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Passive Inerter-Based Network Self-Induced Oscillations Damping for Barge-Type Floating Offshore Wind Turbines

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Abstract— **This contribution analyses the influence of a passive inerter-based network on the 5MW NREL FOWT with a barge-type foundation when the system is subjected to the specific problem of self-induced oscillation. The concept of implementation of an inerter-based network combined with a standard tuned mass damper (TMD) in the nacelle is here presented with the objective of demonstrating the effectiveness of the introduced network in satisfactorily reducing the amplitude of the self-induced oscillations in comparison to the case of the FOWT system fitted with only a TMD. Major improvements in the overall structural stability were found with a suppression rate ranging from 49% to up to 86%, in the wind velocities of interest, when the phenomenon may occur. Moreover, it was shown that the inerter installed in the nacelle reduces the overall natural frequency of the system by over 56%.**

I. INTRODUCTION

The renewable energy sector is one of the fastest growing industries worldwide as the exhaustion of naturally existing resources such as fossil fuels or coal is greatly apparent. Wind energy and offshore wind technology gained relevance in the last decade as a response to climate change such as global warming.

Offshore wind technology is an environmentally friendly, cost-effective and reliable energy source. It is a mature technology due to several years of land-based structure development and investments. However, over 80% of offshore resources are positioned in the coastal waters where fixed foundations are no longer feasible [1], [2]. Floating offshore wind turbines (FOWTs) are energy-generating marine structures that represent a promising solution for the current renewable energy market challenges. One of their greatest advantages is the possibility of installation in deep and open waters where they take full advantage of the offshore natural resources available such as stronger and more constant winds.

The main control objectives for floating offshore wind turbines, among others, are maximisation of power production with simultaneous assurance of the system's stability. However, FOWTs pose new control challenges as the foundation is no longer fixed to the seabed. Besides the exposure to severe environmental conditions such as strong wind and waves, and various dynamic loads, FOWTs could experience self-induced instabilities. Self-induced vibrations are a natural phenomenon experienced by mechanical systems of rotary nature. This phenomenon is particular to FOWTs and it is a direct consequence of the blade-pitch control applied during the power production cycle. It also is influenced by the floating type of foundation.

One of the methods of controlling instabilities in wind turbines is the application of structural control. Structural control for wind technology was adapted from civil engineering with its main objective being the protection from excessive dynamic loadings coming from various external or internal sources [3]–[5]. In onshore and fixed-bottom offshore structures, the structural control is mostly implemented at the nacelle level and usually, such solutions are sufficient as the turbine foundation is fixed. Also until the early 2010s, most of the control in offshore wind turbines was focused on passive control approaches [6]–[10].

Tuned dampers (TDs) are the most widely used structural control devices. In the wind turbine, they can be mounted on either the nacelle, tower or support foundation. The most well-known TD is the tuned mass damper (TMD). TMD is a mechanical device consisting of a mass element, spring and damper. The device can be designed to be passive, semi-active or active. The application of the passive control requires no external energy input and it is based on the absorption of the vibrational energy followed by the energy dissipation which is done by the device's elements tuned to a certain mode of a structure [11], [12].

A mechanical device known as an inerter, developed in the early 2000s, operates by exerting an equal and opposite force to its terminals which is proportional to the relative acceleration between them. The device has the constant of proportionality referred to as inertance, expressed in units of kilogram [13], [14]. The inerter can be implemented as a structural control element [15], [16]. It can be done by the combination of classic TMD with an individual inerter, either in series or parallel, referred to as TMDI [17]–[20], or by combination with various inerter-based networks [21].

The load mitigation of barge-type FOWTs with the classical TMD enhanced by an inerter-based mechanical network was investigated [22] with seven mechanical networks tested. It was shown that the parallel connection of the spring and inerter provides the best improvement of the tower-top fore-aft displacement if only that parameter is considered. Moreover, the series connection of spring and inerter reduces

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the tower-top fore-aft deflection only in the case of that specific arrangement (spring and inerter directly connected).

A passive spring, damper, and an inerter-based parallel connected network were studied by [23] where the authors proposed a linear limited DOF model with a dynamic vibration absorber (DVA) installed in the nacelle of the barge-type FOWT. The main work objective was to reduce wave and wind loadings. It was concluded that passive structural control enhanced by an inerter provides a reduction in the tower-top fore-aft displacement and tower base fore-aft bending moment, and consequently, the reduction of the tower fore-aft damage equivalent load (DEL). In [24] the same authors studied the impact of other inerter-based configurations in the mitigation of wave-wind-induced loads in the barge FOWT. The study focused on the network effect on tower-top fore-aft deflection and the TMD working space due to these external loadings. A time-efficient parameter optimization method with a mixed-performance objective function was selected. It was concluded that there is a trade-off in the inerter-based configurations i.e., inerters require larger TMD stroke space such that for specific configurations it is impossible to provide simultaneous improvement to the tower top displacement and the TMD working space.

The effectiveness of the inerter was also tested for spar-type FOWT for vibration control [25] or in particular in tower load mitigation [26], [27]. It was shown that by inerter incorporation the system's tower fore-arf and side-to-side displacements are reduced.

In this work, the authors present an inerter-based network, that is added to the existing TMD in the nacelle in a floating offshore wind turbine with a barge-type platform. The contribution focuses on the comparison between the behaviour of the FOWT system with classic TMD against a model enhanced by a proposed inerter-based in the case when self-induced oscillations occur. It is demonstrated that a system with classic TMD cannot cope with the phenomenon presented, hence the main objective is to provide vibrational control for the FOWT system and guarantee stability through the incorporation of the introduced inerter-based network.

II. SELF-INDUCED OSCILLATIONS

The origin of the phenomena of self-induced oscillations is the control method used during an operating cycle in wind turbine power production. There are three operating regions (OPs) defined for the 5MW NREL wind turbine. In Region I ($0m/s$ to V_{cut-in}) there is no power production and the structure stays in the parked condition. In Region II (V_{cut-in} to V_{rated}), referred to as the below-rated wind region, up to the rated value of 11.4m/s the main control objective is the maximisation of power generation which is done by generator torque control. In Region III (V_{rated} to $V_{cut-off}$), referred to as the above-rated wind region, up to cut-off wind velocity of 25m/s the control objective is changed from maximum to optimal power generation. It is done by activation of the blade pitch control which works together with generator torque control [28].

Self-induced oscillations in floating offshore wind turbines are a direct consequence of the implementation of the blade pitch control in the above-rated wind velocity i.e., conventional onshore pitch-to-feather control results in the reduction of the steady-state rotor thrust in Region II and may cause the decrease of the overall damping of the turbine's platform. These instabilities originate from changes in the derivative of the thrust sensitivity dependent on the relative wind speed at the hub height that causes the overall damping coefficient to reach negative values [29].

The effects of negative damping in FOWT can be analysed by considering the problem as a rigid-body platform-pitch single-degree-of-freedom system [29]. The equation of motion governing this system is presented in (1).

$$(I_{mass}+A_{radiation})\ddot{\zeta}+(B_{radiation}+B_{viscous})\dot{\zeta}+ (C_{hydrostatic}+C_{lines})\zeta = L_{HH}T \quad (1)$$

The parameters in (1) are as follows: platform pitch angle ζ, platform pitch rotational velocity $\dot{\zeta}$, platform pitch rotational acceleration $\ddot{\zeta}$, pitch inertia associated with wind turbine and barge mass I_{mass}, added inertia (added mass) associated with hydrodynamic radiation in pitch $A_{radiation}$, damping associated with hydrodynamic radiation in pitch $B_{radiation}$, linearized damping associated with hydrodynamic viscous drag in pitch $B_{viscous}$, hydrostatic restoring in pitch $C_{hydrostatic}$, linearized hydrostatic restoring in pitch from all mooring lines C_{lines}, hub height L_{HH} and aerodynamic rotor thrust T.

Equation (1) can be rewritten in terms of the transnational displacement of the hub ($x = L_{HH} \times \zeta$) and the thrust sensitivity $\frac{\partial T}{\partial V}$.

$$\frac{I_{mass}+A_{radiation}}{L_{HH}^2}\ddot{x}+\underbrace{\left(\frac{B_{radiation}+B_{viscous}}{L_{HH}^2}+\frac{\partial T}{\partial V}\right)}_{C_x}\dot{x} +\frac{C_{hydrostatic}+C_{lines}}{L_{HH}^2}x = T_0 \quad (2)$$

Looking at (2), T_0 is the aerodynamic rotor thrust at the linearization point and V is rotor-disk-averaged wind speed. It is evident that the thrust sensitivity term is present in the overall damping coefficient C_x. If the rotor thrust decreases for increasing wind speeds ($\frac{\partial T}{\partial V} < 0$) in Region III, the overall damping coefficient of the system may become negative if

$$\left|\frac{B_{radiation}+B_{viscous}}{L_{HH}^2}\right| < \left|\frac{\partial T}{\partial V}\right|.$$

Self-induced oscillations are a unique challenge affecting floating offshore wind turbines that may lead to the appearance of structural instabilities. Hence, there is a need to design new control strategies for vibrational control in the nacelle/tower/platform subsystems [29]–[31].

III. MODEL DESCRIPTION

In this work, a 5MW NREL wind turbine with a barge-type foundation is used. The 5MW baseline wind model is a conventional three-bladed upwind variable-speed blade-pitch-to-feather-controlled turbine [32]. This benchmark wind turbine model is applied as the worldwide standard

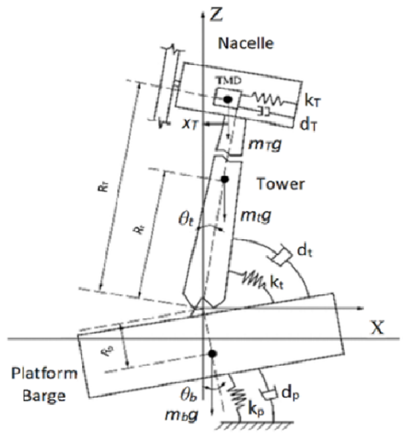

Fig. 1. Schematic diagram of the FOWT with barge-type foundation [40]

baseline model for both land-based and sea-based structures [29], [33]–[39].

A. Baseline Model with TMD

The dynamic equations of motion describing the floating offshore wind turbine system with a barge-type foundation (Fig. 1) are derived from first principles using Lagrange's equations of a non-conservative system with n generalized coordinates [40]. It is assumed that the baseline model is fully isolated from any external environmental factors and tower flexibility is represented by a linear rigid rotating beam hinged at the tower bottom. It can be also assumed that the pitching of the structure platform never exceeds 10 degrees [38], hence (3), represents a linearized 3DOFs model of a FOWT with a barge-type foundation.

$$\begin{cases} I_p\ddot{\theta}_p = -d_p\dot{\theta}_p - k_p\theta_p - m_p g R_p\theta_p + k_t(\theta_t-\theta_p) + d_t(\dot{\theta}_t-\dot{\theta}_p) \\ I_t\ddot{\theta}_t = m_t g R_t\theta_t - k_t(\theta_t-\theta_p) - d_t(\dot{\theta}_t-\dot{\theta}_p) - m_T g(R_T\theta_t - x_T) \\ -k_T R_T(R_T\theta_t - x_T) - d_T R_T(R_T\dot{\theta}_t - \dot{x}_T) \\ m_T\ddot{x}_T = k_T(R_T\theta_t - x_T) + d_T(R_T\dot{\theta}_t - \dot{x}_T) + m_T g\theta_t \end{cases} \tag{3}$$

In (3), the model parameters are as follows: platform inertia I_p, mass of the platform rigid body m_p, barge centre of mass R_p, platform flexibility k_p, platform torsion properties d_p, tower inertia I_t, mass of the tower rigid body m_t, tower centre of mass R_t, tower flexibility k_t, tower torsion properties d_t, mass of the TMD inside nacelle m_T, TMD centre of mass R_T, TMD spring coefficient k_T, TMD damping coefficient d_t, and gravitational acceleration g.

A free decay test of the 2-degrees-of-freedom (2DOFs) model, consisting of the platform pitch and tower top displacement (TTD) is performed to estimate model parameters and validate against the 5MW NREL benchmark. Some parameters are known and can be taken from the OpenFAST input file i.e., R_p= -0.281m, R_t= 64.2m, m_p= 5452000kg, m_t= 697460kg and g= 9.81m/s^2. Hence, the model parameters necessary to be estimated are I_p, k_p, d_p, I_t, k_t and d_t. The system parameter identification process was done by application of the Nelder-Mead simplex algorithm as described in [41] with the objective function being a sum of squares between the author's model tower top displacement and tower top displacement TTDspFA from the OpenFAST output file:

$$objective = \sum(TTD - TTDspFA)^2.$$

TABLE I
TMD MODEL PARAMETERS

Parameter	Estimation	Optimization
k_T $[kg\cdot m^2/s^2]$	4851.608	8947.108
d_T $[kg\cdot m^2/s]$	10813.0489	8961.0353

After the model validation, TMD parameter estimation is performed by identification with the OpenFAST 5MW NREL benchmark model when TMD in the nacelle is enabled. The TMD parameters to be estimated are k_T and d_T where the centre of the TMD mass is known as R_T= 90.6m and the TMD mass is fixed at m_T= 40000kg.

Once the parameters are known, the TMD spring and damper coefficients are optimised with the objective of minimizing the tower top displacement. The optimization algorithm used was the generalized pattern search (GPS) [42]. Pattern search is a direct search optimization method where the algorithm does not require information about the objective function gradient. The optimal solution is found by looking through search polls and a sequence of points that approach an optimal point is found [43]. Table I shows the results of the TMD estimation and optimisation.

B. Structure with Inerter-Based Network

The inerter-based network proposed in this work is an enhancement to an already existing, in the nacelle, classic TMD in the 5MW FOWT with a barge-type foundation. The main objective of the network is to suppress the self-induced oscillations appearing in the structure as a result of the loss of the platform damping as explained in Section II. The force produced by an inerter is proportional to the relative acceleration between two connected terminals [13]. The relationship is shown in (4) where b is the inertance with a_2 and a_1 are two corresponding accelerations.

$$F_{inerter} = b(a_2 - a_1) \tag{4}$$

The schematic diagram of the nacelle with the inerter-based network added is presented in Fig. 2, with newly introduced parameters being inertance b, spring stiffnesses k_1, k_2 and damper coefficient d. The mass of the new network m is kept unchanged and equal to m_T, hence the centre of the network mass also remains as R_T. Due to the nature of the network, an additional degree of freedom is introduced. The equations of motion of the FOWT with classic TMD derived in (3) are modified accordingly with the new relationship shown in (5) producing the 4th degree-of-freedom.

$$F_{inerter} + F_{spring,1} = F_{damper} + F_{spring,2} \tag{5}$$

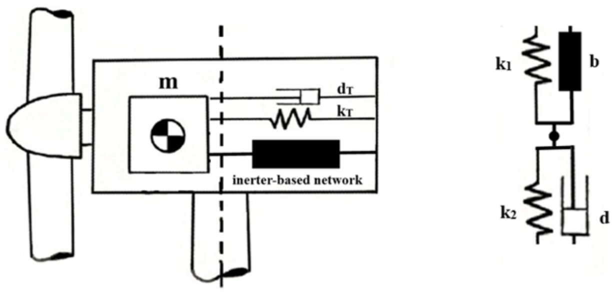

Fig. 2. Inerter-based network in the nacelle

TABLE II
INERTER-BASED NETWORK MODEL PARAMETER OPTIMIZATION

Parameters	Values
b $[kg]$	10000.73
d $[kg \cdot m^2/s]$	65472.46
k_1 $[kg \cdot m^2/s^2]$	10000.004
k_2 $[kg \cdot m^2/s^2]$	999.99
d_T $[kg \cdot m^2/s]$	10315.56
k_T $[kg \cdot m^2/s^2]$	1388.45

The unknown parameters of the inerter-based network are identified by the application of the generalized pattern search (GPS) algorithm. Table II shows the resultant inerter-based network parameters where initial guesses were obtained by the interior-point method (IPM) [44]. The previous TMD parameters k_T and d_T are included in the minimisation loop and the optimization is performed for a whole structural control system in the nacelle. Fig. 3 shows the resultant response plot.

IV. ANALYSIS OF MODEL RESPONSE UNDER SELF-INDUCED OSCILLATIONS

The response of the model with the inerter-based network compared to the classic TMD demonstrates the reduction

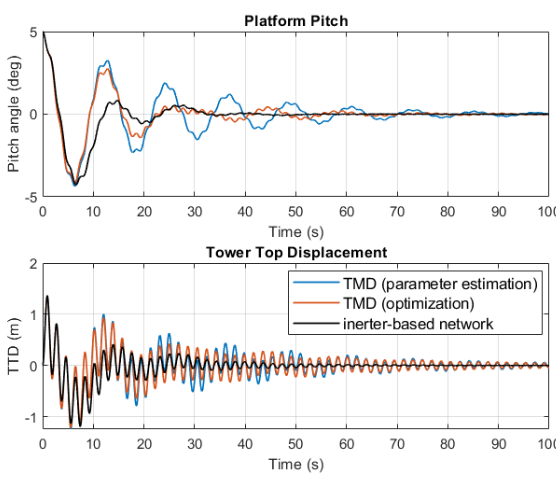

Fig. 3. Comparison free decay response of the model with inerter-based network vs model with TMD-only.

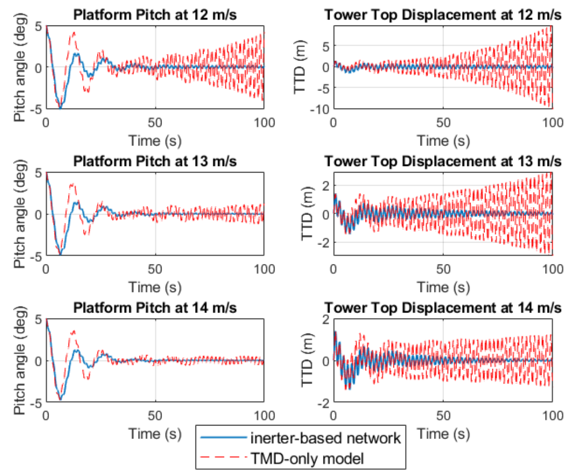

Fig. 4. Comparison response plots of the model with the inerter-based network vs model with TMD-only under self-induced oscillations.

TABLE III
SUPPRESSION RATE BETWEEN INERTER-BASED NETWORK AND CLASSIC TMD

Wind velocity	12m/s	13m/s	14m/s	15m/s
Suppression rate [%]	85.91	73.13	58.77	49.14

of the oscillation amplitude for both platform pitch and TTD. Following these results, the model enhanced by the network is tested against self-induced oscillations. Initially, the phenomenon is replicated by modifying the overall platform damping coefficient as explained in II based on the estimation of the damping ratios during the wind turbine power production cycle [29].

A. Time Domain Analysis

The performance of the model with the inerter-based network is simulated against the case of self-induced vibrations when the phenomenon is most prominent i.e., wind velocities from V_{rated}, when the change of the control objective takes place, up to approximately 14-15m/s. Fig. 4 shows the results of the simulations when the system responds due to the initial platform pitch of 5°.

It is visually evident that the introduced network provides a good oscillation amplitude reduction. The suppression rate is used to quantify the performance improvement, as shown in Table III, and can be defined as:

$$\frac{SD(TTD_{TMD}) - SD(TTD_{\text{inerter-based network}})}{SD(TTD_{TMD})} \times 100\%$$

where SD stands for standard deviation.

Moreover, the eigenvalue evolution of the model is assested throughout the full power production cycle (wind velocity range from 4m/s to 24m/s). Fig. 5 shows the resultant evolution where the red square markers represent the eigenvalues corresponding to velocity 4m/s and pink diamonds are the eigenvalues at 24m/s. The model with TMD-only under self-induced oscillations is indicated in blue with the

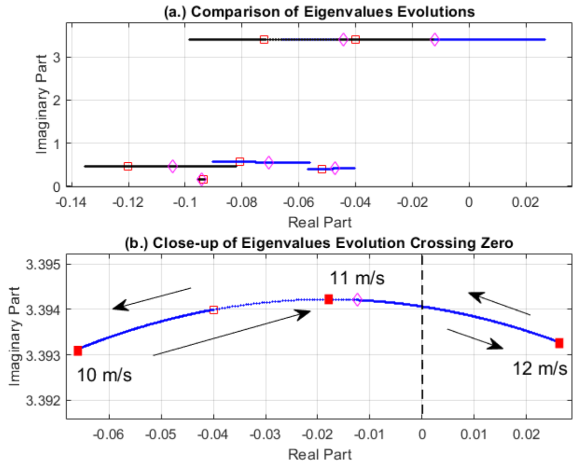

Fig. 5. (a.) Eigenvalues evolutions of classic TMD (blue line) and model enhanced by the interter-based network (black line); (b.) Close-up of the eigenvalue of interest that crosses the stability axis at V_{rated} when the system is fitted with classic TMD only.

TABLE IV
REDUCTION OF THE NATURAL FREQUENCY WITH INERTER-BASED NETWORK

Wind velocity	11m/s	12m/s	13m/s	14m/s	15m/s
Reduction TF_1 [%]	15.02	13.87	14.27	14.52	14.66
Reduction TF_2 [%]	56.25	56.81	56.62	56.49	56.43

model enhanced by the inerter-based network in black. It is evident from Fig. 5 that the black line does not cross the zero at the real axis which indicates that the proposed inerter-based network remains stable throughout the full wind turbine production cycle regardless of the appearance of self-induced oscillations compared to the model with TMD-only which becomes unstable as the result of the overall platform damping achieving negative values.

B. Frequency Domain Analysis

As the inerter-based network is introduced at the nacelle, it is of interest to analyse the changes in the frequency responses of the model. Fig. 6 shows the Bode Diagrams comparison at wind velocity 12m/s where the effects of self-induced oscillation are the most prominent on the system. $TF_1 = \frac{X_T(s)}{\Theta_t(s)}$ represents the relation between the network mass inside the nacelle and the rotation of the tower and $TF_2 = \frac{\Theta_p(s)}{X_T(s)}$ is the rotation of the platform with respect to the network mass element. To quantify the visible reduction of peaks on the Bode Diagrams, the natural frequency reduction for the model enhanced by the inerter-based network in comparison to the model with TMD only was obtained (Table IV) in the range of wind velocities of interest where self-induced oscillation occur. It is evident that the implementation of the inerter reduced the overall natural frequency of the system (TF_2) by more than half. The reduction of the natural frequency results in lessened oscillation. As a consequence, the amount of fatigue the structure experiences, due to cyclic stresses, can be considerably decreased.

V. CONCLUSIONS

In this paper, the benefits of using a passive inerter-based network for FOWT structural stability were presented with a special interest in the case of self-induced oscillations. A 4DOFs model was developed by introducing an additional DOF in the model with classic TMD installed in the nacelle. The presented structural control device which is an enhancement to the classic TMD was tested with the main objective of dampening the self-induced oscillations appearing at the wind turbine platform. The key highlights of this work can be summarised as follows:

- In a free decay test, the inerter-based network reduces the amplitude of oscillations of the tower top displacement and platform pitch.
- In the case of self-induced oscillations, the inerter-based network influences the dynamic behaviour of the system by reducing the oscillations amplitude of both tower top and platform pitch, up to 85.91% at wind velocity of 12m/s.
- The root loci analysis shows that the inerter-based network guarantees a stable response of the system throughout a full range of wind speeds, regardless of the occurrence of self-induced oscillations.
- The frequency domain analysis yields that the inerter-based network reduced the natural frequency of the model by over 56% when the relation between the wind turbine's platform and the network's mass element is considered and by up to 15% for the relation between the network's mass element and the wind turbine's tower.

It can be concluded that the incorporation of an inerter in the barge-type FOWT can benefit the general dynamic response of the structure through suppression of the un-

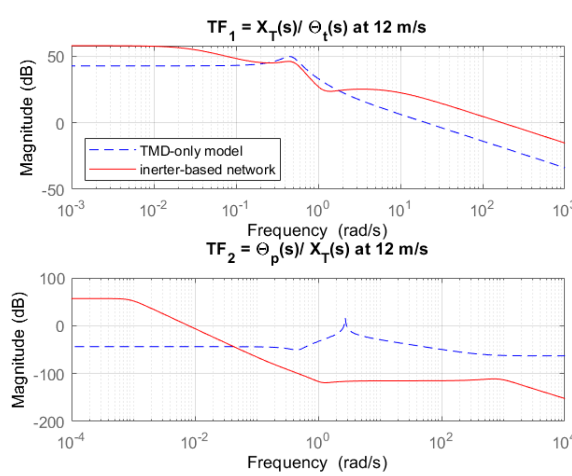

Fig. 6. Comparison of the Bode Diagrams of the model with TMD-only (blue dashed line) and enhanced by the inerter-based network (red solid line).

wanted oscillations appearing at the structure. In the case of an occurrence of self-induced oscillations, the proposed network ensures the stability of the system, reduces oscillation amplitude and also influences natural frequency.

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