2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Centralized and Distributed Economic Model Predictive Control in Water Distribution Networks

B. A. L. M. Hermans[1], C. S. Kallesøe[2,3], S. S. Rathore[2], and S. Weiland[1]

Abstract— **Cost optimal control in water distribution networks is considered. Focus is on distribution networks with tree like structures, that can be separated into sections composed of an elevated reservoirs (water towers), a pumping stations, and a consumer district. Given the structure of the network, a centralized and a distributed economic model predictive controller is developed. The later is expected to improve scalability and easy commissioning of the controller setup. Both control methods are based on first principle models. The controllers performance and effectiveness are tested on a scaled water distribution network and their performances are compared. The tests shows similar performance of the two proposed controllers, and both controllers effectively reduce the operational cost when compared to controllers that disregard energy prices.**

I. INTRODUCTION

Elevated reservoirs are often used in Water Distribution Networks (WDN's) to meet fluctuations in the demand, provide reserve supply in the case of fire and other emergencies, and to stabilize the pressure at the consumers [1]. The strait forward way of controlling the filling of these reservoirs is to pump water into the reservoirs whenever they are empty, without considering energy availability or cost. The transition to sustainable weather-dependent energy sources demands better coordination between energy supply and consumption. Optimal control in water distribution networks can contribute to this. To that end, [2] examen different relations between the water distribution network and the electrical grid, and propose pump scheduling method that enable grid support. In [3] energy prices are proposed as a mean to shape the consumption to the production, and the approach is exemplified on a WDN.

To operate WDN's within pressure bounds, optimal control methods, such as Model Predictive Control (MPC), has been proposed in the literature. Following [4], economic MPC (EMPC) is discussed as a control method for nonlinear systems. Central approaches for optimizing the operation of WDN are presented in [2] with focus on pump scheduling, and in [5] a control structure utilizing EMPC for planing the pump operation to varying electricity prices is considered. The operational cost of water distribution networks is reduced by letting the controller consider economic aspects such as energy cost in [6] and [7]. In our work a central control approach is compared to a decentralized architecture, where the communication structure for the decentralized architecture is designed based on the structure of WDN. Decentralized optimal control is considered in [8] where economic and distributed control is combined. Decentralized MPC with a hierarchical structure has been used for control of WDN in [9], and with bidirectional communication in [10], [11] and [12].

This work presents a distributed EMPC structure that minimized the operational cost for WDN of variable size, but with a tree like structure. Here, a tree like structure implies networks, where pump-reservoirs sections are connected in series. A WDN, where parts of the network has exactly this structure, is found in Barcelona [6]. A EMPC is designed along with a decentralised EMPC, where the later utilizes the tree like structure, to setup an efficient communication strategy with one directional communication only. For the EMPC designs, a non-convex optimization problem, derived from first principles modeling, is converted to a convex problem ensuring robust system operation. The EMPC algorithm is afterward decentralized, leading to a control setup that is expected to improve the scalability and enables the possibility for step-wise commissioning of the the control system. The network model and the water demands are expected to be known along with energy prices 24 hours into the future. Water demand prediction and model identification are presented in [7] for network sections similar to the ones considered in this work, and will not be considered further here. Both the centralized and the distributed EMPC's are tested on a scaled WDN, implemented in the lab environment described in [13]. These tests verify the robustness and usability of the approaches when applied to a real life system.

The main contributions of the paper are 1) the approximation of the WDN model that leads to a convex EMPC, 2) the communication structure derived from the approximated WDN model, and 3) the lab tests, showing that the approach works on an WDN emulation implemented in the Smart Water Lab [13].

The paper states, in Section II by introducing the considered WDN with tree like structure, including the derivation of a model utilized by the controllers presented in Section III. The smart water laboratory implementation and test results are presented in Section IV. The paper ends with concluding remarks in Section V.

II. WATER DISTRIBUTION NETWORK

The WDN considered in this work is fed by a single water treatment plant and consists of a system of pumps, pipes,

[1]Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands b.a.l.m.hermans@student.tue.nl, s.weiland@tue.nl
[2]Department of Electronic Systems, Aalborg University, Aalborg, Denmark {csk,ssr}@es.aau.dk
[3]Grundfos, Bjerringbro, Denmark ckallesoe@grundfos.com

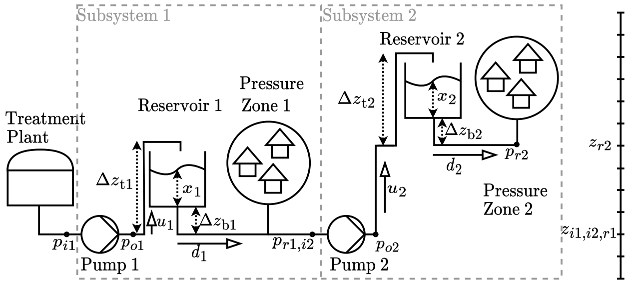


Fig. 1: Schematic representation water distribution network consisting of two subsystems.

and elevated reservoirs feeding civilized areas with different geodetic elevations, cf. Fig. 1. These geodetic levels are here denoted pressure zones. All reservoirs are filled from the top, ensuring tolerable water freshness at all times. One net demand flow per zone represents water distributed among consumers in a pressure zone.

A. Generalized Network Model

Graph theory is used to obtain a generalized system representation. The WDN is split into subsystems, each containing one pump, one piping section, one elevated reservoir, and is connected to one pressure zone, see Fig 1. The relation between the subsystems is described by a rooted tree graph $G = (\mathbb{V}, \mathbb{E})$. Let the WDN be composed by M subsystems, then the graph G has $M+1$ vertices related to the root node and the subsystems, and the M edges related to the connection between the subsystems. In the following, it is assumed that the root note represents an infinite water source. Let index $j \in \{1, \cdots, M\} \subset \mathbb{V}$ denote subsystems ($j = 0$ represents the root node). The index set of downstream subsystems connected to the output of subsystem j (j's children) is denoted $\mathbb{L}_j \subset \mathbb{V}$, while the source ($j$'s parent) is indexed using m_j. With this graph definition, the WDN in Fig 1 contains $M = 2$ subsystems. The absolute geodetic elevation at the inlet of subsystem j is $z_{\mathrm{i}j}$, and the elevation of its reservoir is $z_{\mathrm{r}j}$. The elevation difference between $z_{\mathrm{i}j}$ and the pipe outlet into the reservoir is denoted $\Delta z_{\mathrm{t}j}$, and $\Delta z_{\mathrm{b}j}$ is the elevation change between the bottom of the reservoir and the outlet of the subsystem. It is assumed that $z_{\mathrm{r}j} \geq z_{\mathrm{i}j}$, implying that water is always pumped from lower to higher areas separated by reservoirs and pumps. Let u_j $[\mathrm{m^3/s}]$ be the volumetric flow pumped by the pumping station of subsystem j. The water leaving subsystem j is the sum of the corresponding net water demand d_j and the pump flow $\sum_{l \in \mathbb{L}_j} u_l$.

B. Subsystem Model

Given initial conditions and inputs, the model should predict water levels, pressures, and flows inside the WDN for 24 hours. The expected system energy consumption is derived from these predictions. The model is derived based on [14].

1) Reservoir model: The dynamic water level x_j $[m]$ in reservoir j can be described by the first-order differential equation

$$\frac{dx_j}{dt} = \frac{1}{A_j}\left(u_j(t) - d_j(t) - \sum\nolimits_{l \in \mathbb{L}_j} u_l(t)\right) \tag{1}$$

where A_j is the reservoir surface area in subsystem j.

2) Pipe model: The pressure drop Δp_j [Pa] over the main pipe in subsystem j is approximated using

$$\Delta p = \lambda_j u_j^2 + \rho g \Delta z_j$$

where u_j is the pumping station flow, g $[\mathrm{m/s^2}]$ is the gravitational constant, ρ $[\mathrm{kg/m^3}]$ is the water density, and Δz_j is the elevation change between the inlet and outlet of the pipe. The flow-dependent pressure drop $\lambda_j u_j^2$ is based on the Darcy-Weisbach equation [14, ch 2] and is only considered in piping segments that feed reservoirs.

3) Pressure equations: Determining water pressure at the inlet $p_{\mathrm{i}j}$ [Pa] and outlet $p_{\mathrm{o}j}$ [Pa] of pump j is necessary to estimate power consumption. The outlet pressure $p_{\mathrm{o}j}$ is approximated using $p_{\mathrm{o}j} = \rho g(z_{\mathrm{r}j} - z_{\mathrm{i}j} + \Delta z_{\mathrm{t}j}) + \lambda_j u_j^2$ for positive flow $u_j > 0$. This approximation does not hold for $u_j = 0$. However, pressure is only used for determining the power consumption, which equals 0 when $u_j = 0$. The reservoir outlet pressure $p_{\mathrm{r}j}$ [Pa] is given by $p_{\mathrm{r}j} = \rho g(\Delta z_{\mathrm{b}j} + x_j)$ for $j \geq 1$. For $j = 0$ (the root node) $p_{\mathrm{i}0} = p_{air} = 0$.

4) State-space system: The reservoir dynamics of subsystem j is described by (1). The corresponding outputs $y_j \in \mathbb{R}^3$ contains observable nodal pressures and the water level in the reservoir $y_j = \begin{bmatrix} p_{\mathrm{o}j} & p_{\mathrm{r}j} & x_j \end{bmatrix}^\top$. Let the output map be $y_j(t) = Cx_j(t) + f_j(u_j(t)) + \gamma$, such that

$$y_j(t) = \begin{bmatrix} 0 \\ \rho g \\ 1 \end{bmatrix} x_j(t) + \begin{bmatrix} \lambda_j u_j^2(t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \rho g(z_{\mathrm{r}j}\text{-}z_{\mathrm{i}j} + \Delta z_{\mathrm{t}j}) \\ \rho g \Delta z_{\mathrm{b}j} \\ 0 \end{bmatrix}. \tag{2}$$

The state space is bounded by the reservoir sizes and pressure requirements for the zones. The input is bounded by pump limits, and the disturbance is bounded by pressure zone flow limits. This yields the feasible sets

$$\mathbb{X}_j = \{x \in \mathbb{R} \,|\, x_{j\min} \leq x_i \leq x_{j\max}\} \tag{3a}$$
$$\mathbb{U}_j = \{u \in \mathbb{R} \,|\, 0 \leq u_j \leq u_{j\max}\} \tag{3b}$$
$$\mathbb{D}_j = \{d \in \mathbb{R} \,|\, 0 \leq d_j \leq d_{j\max}\} \ . \tag{3c}$$

5) Power consumption: This project focuses on the effective energy fed into the WDN, meaning that electrical and mechanical pump efficiencies are not considered. Determining the pump pressure boost and assuming unidirectional flow, the pump power is approximated by $P_j = (p_{\mathrm{o}j} - p_{\mathrm{i}j}) q_{\mathrm{p}j}$, so that

$$P_j = \begin{cases} u_j y_{j,1} & \text{if } j = 1 \\ u_j(y_{j,1} - y_{m_j,2}) & \text{if } j \in \mathbb{V}' \end{cases} . \tag{4}$$

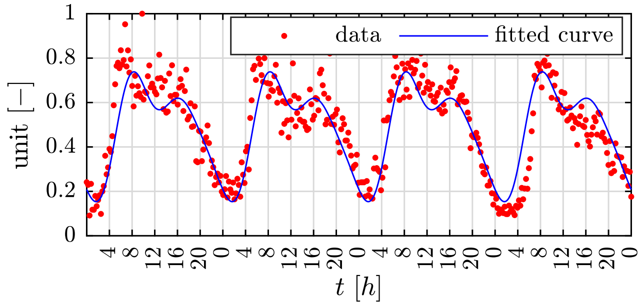


Fig. 2: Normalized demand profile fit (4/88 days)

C. WDN operations

1) Cost of energy: With today's fluctuating electrical energy market, energy prices can be negative. However, in this work the energy price $c(t) \in \mathbb{R}^+$, that is, the energy price is assumed to be positive and has an arbitrary unit.

2) Demand profile: The WDN must be able to deliver the maximum possible demand flow at any time. Data suggests that water demand exhibits noisy periodic behavior with peak demand in the morning and lowest demand at night. The minimal Fourier series order with which the desired periodicity can be described is 3. Based on 88 days of normalized data from a Danish water utility, and utilizing the function `fit` [15], the demand model is given by

$$\tilde{d}_j(t) = \begin{cases} 0 & \text{if} \quad x_j = 0 \\ \kappa_j \big(a_0 + \sum_{i=1}^{3} (a_i \cos(2\pi i t/T_f) \\ \qquad + b_i \sin(2\pi i t/T_f)\big) & \text{if} \quad x_j > 0 \end{cases} \tag{5}$$

where $\kappa_j = {}^{q_{cj,\text{mean}}}/{a_0}$ is a scaling factor dependent on the mean demand $q_{cj,\text{mean}}$. The normalized data-fit is plotted for four out of eighty-eight days in Fig. 2. The fundamental period $T_f = 24$ [hrs] in the data but is scaled in the laboratory tests.

III. OPTIMAL CONTROL

An EMPC approach is adopted to minimize the operational energy cost of the WDN. The EMPC is implemented as a supervisory controller generating pump water flow references. A centralized version of the EMPC is presented first, followed by a distributed version. Both controller implementations operate in discrete time and follow a receding horizon approach, ignoring any data processing or optimization delays. The power price $\tilde{c}(t)$ is assumed to be known over prediction horizon T_{H} and is equal for all pumps. The water demand is assumed to be bounded by (3c), but expected to follow (5), given $q_{cj\text{mean}}$.

An on/off controller is used to compare the optimal control solutions to a conventional one. This controller turns the pumps on if the water level in the connected reservoir falls under a lower threshold and turns them off if the water level exceeds an upper threshold.

A. Centralized EMPC

Following the system model and power calculations presented in Section II, let $u(t) \in \mathbb{R}^M$ denote the input to all subsystems at time t. The continuous version of the constraint optimization problem, to be solved at each iteration, is

$$\min_{u(t)} \quad \int_{t_0}^{t_0+T_{\text{H}}} c(\tau) P(u(\tau), y(\tau))\, d\tau \tag{6}$$

$$\text{s.t.} \quad P(u(t), y(t)) = u_1(t) y_{1,1}(t) + \sum_{j\in\mathbb{V}'} u_j(t)(y_{j,1}(t) - y_{m_j,2}(t))) \tag{6a}$$

$$y_j(t) = C x_j(t) + f_j(u_j(t)) + \gamma_j \qquad \forall j \in \mathbb{V} \tag{6b}$$

$$\dot{x}_j(t) = {}^1/{A_j}(u_j(t) - \tilde{d}(t) - \sum_{l\in\mathbb{L}_j} u_l(t)) \qquad \forall j \in \mathbb{V} \tag{6c}$$

$$x_j(t_0 + T_{\text{H}}) = x_{j,\text{T}} \qquad \forall j \in \mathbb{V} \tag{6d}$$

$$x_j(t) \in \mathbb{X}'_{j,d_{\max},u(t)} \subseteq \mathbb{X}_j \qquad \forall j \in \mathbb{V} \tag{6e}$$

$$u_j(t) \in \mathbb{U}_j \qquad \forall j \in \mathbb{V} \tag{6f}$$

where $\mathbb{X}'_{d_{\max},u(t)} \subseteq \mathbb{X}$ incorporates erroneous demand predictions and ensures adequate water levels if $d(t) = d_{\max}$. Terminal state constraint (6d) enforces predefined reservoir water levels at the end of the prediction horizon.

B. Convexity

For the implementation of the optimization problem (6), it is discretized and solved using standard optimization methods. However, it can be shown that there exist operating conditions in which the discrete version of optimization problem (6) is not convex. The relation in (6) that affect convexity is that the power P depends negatively on parent system state x_{m_j} through $y_{m_j,2}$. see (6a) and (2).

1) Convexification: A convex optimization problem is desired to produce global optima in all situations, ensuring consistent system operations. Convexification of the optimization problem (6) can be obtained by removing the dependency of x_{m_j} in the expression of the power (6a). This is achieved by approximating y_{m_j} in (6a) without changing the model dynamics constraints in (6c), such that $\tilde{P}_j = u_j(y_{j,1} - \tilde{y}_{m_j,2}(\tilde{x}_{m_j}))\ \forall j \in \mathbb{V}'$, where $\tilde{x}_{m_j}$ is constant.

To obtain an optimal $\tilde{x}_{m_j}$, the absolute maximum approximation error for each P_j, $j \in \mathbb{V}'$, is minimized. The error

$$e_j(\tilde{x}_{m_j}, x_{m_j}, u_j) := \frac{P_j(x_{m_j}, u_j) - \tilde{P}_j(\tilde{x}_{m_j}, u_j)}{P_j(x_{m_j}, u_j)} \tag{7}$$

is bounded by the reservoir water limits and minimum input. Specifically, $e_j(x_{m_j}) \le e_j(x_{m_j\min})$, $e_j(x_{m_j}) \ge e_j(x_{m_j\max})$, $|e_j(u_j)| \le |e_j(0)|\ \forall\, x_{m_j} \in \mathbb{X}_{m_j}$, $u_j \in \mathbb{U}_j$. Equating $e_{\min}$ and $-e_{\max}$ yields $e_j(\tilde{x}_{m_j}, x_{m_j\max}, 0) = -e_j(\tilde{x}_{m_j}, x_{m_j\min}, 0)$, which is solved for $\tilde{x}_{m_j}$, resulting in

$$\tilde{x}_{m_j} = \frac{(x_{\min} + x_{\max})(\gamma_{m_j,2} - \gamma_{j,1}) + 2\rho g x_{\max} x_{\min}}{2(\gamma_{m_j,2} - \gamma_{j,1}) + \rho g(x_{\min} + x_{\max})}$$

where subscript notation m_j is omitted in the bounds. Problem (6) is converted to a convex optimization problem if (6a) changes to

$$P(u, y) = u_1 y_{1,1} + \sum_{j \in \mathbb{V}'} u_j (y_{j,1} - \tilde{y}_{m_j,2})) \quad (8)$$

where $\tilde{y}_{m_j,2} = \rho g \tilde{x}_{m_j} + \gamma_{j,2}$. Note that parent water level $\tilde{x}_{m_j}$ is a constant approximation of the actual water level. The constant approximation is not used in the dynamics constraints of (6).

C. Distributed EMPC

In distributed control it is desired to control each subsystem individually using convex cost-functions. Using the convexification method presented in (8), implies that local cost-functions always are independent from their parents' state. Given this independence from parents' state in combination with the tree structure of the network, the choice of solving strategy, e.g. iteratively or sequentially (presented by [16]), naturally goes to a sequential approach. Locally, the following optimization problem is solved

$$\min_{u_j(t)} \int_{t_0}^{t_0+T_{\mathrm{H}}} c(\tau) P_j(u_j(\tau), y_j(\tau))\, d\tau \quad (9)$$

$$\text{s.t.} \quad P_j(u_j, y_j) = \begin{cases} u_j y_{j,1} & \text{if } j = 1 \\ u_j (y_{j,1} - \tilde{y}_{m_j,2}) & \text{if } j \in \mathbb{V}' \end{cases} \quad (9a)$$

$$y_{j,1} = f_{j,1}(u_j(t)) + \gamma_{j,1} \quad (9b)$$

$$\dot{x}_j(t) = B_j (u_j(t) - d_j(t) - \textstyle\sum_{l \in \mathbb{L}_j} u_l(t)) \quad (9c)$$

$$x_j(t_0 + T_{\mathrm{H}}) = x_{j\mathrm{T}} \quad (9d)$$

$$x_j(t) \in \mathbb{X}'_{j, d_{j\,\max}, u_j(t)} \subseteq \mathbb{X}_j \quad (9e)$$

$$u_j(t) \in \mathbb{U}_j \quad (9f)$$

At each control iteration, the controllers, optimize sequentially, starting at the leaf subsystem(s) and ending at the root subsystem. All leaf subsystem controller(s) independently solve the local optimization problem(s). The current and expected future inputs are communicated to the corresponding parent subsystem(s). Each parent subsystem controller optimizes, considering the expected future inputs of the child subsystem(s) as a disturbance. This process continues up to the root subsystem and starts over each control iteration. Any time delays are ignored. With this setup, only backward communication from the leaf subsystem(s) toward the root is necessary.

IV. RESULTS

The convex centralized and distributed controllers are both implemented in discrete time using the function `fmincon` [15] . Slack variables are added to the terminal constraint (9d) and thereby softening the terminal constraint. These slack variables help ensuring convergence and have not shown to introduce stability issues. An input delay is present due to the nature of the control structure. However, $T_{delay} \leq 1$ second is observed in all optimizations, which is negligible if the sample time T_s is considerable longer than this. The sample time $T_s = 1$ hour in real-life and $T_s = 60$ sec in the accelerated lab tests. Energy prices are considered repetitive and and 50% lower between 07:00 and 22:00 compared to the other hours of the day. Even though, this pattern is artificial compared to what is expected in real life applications, it is chosen to recognize if the controllers exhibit the desired behavior. The price is shown for one day in Fig. 5a and Fig. 6a.

Laboratory tests are performed using the bounds, consumer demands, and empirically obtained parameters listed in the tables in I. All tests start at virtual midnight with $x_j = 300 \pm 6$ mm, and are time accelerated by a factor 60 such that 24 minutes represents 24 hours in a real WDN. The supervisory control operates with $T_s = 60$ seconds and $N = 24$, meaning that a prediction horizon of 24 hours is used in the economic MPC controllers. The terminal equality state constraint is chosen to be $x_{j\mathrm{T}} = x_{j\max}\ \forall\ j \in \mathbb{M}$.

A. Laboratory Implementation

A spatially scaled WDN containing two subsystems, is implemented in the smart water infrastructure laboratory presented by [13], using three generic modules types: pumping stations, consumer units, and piping units. The lab can be seen in Fig. 3.

Fig. 3: Picture of the Smart Water Lab used for the tests.

The physical system dimensions and elevations are listed in Table Ia. Pressurizing reservoirs mimics the effect of physical elevation. The designed EMPC acts as a supervisory controller that sets reference flow signals to local PI controllers that regulate the local pumping stations flows to desired values. Consumer demands are determined by (5), and local PI controllers control valve flows to act as consumers. All local PI controllers operate at $f_s = 1$ Hertz.

1) Parameter estimation: A select number of parameters, listed in Table Ic, is estimated to minimize the difference between the physical system and the first principle model.

B. Controller behavior

Five days are emulated in the laboratory using the centralized EMPC, distributed EMPC, and the on/off controller. Presimulation studies suggest that, using the model predictive controllers, the system exhibits periodic (limit-cycle-like) behavior in the states and inputs after transient.

1) Centralized EMPC: Relevant signals for one emulated day are presented in Fig. 5, where the total system passed the transient time. The main observation that can be made is that pump flow is significantly higher when the power

parameters	$z_{1:2}$	z_{r1}	z_{r2}	$\Delta z_{b1:2}$	$\Delta z_{t1:2}$	$A_{1:2}$
unit	m	m	m	m	m	m^2
value	0	0	1	2	3	$\pi 0.3^2$

(a) Physical parameters

parameter	$x_{1\min}$	$x_{1\max}$	$x_{2\min}$	$x_{2\max}$	$u_{1\min}$	$u_{1\max}$
unit	mm	mm	mm	mm	$^{l}/_{\min}$	$^{l}/_{\min}$
value	110	440	110	440	0.5	14.0
parameter	$u_{2\min}$	$u_{2\max}$	$q_{c1\text{mean}}$	$q_{c1\max}$	$q_{c2\text{mean}}$	$q_{c2\max}$
unit	$^{l}/_{\min}$	$^{l}/_{\min}$	$^{l}/_{\min}$	$^{l}/_{\min}$	$^{l}/_{\min}$	$^{l}/_{\min}$
value	0.5	8.0	2.0	4.0	3.0	6.0

(b) Bounds and operating points

parameter	unit	expected	emperical	error [%]
$\gamma_{1,1}$	kPa	29.34	27.07	7.74
$\gamma_{1,2}$	kPa	19.56	18.58	5.04
$\gamma_{2,2}$	kPa	39.12	38.82	0.77
ρg	$^{kg}/_{m^2 s^2}$	9781	9627	1.57
λ_1	$^{kg}/_{m^7}$		$1.990e11$	
λ_2	$^{kg}/_{m^7}$		$2.080e12$	

(c) System parameters in y

TABLE I: Laboratory system specifications

is cheap than when the power is expensive, indicating that the controller behaves as desired. Reservoir 2 exceeds its upper limit, which is explainable by the dynamics of the flow controllers, which means that the desired flow reference is not followed exactly. Poor flow control in the lower flow regimes is visible in Fig. 5c and Fig. 5d. A peak in operational cost is visible in Fig. 5f and is caused by the PI settling time delay.

2) Distributed EMPC: Relevant signals for one emulated day are presented in Fig. 6, where the total system passed the transient time. Comparable behavior to the centralized EMPC can be observed, such as limit-exceeding water levels, poor flow control in the lower flow regime, and a peak in cost every morning. A small difference between the distributed EMPC and the centralized EMPC can be observed in the water levels of reservoir 1. On average, reservoir 1 water level is 1.25% lower in the distributed EMPC tests.

3) Cumulative cost: The cumulative theoretical operating cost for the three presented control methods, based on laboratory pressure measurements, is plotted over time in Fig. 4. After five days, the cumulative cost is 23.2% and 21.2% lower for the centralized EMPC and the distributed EMPC compared to the on/off controller, respectively. In the distributed EMPC case, the child subsystem disregards the water level, water demand, and other disturbances of its parent subsystem. As one would expect, this results in a sub-optimal outcome compared to centralized EMPC, which considers all available information. This is evident in the test results by the 2% less saving when applying the distributed EMPC.

4) Approximation errors: The theoretical maximum error of making the cost function convex (7), is $\leq 9\%$. Empirically, using combined measurement data, pump 2 mean power approximation error is 4.2%.

V. CONCLUSION

In this paper a centralized and distributed EMPC are developed for a type of serial connected pumping stations

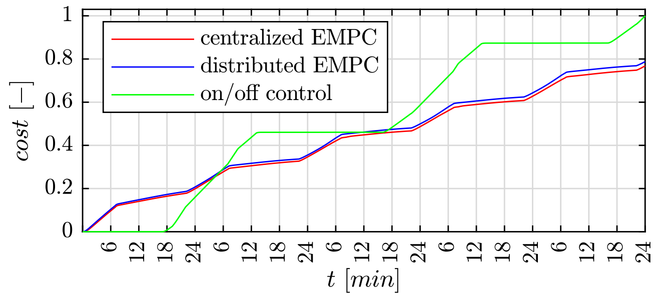


Fig. 4: Normalized cumulative cost different control types based on 5 emulated days in the laboratory

found in water distribution networks. To ensure a robust solution to the embedded optimization problem, convexity is forced upon the problem by adjusting the underlying model. Beside imposing convexity, this adjustment also leads an efficient extension to a distributed setup, with a simple communication structure between the local controllers. A bound on the cost of introducing this adjustment is derived, and is calculated to be less than 9% in the presented experimental results. The laboratory test results show that the centralized and distributed EMPC are both effective solutions. Considering the specific power price structure from Fig. 5a, the distributed EMPC methods reduce operating costs by approximately 21% compared to the presented conventional control. Moreover, both controllers exhibit robust behavior and prove capable of handling high, unexpected water demands.

Future work include stability analysis and further analysis of the impact on the saving potential imposed by forcing convexity on the model.

ACKNOWLEDGMENT

The authors would like to thank Poul Due Jensen Foundation (Grundfos Foundation) for supporting the lab, and Jorge Val Ledesma for valuable discussions.

REFERENCES

[1] the National Academy of Sciences, *Drinking Water Distribution Systems, Assessing and reducing risks*. National Academies Press, 2006.

[2] R. M. Menke, "Sustainable energy management of water distribution systems through optimised pump scheduling," Ph.D. dissertation, Imperial College London.

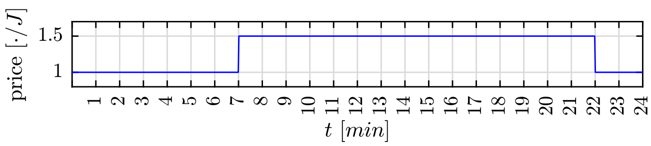
[3] R. G. Junker, C. S. Kallesøe, J. P. Real, B. Howard, R. A. Lopes, and H. Madsen, "Stochastic nonlinear modelling and application of price-based energy flexibility," *Applied Energy*, vol. 275, 2020.

[4] M. Ellis, H. Durand, and P. D. Christofides, "A tutorial review of economic model predictive control methods," *Journal of Process Control*, vol. 24, no. 8, pp. 1156–1178, 2014. [Online]. Available: http://dx.doi.org/10.1016/j.jprocont.2014.03.010

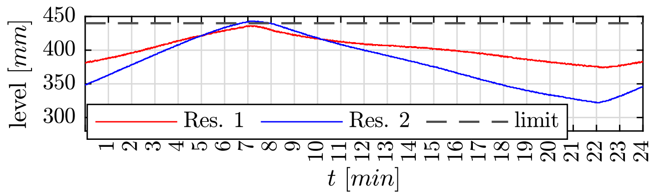
[5] Y. Wang, V. Puig, and G. Cembrano, "Non-linear economic model predictive control of water distribution networks," *Journal of Process Control, Vol. 56, Page 23-34*, 2017.

[6] C. Ocampo-Martinez, V. Puig, G. Cembrano, and J. Quevedo, "Application of predictive control strategies to the management of complex networks in the urban water cycle," *IEEE Control Systems*, vol. 33, no. 1, pp. 15–34, 2013.

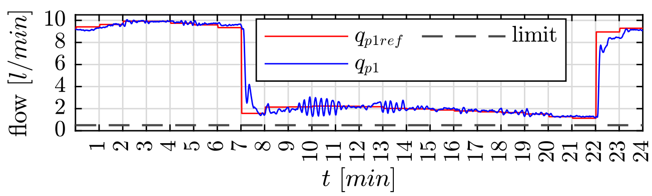
[7] C. S. Kallesøe, T. N. Jensen, and J. D. Bendtsen, "Plug-and-Play Model Predictive Control for Water Supply Networks with Storage," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 6582–6587, 2017.



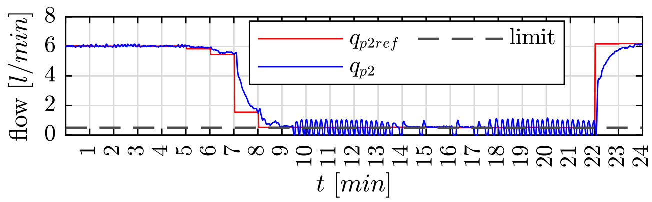
(a) Power price



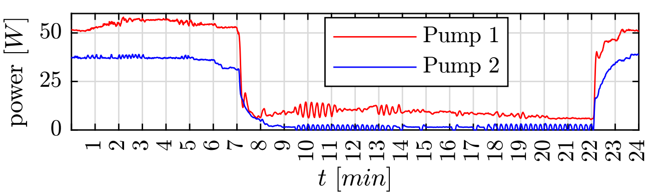
(b) Reservoir water levels



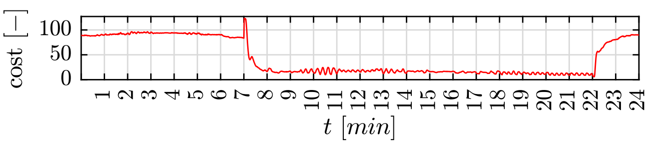
(c) Pump 1 flow



(d) Pump 2 flow

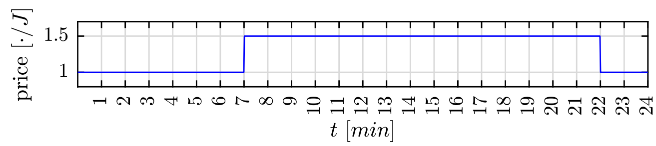


(e) Energy consumption

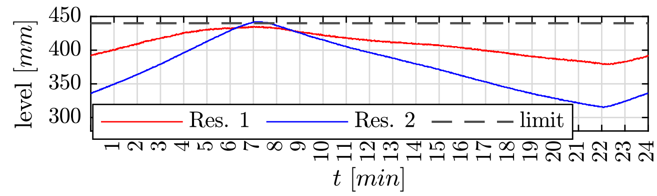


(f) Total cost over over time

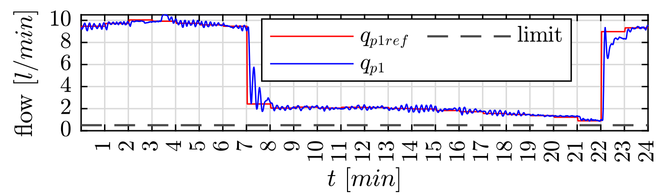
Fig. 5: Centralized EMPC laboratory test single period



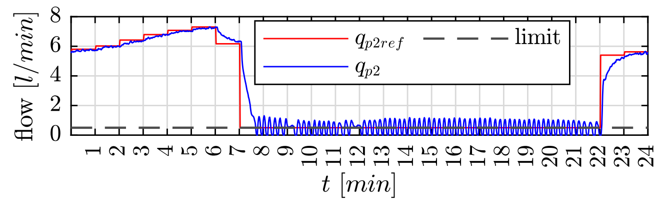
(a) Power price



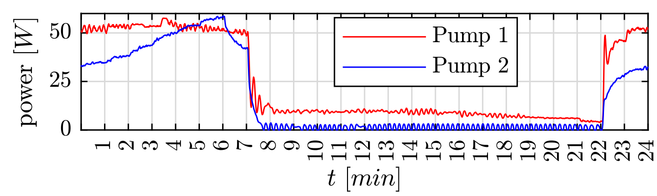
(b) Reservoir water levels



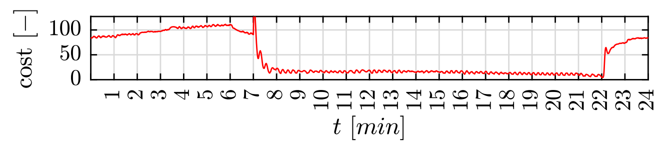
(c) Pump 1 flow



(d) Pump 2 flow



(e) Energy consumption



(f) Total cost over over time

Fig. 6: Distributed EMPC laboratory test single period

[8] M. A. Müller and F. Allgöwer, "Distributed economic MPC: A framework for cooperative control problems," *IFAC Proceedings Volumes (IFAC-PapersOnline)*, vol. 19, pp. 1029–1034, 2014.

[9] C. Ocampo-Martinez, V. Puig, and S. Bovo, "Decentralised MPC based on a graph partitioning approach applied to the Barcelona drinking water network," *IFAC Proceedings Volumes (IFAC-PapersOnline)*, vol. 44, no. 1 PART 1, pp. 1577–1583, 2011.

[10] R. R. Negenborn, B. De Schutter, and H. Hellendoorn, "Multi-agent model predictive control of transportation networks," *Proceedings of the 2006 IEEE International Conference on Networking, Sensing and Control, ICNSC'06*, pp. 296–301, 2006.

[11] S. Leirens, C. Zamora, R. R. Negenborn, and B. De Schutter, "Coordination in urban water supply networks using distributed model predictive control," *Proceedings of the 2010 American Control Conference, ACC 2010*, vol. 19, pp. 3957–3962, 2010.

[12] D. Liu, S. Li, Y. Zheng, and J. Wu, "Distributed zone MPC of pressure management for water distribution network systems," *IET Control Theory and Applications*, vol. 13, no. 11, pp. 1704–1717, 2019.

[13] J. Val Ledesma, R. Wisniewski, and C. S. Kallesøe, "Smart water infrastructures laboratory: Reconfigurable test-beds for research in water infrastructures management," *Water (Switzerland)*, vol. 13, no. 13, pp. 1–24, 2021.

[14] P. K. Swamee and A. K. Sharma, *Basic Principles of Pipe Flow*. John Wiley & Sons, Ltd, 2008.

[15] Matlab, *MATLAB version 9.11.0.1769968 (R2021b)*, The Mathworks, Inc., Natick, Massachusetts, 2021.

[16] P. D. Christofides, R. Scattolini, D. Muñoz de la Peña, and J. Liu, "Distributed model predictive control: A tutorial review and future research directions," *Computers and Chemical Engineering*, vol. 51, pp. 21–41, 2013.