dynamic environment," *IEEE Transactions on Control Systems Technology.*, vol. 31, pp. 1507–1520, 2023.

[15] T. G. Molnar, W. B. Qin, T. Insperger, and G. Orosz, "Application of predictor feedback to compensate time delays in connected cruise control," *IEEE Trans. Intel. Transp. Syst.*, vol. 19, pp. 545–559, 2018.

[16] S. Öncü, J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Cooperative adaptive cruise control: Network-aware analysis of string stability", *IEEE Trans. Intell. Transp. Syst.*, vol. 15, pp. 1527–1537, 2014.

[17] A. Petrillo, A. Salvi, S. Santini, and A. S. Valente, "Adaptive multi-agents synchronization for collaborative driving of autonomous vehicles with multiple communication delays", *Transportation Research Part C: Emerging Technologies*, vol. 86, pp. 372–392, 2018.

[18] J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Lp string stability of cascaded systems: application to vehicle platooning", *IEEE Transactions on Control Systems Technology*, vol. 22, pp. 786–793, 2014.

[19] A. Salvi, S. Santini, and A. S. Valente, "Design, analysis and performance evaluation of a third order distributed protocol for platooning in the presence of time-varying delays and switching topologies", *Transportation Research Part C: Emerging Technologies*, vol. 80, pp. 360–383, 2017.

[20] A. Samii and N. Bekiaris-Liberis, "Robustness of string stability of linear predictor-feedback CACC to communication delay", *IEEE International Conference on Intelligent Transportation Systems*, Bilbao, Spain, 2023.

[21] M. Wang, S. P. Hoogendoorn, W. Daamen, B. van Arem, B. Shyrokau, and R. Happee, "Delay-compensating strategy to enhance string stabilityof autonomous vehicle platoons", *Transportmetrica B: Transport Dynamics*, vol. 6, pp. 211–229, 2016.

[22] L. Xiao and F. Gao, "Practical string stability of platoon of adaptive cruise control vehicles", *IEEE Transactions on intelligent transportation systems*, vol. 12, pp. 1184–1194, 2011.

[23] H. Xing, J. Ploeg, and H. Nijmeijer, "Smith predictor compensating for vehicle actuator delays in cooperative ACC systems", *IEEE Transactions on Vehicular Technology*, vol. 68, pp. 1106–1115, 2018.

[24] H. Xing, J. Ploeg, and H. Nijmeijer, "Compensation of communication delays in a cooperative ACC system", *IEEE Transactions on Vehicular Technology*, vol. 69, pp. 1177–1189, 2019.

[25] H. Xing, J. Ploeg, and H. Nijmeijer, "Robust CACC in the presence of uncertain delays", *IEEE Transactions on Vehicular Technology*, vol. 71, pp. 3507–3518, 2022.

[26] Y. Zhang, Y. Bai, J. Hu, and M. Wang, "Control design, stability analysis, and traffic flow implications for cooperative adaptive cruise control systems with compensation of communication delay", *Transportation Research Record*, vol. 2674, pp. 638-–652, 2020.

[27] Y. Zhang, Y. Bai, J. Hu, D. Cao, and M. Wang, "Memory-anticipation strategy to compensate for communication and actuation delays for strings-stable platooning", *IEEE Transactions on Intelligent Vehicles*, vol. 8, pp. 1145–1155, 2022.