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Adaptive optimal control of membrane processes to minimize fouling

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Abstract— **This paper presents an adaptive optimal control algorithm to minimize membrane fouling and energy to deal with model uncertainty and disturbances in the characteristics of the water to be filtered. The Membrane BioReactor under interest is operated at constant flux and the objective is to minimize energy requirements such that the quantity of water filtered over a given period of time t_f equals p^*. The algorithm is based on the iterative application of the optimal strategy initially proposed in [1] to account on input water characteristics changes. With such a modified control approach, it is shown in simulations that the ratios between filtration and backwash time periods adapt over the time to take account on unknown water input variations.**

I. Introduction

Membrane BioReactors (MBR) are more and more used in the wastewater industry due to their high treatment performance and the recent and continuous decrease in their investment and functioning costs, [2]. However, their main drawback remains the risk for membrane fouling which must be managed. A number of strategies and technologies have been proposed to deal with this problem for both systems functioning at constant flux (and variable TransMembrane Pressure) or at variable flux (and constant TMP). Among others, methods based on models, and notably optimal controls have shown to exhibit very interesting results in practice, *cf.* for instance [3-5]. With respect to other methods, these approaches are interesting because they allow to quantify, for the simplified models used, how far the actual functioning evolves from the theoretical optimum.

Dealing with such model-based approaches require using models of MBR. Such models can be obtained from data (as in our approach) or from mass-balance models coupling biokinetics models (based on the well-known ASM models) with models describing the dynamics of membrane fouling, *cf.* for instance [6]. In this work, such detailed models are used for simulating the actual system (as a "virtual process") while a simple control design model is obtained using the data generated with the detailed model. In such a way, the detailed model is seen as a virtual process that can be used to evaluate the control approaches and their robustness prior to apply them in real life.

This paper is organized as follows. First, the optimization problem when the systems inputs are constant is recalled together with its general solution. Then, its adaptive form to deal with varying inputs is proposed. For testing the adaptive optimal control, a virtual process coupling a biological process model with a model describing the fouling dynamics is proposed. Then, the proposed control algorithm is applied to this virtual process: the results obtained are compared to those obtained assuming constant inputs. Finally, conclusions and perspectives are drawn.

II. Optimal Control of Filtration Systems

A. Optimal time control with a constant input flux

In this section, we recall the optimal control problem that has been solved in [1]. Let us consider a membrane system operated at constant flux. It is supposed that the dynamic of the system may be characterized by a hidden variable $x(t)$ – for the sake of simplicity let interpret it as the quantity of matter attached onto the membrane - this dynamic being modeled as:

$$\dot{x} = \frac{1+u}{2} f_p(x) - \frac{1-u}{2} f_r(x) \qquad (1)$$

where u in [-1,1] is the control (by convention, u=1 during filtration phase and u=-1 in backwash phase).

The dynamic of the water produced over time is given by:

$$\dot{p} = \frac{1+u}{2} L_p - \frac{1-u}{2} L_r \qquad (2)$$

where L_p and L_r are the fluxes during filtration and backwash respectively.

It was further assumed that the total energy demand is the sum of the energy needed during the filtration phase and the one required during the backwash phase: this energy, denoted E_T depends on $x(t)$ and is expressed by (3):

$$E_T(x_0, u(.)) = \int_0^{t_f} \left(\frac{1+u}{2} l_p(x(t)) + \frac{1-u}{2} l_r(x(t)) \right) dt \qquad (3)$$

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where x_0 is the initial state while l_p and l_r are functions modeling the required energy during the filtration and the backwash phases, respectively.

Under appropriate general hypotheses and using the Pontryagin Maximum Principle, Aichouche *et al*. solved analytically the problem of minimizing the total energy E_T to attain a given quantity of water treated at the free final time t_f in computing the best sequence of filtration and backwash phases, and their optimal lengths for the specific following functions (available studies show that first-order functions are good candidates, [1]):

$$f_p = a_p x + b_p$$

$$f_r = a_r x$$

$$l_p = c_p x + d_p$$

$$l_r = c_r x + d_r$$

Formally speaking, the following problem was solved:

$$\inf_{u(\cdot)} \int_0^{t_f} \left(\frac{1+u}{2} l_p(x(t)) + \frac{1-u}{2} l_r(x(t)) \right) dt$$

subject to (1) and (2) with $u \in [-1,1]$ and where t_f is the first entry time in the target :

$$T = \{(x,p) \ s.t. \ p \geq p^*\}$$

where p^* is the desired quantity of water to be treated at t_f.

In the most general case, it was shown that the optimal control consists in first applying either filtration or backwash (depending on the position of the initial condition x_0 with respect to a specific value $\underline{x}$, which only depends on the parameters of the model) until a singular arc is attained (corresponding to a specific value $\underline{u}$), then stay on this singular arc until a specific instant (which also only depends on the model parameters), and finally apply a filtration phase until the desired quantity of water p^* be attained. Notice that such a strategy involves a singular arc: it means that the system is assumed to be able to function at a value of u which is neither -1 (backwash) nor +1 (filtration). Since a filtration system can only function physically in these two distinct modes, applying a singular arc in practice on a given period of time is equivalent in fixing the filtration over backwash times ratio and alternating between these two controls as often as possible. This theoretical optimal strategy is illustrated in Figure 1.

Remark: Figure 1 presents the very general optimal control synthesis for the problem considered. Notice however that for some very specific initial conditions, the optimal control consists in switching from -1 to +1 instead of staying on the singular arc (cf. [1]). This case corresponds to the yellow curve, which is called a "switching curve" in optimal control theory. In practice, this case is very particular and in most cases does not occur, unless at initial time the variable p is relatively close to p^* while x is already large, which makes not really sense in real applications.

More precisely, for the functions (4-7) it was shown that the optimal control parameters to minimize E_T are given by:

$$\underline{x} = \left(b_p L_r + \frac{\sqrt{b_p^2 L_r^2 + \frac{a_r b_p (d_p L_r + d_r J_v) + b_p^2 c_r L_r}{a_r c_p - a_p c_r}}}{a_p L_r + a_r L_p} \right) \tag{8}$$

$$\underline{u} = \frac{-(a_p + a_r)\underline{x} + b_p}{(a_r - a_p)\underline{x} + b_p} \tag{9}$$

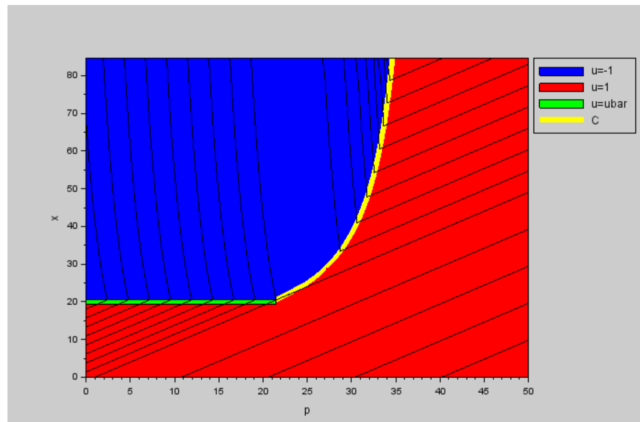

Figure 1. Optimal synthesis for the considered parameters in the $(p; x)$ plane. The singular arc is in green and the switching curve in yellow (from [1])

B. Main contribution: an adaptive strategy to deal with uncertainty and input disturbances

Notice that the above recalled optimal control parameters (the ratio of filtration/backwash phase lengths and the switching instant) only depend on the initial condition and on the model parameters. It should be underlined here that these parameters are supposed to be constant. Of course, functions f_p and f_r in (1) do depend on input concentrations. In other words, the proposed control is feasible and indeed optimal as long as the input water characteristics are constant. Thus, such a strategy is of interest when the filtration system is considered as an unitary separation system. However, it is the rule rather than the exception that input water characteristics change in time, notably its concentration in total suspended solids (TSS). It is notably the case when the filtration system is coupled with a biological process and form a MBR treatment process. This situation was well motivated in [6] where it is mentioned: "[...] such a system cannot be modelled by a "biological compartment" followed by a model describing the physical behavior of the membrane: the coupling of both must be necessarily taken into account to finally come up with what will be named hereafter an "integrated model". Thus, when the inputs of the whole system vary with time, the parameters of the functions f_p and f_r in (1) would also change with time. This makes most of the optimal control approaches developed in the literature for the optimal control of MBR – and that are often applied in open loop - not very robust with respect to process inputs.

To deal with these characteristics, the following adaptive optimal control is proposed. This adaptive control is still based on the previously recalled algorithm: however, instead of computing control parameters once and for all, model – and control parameters - are re-identified after each filtration/backwash sequence assuming the TMP is measured online. That way, the control continuously adapts control parameters to the actual fouling dynamics in case of input changes. It is an important step forward with respect to the initial work proposed in [1] since instead of an open-loop control, the optimal control works now in closed-loop.

The hereabove described algorithm is modified as follows:

- The initialization of the control is unchanged: a first identification is realized using data available during one (or more) filtration/backwash sequence(s). Then, based on the identified kinetics, the computation of the optimal control parameters is realized and one obtains in particular $\underline{x}$. At that instant, we apply $u_0 = +1$ (filtration) if $x < \underline{x}$ or $u = -1$ (backwash) if $x > \underline{x}$ until $x = \underline{x}$ (and we denote t_e the corresponding instant);
- At t_e, using the last available data, we identify again the model dynamic and compute a new control u_1. To allow for the system to attain a pseudo steady state, the new computed control is applied for a number of cycles, typically 3 (and we denote t_c one cycle length);
- From that time (t_e+3t_c) until the time at which the objective will be attained (a given quantity of treated water has been produced), we i) re-identify the dynamics of the system, ii) re-identify control parameters and iii) apply it to the system after each cycle period.

This algorithm is schematically represented in Figure 2:

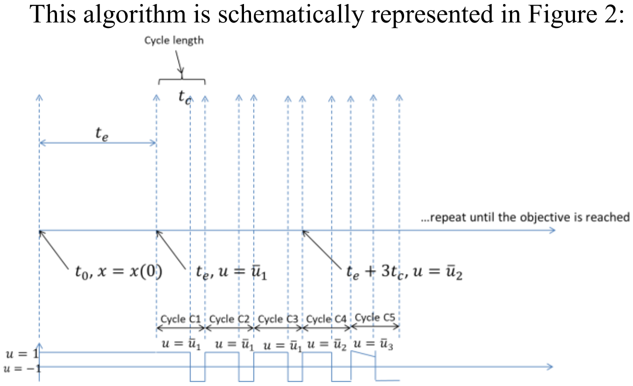

Figure 2. An adaptive optimal control of the filtration membrane system to adapt the optimal control in the presence of input variations

Remark: Notice that the variations of the parameters over the time are not known in advance. Therefore, the optimization problem under uncertainty could be defined under some statistical knowledge of the uncertainty and then consists in optimizing in average, or as a robust optimization "in the worst case", which might be very conservative. Here, we implicitly assume that the variations of the parameters are slow so that we expect that applying the optimal feedback law as if the parameters were constant would not be so far from the optimal solution as if the variations of parameters were known in advance. Of course, we do not have a mathematical proof of this property, but we show the efficiency of the approach as a proof of concept. Let us underline that this approach is possible because we have a control strategy in feedback loop.

To test this new adaptive algorithm, a model coupling the biological and fouling dynamics is necessary. We present such a coupled model in the following section inspired from the model initially proposed in [7] and further validated by [8].

II. A VIRTUAL PROCESS: COUPLING A BIOLOGICAL PROCESS MODEL WITH A FILTRATION MODEL

A. Model of a virtual Anaerobic Membrane BioReactor

To simulate the functioning of a biological system, we chose to couple a biokinetic model of the anaerobic digestion including the dynamics of the SMPs (*cf.* [9]) to a fouling model initially proposed in [7]. This model describes the dynamics of a simple two step model of the anaerobic digestion that has been validated several times on real data, including the dynamics of the so-called Soluble Microbial Products (this model is named hereafter "AM2b", *cf.* [8]). An interesting point is that this last model potentially allows us to take into account the influence of both the Total Suspended Solids that attach onto the membrane (the "cake") and the SMP (that are smaller components and that can block the pores of the membrane).

Based on a mass balance for the different model components, the dynamic equations of the coupled system, during the filtration phase, are given by:

$$\dot{X}_1 = (\mu_1(S_1) + \mu_{PMS}(SMP) \ldots \\ \ldots - k_{d1} - \frac{Q_w}{V} - C_{x1})X_1 \quad (10)$$

$$\dot{X}_2 = (\mu_2(S_2) - k_{d2} - \frac{Q_w}{V} - C_{x2})X_2 \quad (11)$$

$$\dot{S}_1 = \frac{Q_{in}}{V} S_{1in} - \left(\frac{Q_{out}}{V} + \frac{Q_w}{V}\right) S_1 \ldots \\ \ldots - k_1\mu_1(S_1)X_1 - C_{s1}S_1 \quad (12)$$

$$\dot{S}_2 = \frac{Q_{in}}{V} S_{2in} - \left(\frac{Q_{out}}{V} + \frac{Q_w}{V}\right) S_2 - k_3\mu_2(S_2)X_2 \ldots \\ \ldots + \left(k_2\mu_1(S_1) + b_2\mu_{PMS}(SMP)\right)X_1 - C_{s2}S_2 \quad (13)$$

$$S\dot{M}P = -M.SMP \ldots \\ \ldots + \left(b_3\mu_1(S_1) + k_{d1} - b_1\mu_{PMS}(SMP)\right)X_1 \ldots \\ +(b_4\mu_2(S_2) + k_{d2})X_2 - C_{SMP}SMP \quad (14)$$

$$\mu_1(S_1) = \mu_{1max}\frac{S_1}{K_1 + S_1} \quad (15)$$

$$\mu_2(S_2) = \mu_{2max}\frac{S_2}{K_2 + S_2 + \frac{S_2^2}{K_i}} \quad (16)$$

$$\mu_{PMS}(SMP) = \mu_{SMPmax}\frac{SMP}{K_3 + SMP} \quad (17)$$

$$M = \beta\frac{Q_{out}}{V} + (1-\beta)\frac{Q_w}{V} \quad (18)$$

where Q_{in} is the input flow rate (L/h), Q_{out} the permeate flow rate (L/h), X_1 the acidogenic biomass concentration, X_2 the methanogenic biomass concentration, S_1 the COD concentration, S_2 the VFA concentration, k_{d1} and k_{d2} are the mortality of X_1 and X_2, respectively (h^{-1}), $k_i, i = 1..3$ and $b_i, i = 1..4$ are yields coefficients (g/g), μ_1 the growth rate of the acidogenic biomass growing on S_1, μ_{PMS} the growth rate of acidogenic biomass growing on the SMP, μ_2 the growth rate of methanogenic biomass growing on S_2, μ_{max1} the maximum growth rate of X_1 on S_1 (h^{-1}), μ_{max2} the maximum growth rate of X_2 on S_2 (h^{-1}), μ_{PMSmax}, the maximum growth rate of X_1 on the SMP (h^{-1}), K_1 the semi-saturation constant associated to S_1, K_2 the semi-saturation constant associated to S_2, K_3 the semi-saturation constant associated to the SMP and K_i the inhibition constant associated to the consumption of S_2. The contribution of the soluble (S_T=S_1+S_2 and SMP) and particulate components (X_T=X_1+X_2) to the membrane fouling is supposed to be modeled by two different phenomena, that are the cake formation (mostly by particulate components but also by soluble components) and the pores blocking (mostly by the SMP but also by other soluble components) following the dynamics:

$$\dot{m} = \delta Q_{out}\left(\sum_i C_{Si}S_i + C_{SMP}SMP + \sum_i C_{Xi}X_i\right) \quad (19)$$

$$\dot{S}_p = \delta' Q_{out}(\beta SMP + \beta' S_T) \quad (20)$$

where C_{Si}, C_{Xi}, C_{SMP}, β and β' are weighted functions while δ and δ' are parameters used to balance the rates of the two fouling phenomena.

B. Dynamic of the system during the backwash

During backwash, the only phenomena of importance is supposed to be the detachment of the attached matter. Under the assumptions *i*) that the dynamics of biological phenomena are supposed to be very slow with respect to the dynamics of the detachment and ii) that the mass of matter detached is small with respect to the matter present in the reactor, the dynamic equations for the backwash period are simply given by:

$$\dot{m} = -f_m(\mathrm{m}) = -w.m \quad (21)$$

$$\dot{S}_p = -f_s(S_p) = -w'.S_p \quad (22)$$

where w and w' are the backwashing efficiencies related to cake layer and to pores blocking, respectively.

C. Open loop simulation: Effect of an input step concentration on the simulated state

In this section, the model is simulated using an open-loop strategy (e.g. with constant filtration and backwash time periods). It is simulated over a time horizon of 68 hours. The filtration and backwash periods lengths are 36 minutes and 3 minutes respectively. The model inputs together with the parameter values used for the simulations are given in the following Table. In Figure 3, the input substrate concentrations S_{1in} and S_{2in} vary from 90 to 180, and from 20 to 60, respectively, at time t=33h30min.

Even if the filtration/backwash length were initially optimally computed (assuming inputs were known), in the presence of such unknown input changes, the constant control will not be optimal anymore.

TABLE 1. INPUT AND MODEL PRAMETER VALUES USED FOR THE SIMULATIONS

Input/parameter	unit	Value
μ_{max1}, μ_{max2}, μ_{maxSMP}	(1/h)	1.2; 1.5; 0.14
K_1, K_2, K_3, K_i	(g/L)	10; 5; 3; 15
k_i, $(i = 1..3)$	-	25; 15; 16.08
b_i, $(i = 1..4)$	-	40; 0.6; 3; 1.3
k_{d1}=k_{d2}	(1/h)	0.18
Q_{in}, Q_{out}, Q_w	(L/h)	10; 8.5; 1.5
V	(L)	50
$C_{x1} = C_{x2}$ $C_{s1} = C_{s2} = C_{SMP}$	-	0.05; 0.005
β, β', δ, δ'	-	0.6; 0.04; 0.7; 0.06
w, w'	-	10; 1

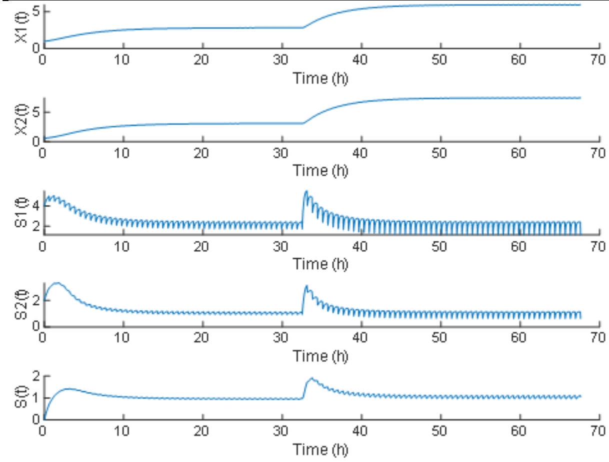

Figure 3. Open-loop simulations of the MBR with step input concentrations. S stands for S_p. "Oscillations" (high variance) observed on some signals are due to the succession of filtration/backwash cycles which have more important repercussions on some variables than on others.

III. NUMERICAL APPLICATIONS OF THE OPTIMAL CONTROL

In this section, we apply the adaptive optimal control presented in Section II.B to the model recalled in the previous section. The control designs are performed with unknown input concentrations. The available measurement is the variable m. In practice, this measurement is not possible. It is thus assumed here that this "hidden" variable is simply proportional to the TMP. This measurement is known to be very robust not really subject to noise. To concentrate on the main important point of the paper (comparisons of the performances of the three control strategies) the simulations shown were realized without any noise.

A. Adaptive optimal control of membrane fouling for constant input concentrations

Recall that even if the input substrate concentrations are constant, they can be unknown. The algorithm described in Section II.B is applied to the MBR. The inputs substrates S_{1in} and S_{2in} are constant and equal 90 and 20 respectively. The closed loop control system is simulated over 68h and 15 minutes. The first three filtration/backwash phase lengths, used for initial system identification are constant (t_F=36 minutes and t_{bw}= 3 minutes).

Figure 4 shows the automatic adaptation of the ratio of the length of the filtration phase over the length of the backwash phase (on the left Figure). On the right Figure, it is observed that the length of the backwash phase increases until the beginning of the 16th hour of the experiment and then stabilizes around 216 seconds. The stabilization observed is due to the convergence of the system towards a pseudo-steady state.

B. Adaptive optimal control of membrane fouling in the presence of input disturbances

Now, we simulated the closed-loop control of the MBR in the presence of input steps. At t=50, S_{1in} and S_{2in} vary from 90 to 180, and from 20 to 60, respectively. As can be seen in Figure 5, the control adapts to these changes in automatically increasing the backwash phase length.

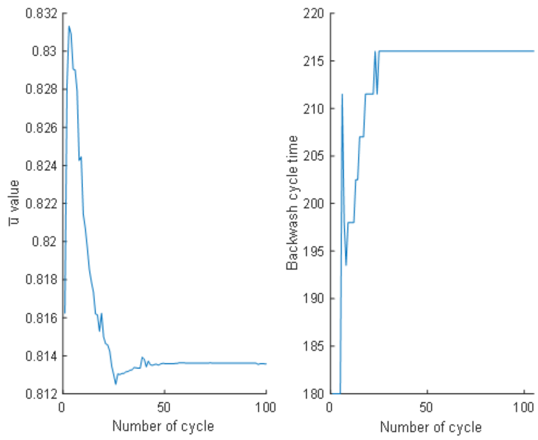

Figure 4. Adaptation of the optimal lengths of the filtration and backwash phases in the presence of unknown constant input substrate concentrations. On the left the ratio (t_F/t_{bw}), on the right t_{bw} (in seconds)

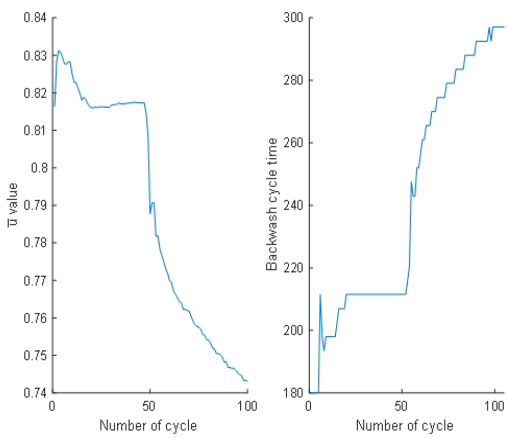

Figure 5. Adaptation of the optimal lengths of the filtration and backwash phases in the presence of an unknown magnitude step in the input substrate concentrations. The ratio (t_F/t_{bw}) is plotted on the left and t_{bw} on the right (in seconds)

IV. PERFORMANCE ANALYSIS

In order to evaluate the performance of the new adaptive control, we compare three different strategies:

- The first strategy consists in applying a constant filtration/backwash ratio as if inputs were not varying with time: it is a kind of industrial reference control (results plotted in blue);
- The second tested strategy consists in applying the original optimal control proposed in [1] without updating model parameters over the entire operation time (results plotted in red);
- The third control strategy consists in applying the actual adaptive optimal control approach (results plotted in green).

A. Performance comparaison for constant input concentrations

Again, consider constant inputs S_{1in}=90 and S_{2in}=20. The first open loop strategy is plotted in blue: it is obtained in applying constant filtration/backwash phase lengths (t_F=36 minutes and t_{bw}= 3 minutes) over a period of 132.6 hours. The second simulated strategy is open loop: the optimal control (the ratio of filtration phase length over backwash phase length) is constant over the whole time horizon, equal to the optimal value found after the very first identification of the system realized using the data acquired on the first three functioning phases. In the actual case, it was found to be u_i=0.82, corresponding to a backwash duration for each cycle i (i>3) equal to 207 seconds.

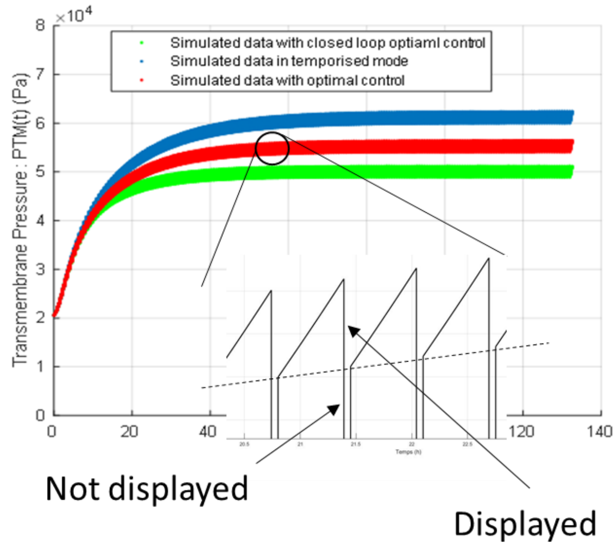

Figure 6. TMP simulations using the three control strategies with constant input concentrations (constant phase length open-loop control in blue, open-loop optimal control strategy in red and adaptive – closed-loop – control strategy in green)

Finally, the last strategy is the adaptive optimal control approach proposed in Section II.B. To compare the different strategies, we plotted in Figure 6 the TMP simulated for these three strategies (we only displayed the filtration and the very first instants of backwash dynamics for not having too large coloured areas) and in Table 1 both the total energy required by the system over the total time period (on the left) and the total quantity of water filtered (on the right).

Table 1: Total Energy (in W.s) and water volume (in m^3) for the three tested strategies (constant phase length open-loop control #1, open-loop optimal control strategy #2 and adaptive #3)

	#1	#2	#3
Energy ET	179800	175400	171800
Water volume	2.535	2.577	2.607

B. Performance comparison for varying input concentrations

Again, the three control strategies are simulated. However, instead of considering constant input strategies, a step in the input substrate concentrations arises at t=35h where S_{1in} and S_{2in} are varied from 90 to 180, and from 20 to 60.

The total energy requirements are given in Table 2 for each strategy.

Table 2: Total Energy (in W.s) and water volume (in m^3) filtered for the three tested strategies (constant phase length open-loop control #1, open-loop optimal control strategy #2 and adaptive #3)

	#1	#2	#3
Energy ET	182600	177800	168300
Water Vol.	2.441	2.481	2.491

C. Discussions

First, it is observed that the TMP is greater during the whole simulation time when the durations of the phases are not optimally controlled. This leads to the fact that The energy requirements when applying the open loop control strategy is greater than the two other control strategies. In addition, when using such an open loop strategy, not only the energy requirement is greater but the quantity of water filtered over the experimental time horizon is smaller. Second, notice that the closed-loop optimal control strategy gives better results than the open loop optimal control. It is due to the fact that using only the data acquired over the first three filtration/backwash periods is not enough to have a satisfying representation of the system over a long period of time. In fact, whatever the input substrate concentrations are varying or not, it is better to use the adaptive control strategy since it is able to better adapt to unknown inputs than the open loop control. In total, the adaptive optimal closed-loop control approach allows to treat 10% more than the optimal open-loop control approach while requiring about 10% less energy.

V. CONCLUSION

In this paper, a new closed-loop adaptive optimal control for membrane fouling control when the system operates at constant flux is presented. It is based on the iterative application of an optimal control strategy initially proposed in [1]. This new control allows to minimize the process energy requirements while accounting for unknown input substrate concentrations. Using simulations, it was shown that this new control is indeed able to minimize energy requirements while maximizing the volume of treated water over a given period of time in adapting the ratio of the filtration and the backwash phase lengths.

The actual control is based on the assumption that the fouling only depends on the formation of a cake onto the membrane. In practice, it is known that there exist other fouling mechanisms such as pore blocking. In such phenomena, the role of the SMP is recognized as being major. Thus, in the future, we will investigate the robustness of the proposed adaptive controller with respect to different kinds of fouling mechanisms and try to develop new optimal controllers for more detailed membrane fouling models.

VI. ACKOWLEDGEMENTS

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