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Multi-Population Approach for Decentralized Control of Urban Drainage Systems with Replicator Dynamics

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***Abstract*— Using information consensus and replicator dynamics (RD), this article presents a distributed algorithm for designing control schemes in urban drainage systems (UDSs). It demonstrates the stability of a closed-loop model with RD in UDSs using passivity arguments for single subsystems. As central models for UDSs, we present two distinct topologies and conduct passivity-based analysis to design appropriate payoff mechanisms. We further extend this to a decentralized scenario in which subsystems within a UDS share information and increase capacity at specific sites in response to intense rainfall. This algorithm with distributed consensus assistance seeks to improve system performance. Several simulations are presented to illustrate the benefits of this method.**

I. INTRODUCTION

Amongst the major problems faced by metropolitan areas, urban flooding is an alarming risk due to several increasing factors causing stress on modern cities infrastructure. The steep growth of urban population, intense rainfall due to climate change and the reduction of green space are some of the growing trends that show this issue is expected to worsen [1], [2]. Guided by sustainability goals, modern cities have decided to reevaluate urban drainage systems (UDSs) in order to adequately respond to this challenge [3]–[5].

An urban drainage system is a key infrastructure in charge of collecting and transporting both rainwater and wastewater away from urban areas to prevent flooding. UDSs form underground networks consisting of interconnected channels that receive external sources of water and also distribute their contents through flows that can be controlled by valves [6]. In ideal cases, these systems work efficiently and allow for all services in large cities to operate uninterruptedly. Yet, in a real-world setting, such systems hold a maximum capacity, which may be exceeded as a consequence of extreme rainfall, natural disasters, or a poor design. To improve their performance, the general goal is to efficiently distribute water such that internal reservoirs can respond to heavy incoming loads. By maximizing its remaining capacity, a UDS can have available empty space to store sudden surges of inflowing water to the network, thus avoiding undesired overflow into populated areas. Therefore, introducing proper control strategies can be a valuable approach to mitigate flooding and collateral negative impact on urban life through an adequate distribution of water, besides being an affordable alternative to expensive renovations of a large infrastructure.

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Lately, there has been an increasing interest in model predictive control (MPC) designs [7], [8] and real-time control techniques [9]–[11] for UDSs. However, a limitation of these methods is the need of a centralized controller in charge of the entire UDS, which is typically a large-scale network, thus requiring a costly implementation. Other current approaches use game theory, setting water tank systems into a closed-loop configuration known as the EDM-PDM model [12]–[14], which consists of two major subsystems: an evolutionary dynamics model (EDM), and a payoff dynamics model (PDM). The EDM incorporates an underlying revision protocol acting on an input payoff to evolve the so-called mean social state that is fed-back to the PDM. In a practical setting, such as this work, one can choose the EDM to act as a controller while the PDM contains a physical model in order to steer the closed-loop system towards equilibrium. In [15], the authors explore the use of game theory for water distribution systems by treating this issue as a resource allocation problem. Here, the benefits of using Brown–von Neumann–Nash and Smith dynamics against other evolutionary dynamics in a single water distribution system are pointed out. Using an EDM-PDM configuration, the evolutionary dynamics serve as controllers acting through the valves of the system. Their desgin shows adequate convergence to wanted equilibrium points and additional disturbance rejection. Alternatively, the work of [16] employs replicator dynamics (RD) attempting to address an extended network with several interconnected water reservoirs. In this case, the target is to evenly distribute water in the reservoirs of two setups of tanks to avoid overflow. Introducing a partitioning algorithm to divide the UDS network into convergent and divergent topologies, their results show a clear advantage of RD dynamics opposed to traditional methods, such as linear-quadratic regulators and model predictive control.

Although the previous works have thoroughly explored the use of evolutionary dynamics for water distribution, a limitation of these studies is their highly local approach. Despite achieving suitable design goals, we highlight that [15] examines a water distribution system with a single inflow and, while simulated random Gaussian rainfall profiles in time are considered in [16], there is no connection between rain intensity and the spatial location of water tanks. Furthermore, the entire network is divided into subsystems, each having single local controllers acting independently from the rest of the system. In real scenarios, flooding is often present in specific areas of a city with heavy rain, rather than occuring randomly or uniformly. Meanwhile, resources from other sectors of the UDS, where there is light rainfall, may remain largely unused despite being capable of sharing their capacity to globally assist the network. The objective of this study is to enhance the efficiency of RD controllers for UDSs in decentralized setups. First, we formulate the control loop as a well known EDM-PDM scheme, to which we will refer to as a single population. Performing a passivity-based analysis of the two models of interest (convergent and divergent topologies), we design suitable payoff mechanisms that ensure convergence to the equilibrium point. Then, we extend this formulation to a distributed network with partially communicated populations, where we use information consensus and modify our payoff mechanism to achieve a better performance taking into account the global spatial distribution of rainfall.

This paper is organized as follows: Section II presents preliminary concepts. Section III describes the models for the two topologies to be considered, and in Section IV we present the control scheme and perform a passivity-based analysis on the models of interest to design suitable payoff mechanisms. Section V proposes augmented dynamics for decentralized control based on information consensus and we present simulations showing an advantage to previous techniques. Finally, conclusions are presented in Section VI.

II. Preliminaries

A. EDM-PDM Model

The closed-loop model in Fig. 1 shows the feedback connection of a payoff dynamics model (PDM) and an evolutionary dynamics model (EDM). Inspired by traditionally static population games [17], this scheme extends the notion to dynamic games, in which players interact through strategies that are allowed to evolve in time. Generally, these are nonlinear systems where the PDM's internal dynamics can represent some dynamical system of interest and its output $\boldsymbol{p}(t)$, called the deterministic payoff, is associated to a reward for the possible strategies the system can adopt. Here, the different strategies are related to control actions to be performed over the system. The EDM acts as a controller and its output $\boldsymbol{x}(t)$, known as the mean social state, evolves according to the received payoffs for each strategy.

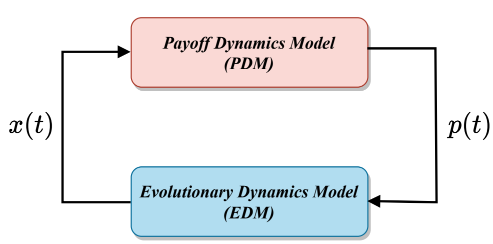

Fig. 1: Diagram of the closed-loop model between a PDM and an EDM.

Designing a control system under this scheme will require a careful choice of the payoff mechanism, which steers the system of interest towards a desired state that coincides with an equilibrium point of the selected EDM. From a systems perspective, the diagram in Fig. 1 allows for a deeper mathematical analysis and design. In this work, we will exploit passivity properties of the replicator dynamics, to design the payoff mechanism for the UDS models of interest. We consider the dynamics described in Section III to be part of a PDM and, in this section, we design the payoff mechanism, based on passivity arguments for stability in closed-loop with a replicator dynamics EDM.

B. Replicator Dynamics

Emerging from underlying imitative revision protocols, the replicator equation is an important example of a biologically inspired mathematical model that captures interspecies competition based on their fitness [17], [18]. Applied to a game-theoretic scenario, the RD are suitable to model the migration between strategies adopted by agents in a large population driven by their received payoff. Let $\mathcal{S} = \{1, 2, \ldots, s\}$ be the set of available strategies for a population, and $x_i(t) \in [0, 1]$ the proportion of agents playing strategy $i \in \mathcal{S}$ at time t. The time evolution of such proportions is determined by the replicator equation

$$\dot{x}_i(t) = \beta x_i(t)(p_i(t) - \overline{p}(t)), \text{ for all } i \in \mathcal{S}, \qquad (1)$$

where $\beta \in \mathbb{R}_{>0}$ is the population growth rate, and $\overline{p}(t) = \sum_{j\in\mathcal{S}} x_j(t)p_j(t)$ is the population's average fitness or payoff. Eq. (1) reproduces the expected behaviour of natural selection, since strategies fitter than the average tend to increase in size, while less fit decrease. An important property fulfilled by RD is stated in the following Lemma.

Lemma 1. *The simplex* $\Delta = \left\{ \boldsymbol{x} \in \mathbb{R}^s_{\geq 0} \,\middle|\, \sum_{i\in\mathcal{S}} x_i = 1 \right\}$ *is positively invariant under RD*[2].

Lemma 1 ensures that for any initial condition $\boldsymbol{x}(t_0) \in \Delta$, the state of the population $x(t) \in \Delta$ for later times $t > t_0$, as expected for actual proportions. Furthermore, at the equilibrium $p_i^* = p_j^*$, for all $i, j \in \mathcal{S}$, provided that all strategies are non-extinct, i.e., $x_i^* > 0$ and $x_j^* > 0$. These are also known as interior points, where it immediately follows that $\boldsymbol{x}^\top \boldsymbol{p}^* \leq \boldsymbol{x}^{*\top} \boldsymbol{p}^*$ for all $\boldsymbol{x} \in \Delta$. Moreover, RD can be appealing thanks to their passivity properties, as pointed out in the following Lemma (see [19]).

Lemma 2. *The system implementing replicator dynamics (Eq. 1) defined as*

$$\Sigma_{RD} : \begin{cases} \dot{x}_i(t) = \beta x_i(t)(p_i(t) - \sum_{j\in\mathcal{S}} x_j(t)p_j(t)), \\ y_i(t) = x_i(t), \ \forall\, i \in \mathcal{S}, \end{cases}$$

with input $\boldsymbol{p} = \mathrm{col}(p_1, p_2, \ldots, p_s) \in \mathbb{R}^s$ *and output* $\boldsymbol{y} = \mathrm{col}(y_1, y_2, \ldots, y_s) \in \mathbb{R}^s$*, is EIP.*

This result further motivates the use of the aforementioned evolutionary dynamics as controllers by means of exploiting their passivity properties, for example, as we will develop in Section IV. A naturally lossless system can be used to render equilibrium points in a negative feedback system stable,

[2] Proofs are omitted due to lack of space.

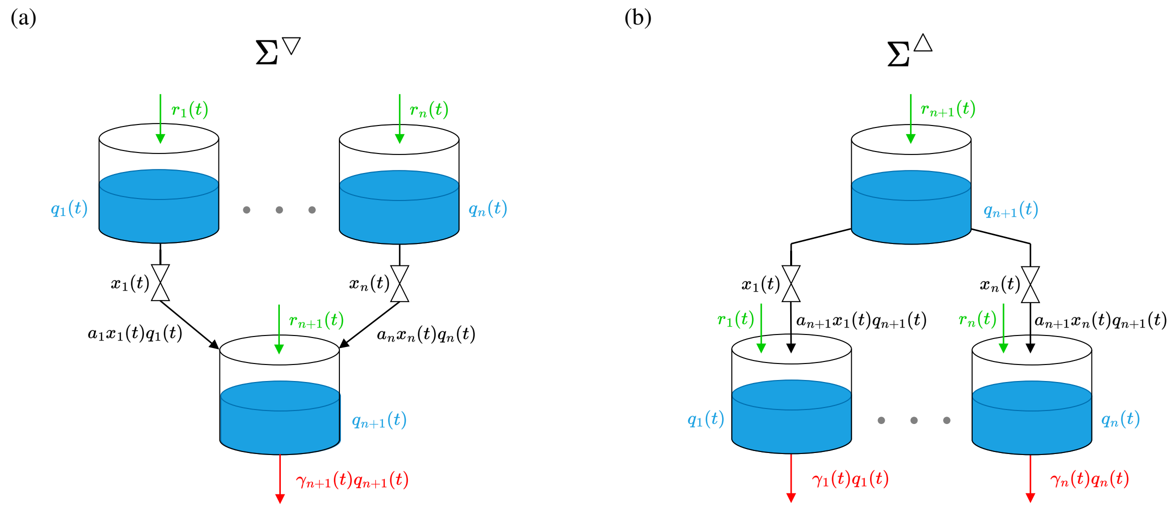

Fig. 2: Two types of subsystems found within a UDS: (a) convergent, and (b) divergent topologies.

for instance, under the conditions provided in the following theorem [20].

Theorem 1. *The equilibrium point of a negative feedback interconnection of a strictly passive system Ψ and a lossless passive system Φ is Lyapunov stable.*

III. Model Description

As outlined in [16], UDSs can typically be partitioned into two types of subsystems: convergent and divergent topologies, which are depicted in Fig. 2. Such division allows for an easier analysis of the entire network by grouping into larger sections that can also represent separate zones from an urban area. Based on the Muskingum model [6], the channels of the UDS are approximated as wastewater reservoirs. For both topologies, the models are derived from continuity equations (i.e., mass conservation) where incompressible flow is assumed.

A. Convergent Topology

In this topology, n source reservoirs receive external inflows of wastewater $r_i(t)$ and discharge their contents to a single receptor reservoir indexed as $n+1$. The outflows of each reservoir is proportional to their current volume $q_i(t)$ through a coefficient a_i which takes into account the reservoir's geometry. Source reservoirs can further manipulate their outflow by controlling the opening percentage $x_i(t) \in [0,1]$ of each valve. For completeness, $x_{n+1}(t)$ can be interpreted as the percentage of a strategy corresponding to closing all valves. Finally, the water loss of the receptor reservoir is characterized by the coefficient $\gamma_{n+1}(t)$. The internal dynamics of this configuration are then given by

$$\Sigma^{\bigtriangledown}: \begin{cases} \dot{q}_i(t) = r_i(t) - a_i x_i(t) q_i(t) \;\;,\;\; i = 1, \dots, n, \\ \dot{q}_{n+1}(t) = r_{n+1}(t) - \gamma_{n+1}(t) q_{n+1}(t) + \dots \\ \qquad\qquad + \sum\limits_{j=1}^{n} a_j x_j(t) q_j(t). \end{cases}$$

The importance of this model, as we discuss in the distributed case, is its frequent use at the topmost layer of UDSs. Usually, several sources of water converge to a larger reservoir and then continue distribution through the network.

B. Divergent Topology

In this topology, a source reservoir indexed as $n+1$ receives an external inflow of wastewater $r_{n+1}(t)$ and discharges its content to n receptor reservoirs. Similarly, the outflow from the source reservoir to each receptor reservoir is proportional to its current volume $q_{n+1}(t)$ through a coefficient a_{n+1} taking into account the source's geometry. Individual flows from the source to receptor reservoirs can be manipulated by controlling the opening percentage $x_i(t) \in [0,1]$ of each valve. Finally, the water loss of receptor reservoirs is characterized by coefficients $\gamma_i(t)$. The internal dynamics of this configuration are then given by

$$\Sigma^{\triangle}: \begin{cases} \dot{q}_i(t) = r_i(t) - \gamma_i q_i(t) + \dots \\ \qquad\quad + a_{n+1} x_i(t) q_{n+1}(t) \;\;,\;\; i = 1, \dots, n, \\ \dot{q}_{n+1}(t) = r_{n+1}(t) - \sum\limits_{j=1}^{n} a_{n+1} x_j(t) q_{n+1}(t). \end{cases}$$

An in depth analysis of the equilibrium points for both subsystems can be found in [16]. For the purpose of this work, we will further assume that no valve is either fully opened or closed at any moment, this is $0 < x_i(t) < 1$, for all $i = 1, \dots, n$, and that external inflows and outflows of wastewater are constant in time, i.e., $r_i(t) = r_i$ and $\gamma_i(t) = \gamma_i$, for all $i = 1, \dots, n+1$.

IV. Passivity-Based Design and Control of Subsystem Topologies

A. A Payoff Mechanism for Convergent Topologies

Taking into account a convergent topology with identical reservoirs such that $a_i = a$, for all $i = \{1, \dots, n\}$, the equilibrium point satisfies $q_i^* = q^*$, for all $i = 1, \dots, n+1$. Now, inspired in the Lyapunov function of [16], consider the storage function $S^{\bigtriangledown}(\boldsymbol{e}_q(t)) = \frac{1}{2aq^*}\sum_{i=1}^{n+1} e_{q_i}^2(t)$, where $\boldsymbol{e}_q(t) = \boldsymbol{q}(t) - \boldsymbol{q}^*(t)$. Time derivatives along $\Sigma^{\bigtriangledown}$ can be shown to have the form

$$\dot{S}^{\bigtriangledown}(\boldsymbol{e}_q(t)) = -\boldsymbol{e}_x^\top(t)\boldsymbol{e}_q(t) - \varphi^{\bigtriangledown}(\boldsymbol{e}_q(t)), \tag{2}$$

where

$$\varphi^{\bigtriangledown}(\boldsymbol{e}_q(t)) = \frac{x_{n+1}(t)}{4q^*}e_{q_{n+1}}^2(t) + \frac{1}{q^*}\left(\frac{4\gamma_{n+1} - an}{4a}\right)e_{q_{n+1}}^2(t) + \frac{1}{q^*}\sum_{i=1}^{n} x_i(t)\left(e_{q_i}(t) - \frac{1}{2}e_{q_{n+1}}(t)\right)^2,$$

is a positive definite function assuming $a < 4\gamma_{n+1}/n$. The obtained form of Eq. (2) allows us to conclude that the convergent topology is a naturally passive system with respect to the negative of the volume state vector. Our analysis suggests a design of a suitable wanted payoff as asserted in the following Lemma.

Lemma 3. *Under the output (payoff) $\boldsymbol{y}(t) = -\boldsymbol{q}(t)$, the convergent subsystem $\Sigma^{\bigtriangledown}$ is strictly EIP.*

Therefore, if $\Sigma^{\bigtriangledown}$ (under the payoff described in Lemma 3) is placed in the PDM connected with an Σ_{RD} EDM, by means of Theorem 1, we can guarantee the stability of the equilibrium point. In addition, an equilibrium point of RD is found when all strategies have equal payoff, matching the design criteria of equal volumes. Finally, using LaSalle's invariance principle, this equilibrium point is locally asymptotically stable.

B. A Payoff Mechanism for Divergent Topologies

Following the approach of the previous case, consider the storage function $S^{\bigtriangleup}(\boldsymbol{e}_q(t)) = \frac{1}{2a_{n+1}q_{n+1}^*}\sum_{i=1}^{n+1} e_{q_i}^2(t)$. It can be shown that time derivatives along $\Sigma^{\bigtriangleup}$ have the form

$$\dot{S}^{\bigtriangleup}(\boldsymbol{e}_q(t)) = \boldsymbol{e}_x^\top(t)\boldsymbol{e}_q(t) - \varphi^{\bigtriangleup}(\boldsymbol{e}_q(t)), \tag{3}$$

where

$$\begin{aligned}\varphi^{\bigtriangleup}(\boldsymbol{e}_q(t)) =& \frac{\gamma_{n+1}}{a_{n+1}q_{n+1}^*}e_{q_{n+1}}^2(t) \\ &+ \frac{1}{q_{n+1}^*}\sum_{i=1}^{n}\left(\frac{4\gamma_i - x_i(t)a_{n+1}}{4a_{n+1}}\right)e_{q_i}^2(t) \\ &+ \frac{1}{q_{n+1}^*}\sum_{i=1}^{n} x_i(t)\left(e_{q_{n+1}}(t) - e_{q_i}(t)\right)^2,\end{aligned}$$

is a positive definite function if $a_{n+1} < 4\gamma_i$, for all $i = 1,\dots,n$. In contrast to convergent topologies, the form of Eq. (3) allows us to conclude that the divergent topology is a passive system with respect to the negative of the volume state vector. The following Lemma states our payoff mehcanism of choice to render the system EIP.

Lemma 4. *Under the output (payoff) $\boldsymbol{y}(t) = \boldsymbol{q}(t)$, the divergent subsystem $\Sigma^{\bigtriangleup}$ is strictly EIP.*

Similarly, if $\Sigma^{\bigtriangleup}$ (under the payoff described in Lemma 4) is placed in the PDM connected with an Σ_{RD} EDM, by means of Theorem 1, we can guarantee Lyapunov and further asymptotic stability of the equilibrium point.

V. Distributed Control of Subsystem Topologies

Up to this point, we have designed control systems with complete knowledge of the subsystem's state. However, this assumption breaks down in large-scale networks as instantaneous information may not be readily available for all sectors of a UDS. In addition, we should emphasize that intense rainfall can cause overflow only in specific areas while other parts of the UDS can have unused remaining capacity. Such remaining free space could be used to relief stress on particular locations through a more efficient distribution across the global network. In this section, we propose augmented dynamics for the previous topologies distributed over a connected communication graph. We will solely focus on convergent topologies, since this configuration is typically found at the topmost layers of a UDS. Nevertheless, the same design can be directly applied to the divergent case.

A. Problem Formulation

Consider a set $\mathcal{K} = \{1, 2, \dots, N\}$ of N populations each comprised of $n+1$ reservoirs arranged in a convergent topology as studied previously. We identify each population as a vertex of a connected and undirected graph $\mathcal{G}_{\mathcal{C}} = (\mathcal{K}, E)$, where $E \subseteq \mathcal{K}\times\mathcal{K}$ is the set of edges, therefore if an edge $(i,j)\in E$ then $(j,i)\in E$. Moreover, let $\mathcal{N}_k \subset \mathcal{K}$ be the set of neighbours with which k shares information, i.e., populations directly connected through an edge. Each population represents a part of the topmost layer of reservoirs in the network, these are those in charge of directly receiving water flow solely from incoming rain. The design objective will be to better distribute the load across populations, such that, even when some sectors of the UDS have relatively small demand, it can relieve the stress caused in other sectors with a greater inflow.

B. Consensus Based Algorithm

Let q_i^k be the volume of water in the $i-$th reservoir of population k, and $\hat{q}_i^{k_1,k_2}$ the volume of water estimated by population k_1 in population k_2's $i-$th reservoir. Additionally, we assume each population to have complete information of their own state at any time, i.e., $\hat{q}_i^{k,k}(t) = q_i^k(t)$ for all $i\in\mathcal{S}$. In this order, we propose the following distributed EDM-PDM dynamics

$$\dot{q}_i^k(t) = r_i^k(t) - a_i^k x_i^k(t) q_i^k(t) \;\;,\;\; i = 1,\dots,n, \tag{4a}$$

$$\dot{q}_{n+1}^k(t) = -\gamma_{n+1}^k(t) q_{n+1}^k(t) + \sum_{j=1}^{n} a_j^k x_j^k(t) q_j^k(t), \tag{4b}$$

$$\dot{\hat{q}}_i^{k,k'}(t) = -\sum_{k''\in\mathcal{N}_k}\left(\hat{q}_i^{k,k'}(t) - \hat{q}_i^{k'',k'}(t)\right), \tag{4c}$$

$$p_i^k(t) = q_i^k(t) \;\;,\;\; i = 1,\dots,n, \tag{4d}$$

$$p_{n+1}^k(t) = \frac{1}{N}\sum_{k'\in\mathcal{K}} \hat{q}_{n+1}^{k,k'}(t), \tag{4e}$$

$$\dot{x}_i^k(t) = \beta x_i^k(t)\left(p_i^k(t) - \overline{p}^k(t)\right), \tag{4f}$$

for all $k' \neq k$ and $k\in\mathcal{K}$. Eqs. (4a) and (4b) simply state the dynamics described in Section III-A for each population. Then, Eq. (4c) incorporates the consensus based approach, according to which each population k tries to reach consensus in its estimates by communicating with its neighbours $k'' \in \mathcal{N}_k$. Eqs. (4d) and (4e) modify the payoff mechanism by averaging across all populations of the set $\mathcal{K}$. While the source tanks $i = 1, \ldots, n$, seek to equal their volume, the $n+1$ tank receives the average volume across receptor reservoirs through all the network. Consequently, a greater flow from source tanks towards the receptor reservoir is to be expected. In high stress demand areas, this will allow for a greater discharge of water from the topmost layers of the UDS. Finally, Eq. (4f) defines an EDM closing the loop model for each population.

C. Simulations

1) Case 1: As a first example of the proposed dynamics in Eq. (4), we present in Fig. 3 the results of this algorithm applied to a distributed setting of convergent topologies under the random graph in Fig. 3(a). The parameters are $n = 2$, $\beta = 10^{-4}$, geometric parameters $a_1^k = a_2^k = 0.1\ \mathrm{s}^{-1}$, $\gamma_3^k = 0.05\ \mathrm{s}^{-1}$, and wastewater inflow $r_1^k = 2r_2^k = 0.5\ \mathrm{m}^3\mathrm{s}^{-1}$, for all $k \in \mathcal{K}$. In this scenario we discuss the performance of the distributed algorithm under homogenous rainfall across the network.

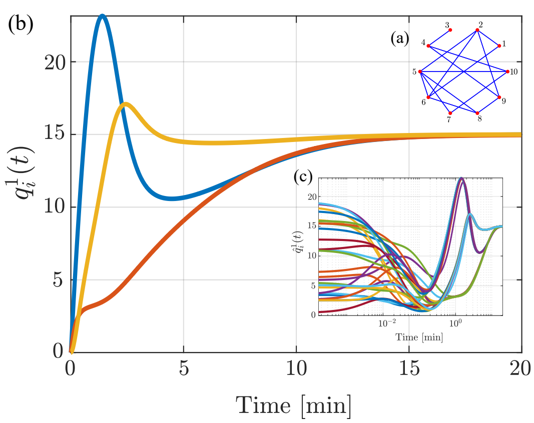

Fig. 3: Simulation of the proposed dynamics with $\beta = 10^{-4}$, over the (a) random connected communication graph $\mathcal{G}_\mathcal{C}$ of $N = 10$ populations of convergent topologies with $n+1 = 3$ reservoirs and $\lambda_2(L) = 0.57$. As a sample, (b) the actual volumes $q_i^1(t)$ and (c) estimated volumes $\hat{q}_i^{1,-k}(t)$ are also shown.

Initially, each population makes a random guess to build their estimates that rapidly achieve consensus with the entire network after the first minute, for this case, as observed in Fig. 3(b). Effectively, all populations converge to an equilibrium point where the volumes in all reservoirs are equal. It is worth noticing the role of the population growth rate β in the convergence of the proposed dynamics. Fig. 4 shows the Euclidean distance between the final social state after $t_f = 15$ min. and the expected equilibrium for different combinations of $(\beta, \lambda_2(L), N)$, where $\lambda_2(L)$ is the algebraic connectivity of graph $\mathcal{G}_\mathcal{C}$. The population growth rate plays the role of a second timescale interfering with the speed at which consensus can be achieved. As shown, larger networks of size N can admit smaller values of β provided that there is enough connectivity $\lambda_2(L)$ in order to converge towards the equilibrium point.

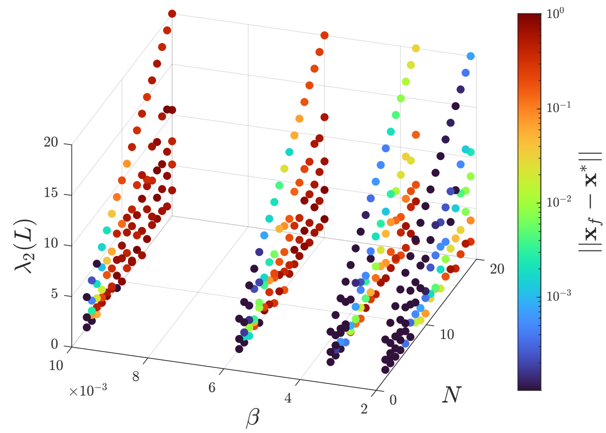

Fig. 4: Numerical convergence to the expected equilibrium point $\boldsymbol{x}^*$ for multiple realizations after $t_f = 15$min.

2) Case 2: In order to highlight the utility of the augmented payoff mechanism in Eqs. (4d) and (4e), we present an example in which the total inflow rate takes a Gaussian form. This represents a dense cloud causing heavy rain at certain sector of the UDS, mainly affecting nearby populations. Consider $N = 14$ convergent subsystems with $n = 2$ connected on a chain as shown in Fig. 5, which are physically distant subsystems communicating with their nearest neighbours. We assume the same parameters as in V-C.1, however, modifying the inflow rates as

$$r_1^k(t) = r_2^k(t) = 0.2e^{-\frac{1}{5}(k-k_0)^2} + 0.01 \quad [\mathrm{m}^3/\mathrm{s}],$$

being maximum for population $k_0 = 7$ and having a uniform background rain as depicted in Fig. 5. We also present the final total water volume held by each population after $t_f = 25$ min. under communicated and independent configurations. Results show the advantage of the proposed algorithm as it effectively reduces the total volume at the sites of heavy rainfall, thus increasing their response capacity. The trade-off is noticeable in populations of lighter rainfall, where it is clear that remaining capacity is reduced. However, these regions can better withstand the incoming flow, showing how the local information sharing assists the global UDS.

Time dynamics of water volume in each reservoir are also compared in Fig. 6, where we focus on the central $k_0 = 7$ population. We observe a faster response in the communicated case due to the modified payoff, which further enhances the reaction to heavy rain. Furthermore, reservoirs 1 and 2 reach equal smaller volumes than the independent case, successfully increasing the remaining capacity as desired.

VI. Conclusions

In this work, we have analyzed two UDS subsystems: convergent and divergent topologies, from an equilibrium-independent passive perspective. We have showed the EIP

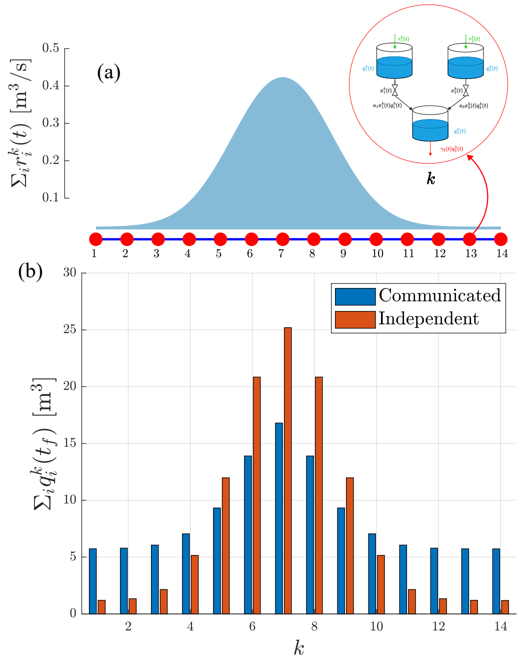

Fig. 5: (a) Non-uniform total rainfall centered at population $k_0 = 7$, and (b) comparison of final total water volume per population in communicated and independent designs.

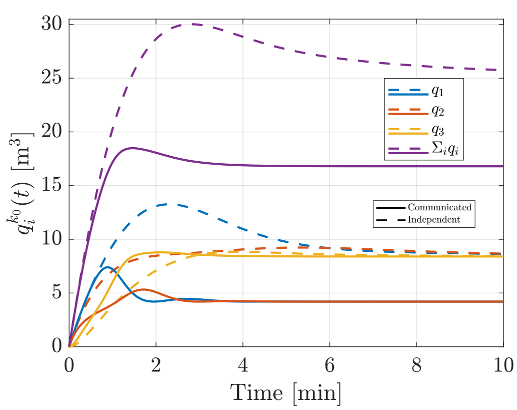

Fig. 6: Comparison of the dynamics of water volume per reservoir in population $k_0 = 7$ between the communicated (solid) and independent (dashed) design.

property of replicator dynamics, which allows us to design a payoff mechanism under the EDM-PDM scheme that ensures stability of the desired equilibrium point. Moreover, we have extended our payoff design to a distributed case using a consensus approach to model the sharing of information on a connected communication graph. Our results show that the proposed decentralized algorithm can be used to alleviate the load on particular populations with high inflow of wastewater in order to mitigate the risk of overflooding.

For future work, numerous improvements can be included to the presented approach. First, we have not yet taken into account the maximum reservoir capacities, which can be relevant for real implementations. Second, analytic conditions for stability of the augmented dynamics in Eq. (4) are still necessary to complement the numerical results on convergence. Finally, combining both the information with physical water flow between subsystems can be an important extension, leading to better performance of UDSs. Since information sharing across the network has been ensured, an improved distribution of water could be achieved by redirecting flow towards subsystems with less load.

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