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An $\mathcal{L}_p$-norm framework for event-triggered control

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***Abstract*— This paper presents a novel event-triggered control (ETC) design framework based on measured $\mathcal{L}_p$ norms. We consider a class of systems with finite $\mathcal{L}_p$ gain from the network-induced error to a chosen output. The $\mathcal{L}_p$ norms of the network-induced error and the chosen output since the last sampling time are used to formulate a class of triggering rules. Based on a small-gain condition, we derive an explicit expression for the $\mathcal{L}_p$ gain of the resulting closed-loop systems and present a time-regularization, which can be used to guarantee a lower bound on the inter-sampling times. The proposed framework is based on a different stability- and triggering concept compared to ETC approaches from the literature, and thus may yield new types of dynamical properties for the closed-loop system. However, for specific output choices it can lead to similar triggering rules as "standard" static and dynamic ETC approaches based on input-to-state stability and yields therefore a novel interpretation for some of the existing triggering rules. We illustrate the proposed framework with a numerical example from the literature.**

I. Introduction

Reducing the usage of communication resources when implementing feedback laws over shared communication networks is often necessary or beneficial for various recent control applications, e.g., in the field of Networked Control Systems (NCS) [1]. There, using shared communication networks to close feedback loops may, e.g., lead to reduced installation cost, more flexibility and better maintainability.

Event-triggered control (ETC) is a popular concept to trade-off control performance and the usage of communication[2], [3]. In ETC, sampling is triggered at runtime based on a triggering rule that depends on the system state, as opposed to time-triggered control, where sampling is triggered at predetermined time instants. The benefits of ETC have been investigated for the first time in [2], which initiated significant research activity, leading to many different ETC approaches. ETC triggering rules have been derived based on, e.g., input-to-state stability (ISS) [4], [5], passivity [6] or Lyapunov conditions [7], [8].

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Particularly relevant for the work herein is [9] that proposed a dynamic event-triggering scheme which can be interpreted as a filtered version of the event-triggering rule in [4]. A similar approach was studied in [10], where the integral of an ISS condition is considered and sampling is triggered as soon as the integral satisfies a trigger condition. Note that the approaches from [9] and [10] lead to similar triggering rules. In [11], the framework from [12], [13] is leveraged and a dynamic ETC triggering rule is proposed to guarantee a certain $\mathcal{L}_p$ gain.

In this paper, we propose a novel framework for ETC based on $\mathcal{L}_p$ norms that is conceptually different from other approaches in the literature. Inspired by the framework for the analysis of time-triggered sampling from [14], the considered class of NCS with event-triggered sampling consists of one subsystem that describes the behavior of plant- and controller states, and one subsystem that describes the evolution of the network-induced error. We consider an emulation scenario and assume thus that a controller is given, such that the $\mathcal{L}_p$ gain between the network-induced error and a chosen system output is known. We propose a class of triggering rules, that enforce sampling whenever the $\mathcal{L}_p$ gain from the chosen output to the network-induced error is sufficiently small. Stability guarantees are then obtained using a simple small-gain condition. We derive an explicit expression for the $\mathcal{L}_p$ gain for the chosen output of the resulting closed-loop system. Moreover, we present a time-regularization inspired by [14], which can be used to guarantee a lower bound on the inter-sampling times. It thus prevents Zeno behavior, i.e., the occurrence of an infinite number of sampling instants in a finite time interval.

The framework that is proposed in this paper is based on different technical concepts than the known approaches in the literature. Whilst it captures in general different types of dynamical system properties and thus allows different conclusions like a bound on the resulting $\mathcal{L}_p$ gain, it can also be used to derive similar triggering rules as in [4], [5] for static ETC, and in [9], [10] for dynamic ETC. However, the general concept and proofs for our proposed framework are different from existing approaches and in particular from those of [4], [5], [9], [10]. Hence it not only provides an alternative way to obtain stability guarantees and to handle disturbances for the setups and triggering rules from [4], [5], [9], [10], but also offers additional flexibility in the design of novel triggering rules. Moreover, whilst [11] can be seen as the event-triggered variant of the Lyapunov function based framework for stabilization of NCS from [12], [13], our proposed framework is inspired by the $\mathcal{L}_p$ gain framework to analyze NCS with time-triggered sampling from

[14]. Finally, we illustrate the proposed framework and the resulting trade-off between guaranteed $\mathcal{L}_2$ gain and average inter-sampling times for a specific numerical example.

This paper is structured as follows. In the remainder of this section, we introduce some notation and recap definitions from the literature that we use in the paper. In Section II, we present the considered NCS model. Section III summarizes the used small gain condition. The proposed ETC framework is presented in Section IV. We discuss the relations between the proposed framework and the results for static and dynamic ETC from [4], [5], [9], [10] in Section V. In Section VI, we illustrate the proposed framework by a numerical example. Section VII concludes the paper.

Notation and definitions

The nonnegative real numbers are denoted by $\mathbb{R}_{\geq 0}$. The natural numbers are denoted by $\mathbb{N}$, and we define $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. We use $(x, y) = \left[x^\top, y^\top\right]^\top$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. Given $t \in \mathbb{R}$ and a piecewise continuous function $f : \mathbb{R} \to \mathbb{R}^n$, we use the notation $f(t^+) := \lim_{s \to t, s > t} f(s)$. For $\alpha_1, \alpha_2 \in \mathcal{K}$, the condition $\alpha_1 \circ \alpha_2 < \mathrm{Id}$ means $\alpha_1(\alpha_2(s)) < s$ for all $s > 0$. We denote by $|\cdot|$ the Euclidean norm[1] of a vector or respectively the induced matrix norm for the Euclidean norm. Given a measurable, locally integrable signal $\varphi : [t_a, t_b] \to \mathbb{R}^n$ and $p \in [1, \infty)$, we denote its $\mathcal{L}_p$ norm as $\|\varphi\|_{\mathcal{L}_p[t_a, t_b]} := \left(\int_{t_a}^{t_b} |\varphi(s)|^p ds\right)^{\frac{1}{p}}$. For $p = \infty$, we denote the $\mathcal{L}_p$ norm as $\|\varphi\|_{\mathcal{L}_\infty[t_a, t_b]} := \operatorname{ess} \sup_{s \in [t_a, t_b]} |\varphi(s)|$. If $\varphi(\cdot)$ is defined on $[t_0, \infty)$ and for some $p \in [1, \infty]$, there exists $K \geq 0$ such that $\|\varphi\|_{\mathcal{L}_p[t_0, t]} \leq K, \forall t \geq t_0 \geq 0$, then we write[2] $\varphi \in \mathcal{L}_p$. We consider in this paper impulsive systems that are governed by equations of the form

$$\begin{aligned} \dot{x} &= f(x, w), \quad t \in [t_j, t_{j+1}] \\ y &= H(x, w) \\ x(t_j^+) &= h(x(t_j)) \end{aligned} \tag{1}$$

with state $x(t) \in \mathbb{R}^{n_x}$, input $w(t) \in \mathbb{R}^{n_w}$, output $y(t) \in \mathbb{R}^{n_y}$, a jump sequence $(t_j), j \in \mathbb{N}_0$ and with continuous functions $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_w} \to \mathbb{R}^{n_x}$, $H : \mathbb{R}^{n_x} \times \mathbb{R}^{n_w} \to \mathbb{R}^{n_y}$. We use the notion of solutions for these systems as in [14, Section II.B]. and the following definitions that are adopted from [14].

Definition 1: Let $p \in [1, \infty]$ be given. The system (1) is finite-gain $\mathcal{L}_p$ stable from w to y with gain γ if there exists $K > 0$ such that for all $t_0 \geq 0$, $x(t_0) \in \mathbb{R}^{n_x}$, $w \in \mathcal{L}_p$ and each corresponding solution $x(\cdot)$, we have that

$$\|y\|_{\mathcal{L}_p[t_0, t]} \leq K|x(t_0)| + \gamma \|w\|_{\mathcal{L}_p[t_0, t]} \;\; \forall t \in [t_0, t_0 + T]$$

where $[t_0, T]$ is the maximal interval of definition of $x(\cdot)$.

Definition 2: Let $p, q \in [1, \infty]$ be given. The state x of system (1) is said to be $\mathcal{L}_p$ to $\mathcal{L}_q$ detectable from output y with gain γ if there exist $K > 0$ and $\gamma > 0$ such that for all $t_0 \geq 0$, $x(t_0) \in \mathbb{R}^{n_x}$, $w \in \mathcal{L}_p$ and each corresponding solution $x(\cdot)$, we have $\forall t \in [t_0, t_0 + T]$ that $\|x\|_{\mathcal{L}_q[t_0, t]} \leq K|x(t_0)| + \gamma \|y\|_{\mathcal{L}_p[t_0, t]} + \gamma \|w\|_{\mathcal{L}_p[t_0, t]}$ holds, where $[t_0, T]$ is the maximal interval of definition of $x(\cdot)$.

[1]Note that the results of this paper also hold if we consider any other p-norm instead.

[2]In the literature, this is sometimes also denoted as extended $\mathcal{L}_p$ space.

II. Setup

We follow in this paper an emulation-based approach and thus start from a plant with known (continuously evaluated) controller, that guarantees a certain $\mathcal{L}_p$ gain for the closed-loop system with continuous feedback when network effects are ignored. We thus consider nonlinear plants of the form

$$\dot{x}_p = f_p(x_p, \hat{u}, w) \tag{2}$$

with state $x_p \in \mathbb{R}^{n_p}$, most recently received control input $\hat{u} \in \mathbb{R}^{n_u}$ and disturbance input $w \in \mathbb{R}^{n_w}$. The controller is given in the form

$$\begin{aligned} \dot{x}_c &= f_c(x_c, \hat{x}_p) \\ u &= g_c(x_c, \hat{x}_p) \end{aligned} \tag{3}$$

with controller state $x_c \in \mathbb{R}^{n_c}$ and last received value of the plant state $\hat{x}_p \in \mathbb{R}^{n_p}$. Note that the results of this paper apply also for static controllers of the form $u = g_c(\hat{x}_p)$. We suppose the controller is designed such, that the $\mathcal{L}_p$ gain from w to an output $y = H(x, e, w)$, where $y \in \mathbb{R}^{n_y}$ and $H : \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_w} \to \mathbb{R}^{n_y}$, is known. Further, we assume that f_p and f_c are continuous and g_c is continuously differentiable (the latter can be relaxed, see [14]).

The sampling times are described by a sequence $\{t_j\}_{j \in \mathbb{N}_0}$ that will be determined by the ETC triggering rule.

At sampling time t_j, $\hat{x}_p$ and $\hat{u}$ are updated as $\hat{x}_p(t_j^+) = x(t_j)$ and $\hat{u}(t_j^+) = u(t_j)$. Between sampling instants, i.e., for $t \in [t_j, t_{j+1}]$, we set $\dot{\hat{x}}(t) = 0$ and $\dot{\hat{u}}(t) = 0$. We thus consider a zero-order-hold scenario. We define the network-induced error as $e := \begin{pmatrix} \hat{x}_p - x_p \\ \hat{u} - u \end{pmatrix}$ with $e \in \mathbb{R}^{n_e}$ and $n_e = n_p + n_u$. Using in addition $x := (x_p, x_c)$ with $x \in \mathbb{R}^{n_x}$ for $n_x = n_p + n_c$, we can write the overall system in the form

$$\begin{aligned} &\left.\begin{aligned} \dot{x} &= f(x, e, w) \\ \dot{e} &= g(x, e, w) \end{aligned}\right\} t \in [t_j, t_{j+1}] \\ &x(t_j^+) = x(t_j) \\ &e(t_j^+) = 0, \\ &y = H(x, e, w) \end{aligned} \tag{4}$$

where f and g are continuous functions that are determined by f_p, f_c and g_c, see [14, Section III] for more details.

The triggering rule for the proposed ETC framework in this paper will be designed to ensure that the system (4) is finite-gain $\mathcal{L}_p$ stable from w to y and, under additional detectability assumptions, $\mathcal{L}_p$ stable from w to (x, e).

III. A small-gain perspective

In this section, we present a small-gain perspective that we will use within the proposed framework to analyze stability properties of the (4). The overall system can be considered for analysis purposes as a feedback-interconnection of two coupled subsystems

$$\begin{aligned} \dot{x} &= f(x,e,w) \quad t \in [t_j, t_{j+1}] \\ y &= H(x,e,w) \\ x(t_j^+) &= x(t_j) \end{aligned} \tag{5a}$$

and

$$\begin{aligned} \dot{e} &= g(x,e,w) \quad t \in [t_j, t_{j+1}] \\ y_2 &= W(e) \\ e(t_j^+) &= 0. \end{aligned} \tag{5b}$$

Here, (5b) describes the behavior of the sampling induced error e. Further, the continuous function $W : \mathbb{R}^{n_e} \to \mathbb{R}^{n_W}$ with $n_W \in \mathbb{N}$ and $W(0) = 0$ determines a fictional output of subsystem (5b), that is used for the analysis.

The first step of the emulation approach is thus to design a controller that guarantees a finite $\mathcal{L}_p$ gain from w to y. We thus make the following assumption on plant and controller.

Assumption 1: Let $p \in [1, \infty]$. The controller is designed such that subsystem (5a) satisfies $\gamma_x > 0$, $\gamma_w > 0$, $K_x > 0$ and $W : \mathbb{R}^{n_e} \to \mathbb{R}^{n_W}$ with $n_W \in \mathbb{N}$ and $W(0) = 0$ the finite-gain $\mathcal{L}_p$ stability condition

$$\begin{aligned} &\|y\|_{\mathcal{L}_p[t_0,t]} \\ &\leq K_x|x(t_0)| + \gamma_x \|W(e)\|_{\mathcal{L}_p[t_0,t]} + \gamma_{w,1}\|w\|_{\mathcal{L}_p[t_0,t]} \end{aligned} \tag{6}$$

for all $t \in [t_0, T]$ and $w \in \mathcal{L}_p$, where T is the maximal interval of definition of $x(\cdot)$.

Note that Assumption 1 implies that the $\mathcal{L}_p$ gain from the input $(W(e), w)$ to the output y is bounded by $\max\{\gamma_x, \gamma_{w,1}\}$. However, using different gains γ_x and $\gamma_{w,1}$ in (6) is potentially less conservative than using one common gain. Given that a controller has been designed to satisfy Assumption 1, similarly to [14], small-gain arguments can be used to guarantee $\mathcal{L}_p$ stability properties if the e-subsystem (5b) of system (5) satisfies the $\mathcal{L}_p$ stability condition

$$\begin{aligned} &\|W(e)\|_{\mathcal{L}_p[t_0,t]} \\ &\leq K_e|e(t_0)| + \gamma_e \|y\|_{\mathcal{L}_p[t_0,t]} + \gamma_{w,2}\|w\|_{\mathcal{L}_p[t_0,t]} \end{aligned} \tag{7}$$

for some $K_e > 0$, $\gamma_{w,2} \geq 0$, sufficiently small $\gamma_e > 0$ and all $t \in [t_0, T]$. In particular, the following small-gain condition that is adapted from [14, Theorem 1] can be used to conclude a finite $\mathcal{L}_p$ gain from w to y, and, if the states x and e of subsystems (5a) and (5b) are detectable from y and respectively $W(e)$, a finite $\mathcal{L}_p$ gain from w to (x, e).

Proposition 1: Suppose that for some $p \in [1, \infty]$ subsystem (5a) satisfies (6) and subsystem (5b) satisfies (7) for all $t \in [t_0, T]$, where T is the maximal interval of definition of $x(\cdot)$. If the small gain condition $\gamma_x\gamma_e < 1$ holds, then the system (5) is $\mathcal{L}_p$ stable from w to y with gain $\frac{\gamma_{w,1}+\gamma_x\gamma_{w,2}}{1-\gamma_x\gamma_e}$.

Moreover, if the state x of subsystem (5a) is $\mathcal{L}_p$ to $\mathcal{L}_p$ detectable from (y, w) with gain γ_d and the state e of subsystem (5b) is $\mathcal{L}_p$ to $\mathcal{L}_p$ detectable from $(W(e), w)$ with gain γ_d, then the system (5) is $\mathcal{L}_p$ stable from w to (x, e) with gain $\gamma_d \frac{\gamma_{w,1}(1+\gamma_e)+\gamma_{w,2}(1+\gamma_x)}{1-\gamma_x\gamma_e}$.

The main idea of the proposed ETC framework is thus to sample such that (7) is enforced with $\gamma_e < \frac{1}{\gamma_x}$.

Remark 1: Note that the conclusions from Proposition 1 are valid for the maximum interval of definition T for x. If the conditions for $\mathcal{L}_p$ stability from w to (x, e) are fulfilled and $w \in \mathcal{L}_p$, then $T = \infty$. Otherwise, additional conditions on the parts of x that are not captured by $H(x, e, w)$ may be required to guarantee that $T = \infty$.

Moreover, Proposition 1 does not guarantee the existence of a lower bound on $t_{j+1} - t_j$, which would be a sufficient condition that there is no Zeno behavior. Instead, such a lower bound will later be derived independent of Proposition 1 for the proposed ETC framework.

Remark 2: In our analysis, the function W serves a similar purpose as the function W in [14], where it describes a Lyapunov function for the protocol that characterizes the update of the error subsystem at sampling times. Further W must be chosen in a suitable way such that it can be treated as an input to (5a). In [14], a lower bound on $W(e)$ in terms of a $\mathcal{K}$-function in $|e|$ is required to guarantee this. Compared to the setup from [14], there are some differences in the way how W is defined in this paper. Since we consider a sampled-data setup, instead of an assumption on the protocol, we simply use the condition $W(0) = 0$ that ensures that $W(e(t_j^+)) = W(0) = 0$. In addition, we do not require W in our setup to be scalar or positive. Further, we do not make a specific restriction on $W(e)$ in terms of a $\mathcal{K}$-function in $|e|$ as lower bound. Instead, Assumption 1 requires implicitly that W is chosen suitably, such that it can be treated as an input to (5a) for which (6) holds.

IV. A Framework for $\mathcal{L}_p$ Gain Based Sampling

In this section, we present the general idea for the proposed ETC framework and present results on $\mathcal{L}_p$ stability and a lower bound on the inter-event times. To ensure that (7) holds, and thus stability guarantees are obtained using Proposition 1, our goal is to trigger such, that

$$\|W(e)\|_{\mathcal{L}_p[t_j,t]} \leq \gamma_e \|H(x,e,w)\|_{\mathcal{L}_p[t_j,t]} + \gamma_{w,2}\|w\|_{\mathcal{L}_p[t_j,t]} \tag{8}$$

holds for $t \in [t_j, t_{j+1}]$ for all $j \in \mathbb{N}_0$. We discuss first how a sufficient condition for (8) can be implemented as a trigger condition. Then, we focus on how to additionally guarantee a lower bound on the inter-event times using a bound on $\|W(e)\|_{\mathcal{L}_p[t_j,t]}$ that depends on $\|H(x,e,w)\|_{\mathcal{L}_p[t_j,t]}, \|w\|_{\mathcal{L}_p[t_j,t]}$ and the time between sampling instants.

A. General triggering concept

Given a system that satisfies Proposition 1 for specific γ_x, our goal is to give stability guarantees using Proposition 1. This can be ensured by triggering sampling instants such that

$$t_{j+1} \leq t_{j+1}^{**} := \inf\left\{ t > t_j : \frac{\|W(e)\|_{\mathcal{L}_p[t_j,t]}}{\|y\|_{\mathcal{L}_p[t_j,t]}} \geq \gamma_e \right\} \tag{9}$$

holds for $\gamma_e < \frac{1}{\gamma_x}$. Observe that (9) implies that $\|W(e)\|_{\mathcal{L}_p[t_j,t]} \leq \gamma_e \|H(x,e,w)\|_{\mathcal{L}_p[t_j,t]}$ and thus that (8) hold for $t \in [t_j, t_{j+1}]$ and thus by summing (8) up from t_0 until t_j that (7) holds if it holds for any $j \in \mathbb{N}$. The proposed framework thus covers the class of triggering rules that checks (9) for different p and different choices of W and y directly. However, it is not limited to this class. It also allows for triggering rules that lead to sufficient conditions for (9). We will use this fact in Section V to illustrate how the proposed framework also covers static and dynamic ETC triggering rules from the literature.

Remark 3: Implementing condition (9) requires measurements of appropriate signals. In particular, $\|W(e)\|_{\mathcal{L}_p[t_j,t]}$ and $\|y\|_{\mathcal{L}_p[t_j,t]} = \|H(x,e,w)\|_{\mathcal{L}_p[t_j,t]}$ can be determined by integrating $|W(e)|^p$ and respectively $|y|^p$ and taking the p-th root. If y cannot be measured, then it can still be computed based on x and e, if these are measured and H does not depend on w. However, if H depends on w and w cannot be measured, then measurements of y must be available to compute $\|y\|_{\mathcal{L}_p[t_j,t]}$.

Remark 4: If $w(t)$ can be measured for all t, which may be possible for some applications like reference tracking, then we can even consider the less conservative condition

$$t_{j+1} \leq t^*_{j+1} := \inf\left\{ t > t_j : \frac{\|W(e)\|_{\mathcal{L}_p[t_j,t]} - \gamma_{w,2}\|w\|_{\mathcal{L}_p[t_j,t]}}{\|y\|_{\mathcal{L}_p[t_j,t]}} \geq \gamma_e \right\} \tag{10}$$

instead of (9).

Proposition 1 does not make any statement about inter-event times. We will discuss in the next subsection, how a lower bound on the inter-event times can be obtained for triggering rules that ensure (9).

B. A lower bound on the inter-event times

To guarantee a lower bound on the inter-event times and thus Zeno-freeness, time regularization as it was proposed in [15] can be used. Note that a lower bound on the inter-event times is also important for the practical implementability of the approach. We explicitly determine a lower bound on the time for which (8) is satisfied after a sampling time, leveraging again the results from [14]. If the ETC mechanism without time regularization would trigger before this minimum inter-event time has lapsed, then the mechanism with time regularization triggers instead as soon as the minimum inter-event time has lapsed. The triggering rule with time regularization is thus given by

$$t_{j+1} = \max\left\{ t_j + \delta, t^{**}_{j+1} \right\}, \tag{11}$$

where the minimum inter-event time $\delta > 0$ still needs to be determined. To do so, we adapt the following assumption from [14].

Assumption 2 ([14]): For all x, w and almost all[3] e, there exist $L \geq 0$ and $\gamma_{w,3} > 0$ such that

$$\left\langle \frac{\partial |W(e)|}{\partial e}, g(x,e,w) \right\rangle \leq L|W(e)| + |H(x,e,w)| + \gamma_{w,3}|w|. \tag{12}$$

The main difference of Assumption 2 compared to its counterpart in [14] is that we introduce the additional term $\gamma_{w,3}|w|$. Compared to [14], this allows to chose H independent from w. Note that it is not restrictive since H can still be chosen as in [14] if this is desired, as long as y or w can be measured. We will discuss later how Assumption 2 can further be relaxed. Moreover, note that vectornorms are considered in Assumption 2, since different from [14], we do not require W to be scalar. However, we can recover the results from [14] when considering the norm of W.

Based on Assumption 2, we can state the following adapted version of [14, Proposition 6].

Proposition 2: Consider the system (5) and let Assumption 2 hold. If $t_{j+1} \in (t_j, t_j + \delta]$ for $\delta := \frac{\kappa}{L}\ln(1 + \gamma^* L) > 0$ for arbitrary[4] $\kappa \in (0,1)$ where $\gamma^* := \min\left\{ \gamma_e, \frac{\gamma_{w,2}}{\gamma_{w,3}} \right\}$ then (8) holds for $t \in [t_j, t_{j+1}]$.

Thus when Assumption 2 holds, then there is a non vanishing time interval δ such that (8) holds if $t_{j+1} - t_j \leq \delta$. This time interval can be used as minimum sampling interval for time regularization. The original triggering rule is only used if it leads to larger sampling intervals than δ. Due to the time regularization, we can thus still conclude that (7) holds for all times, even if there are sampling intervals for which (9) does not hold but the time regularization is used.

Note that the proposed framework is however not limited to time-regularization. Instead, other approaches to guarantee an upper bound on the inter-event time can be used. For example, the approaches for static and dynamic ETC from [4], [5], [9], [10] can be adopted as well, as discussed in Section V.

C. $\mathcal{L}_p$ stability results

For the triggering rule given by (11) with t^{**}_j according to (9), we can state the following result.

Theorem 1: Consider the system (5) and let Assumptions 1 and 2 hold. If the sequence (t_j) is generated by (11) with t^{**}_{j+1} according to (9) for some $\gamma_e \in \left(0, \frac{1}{\gamma_x}\right)$, $\gamma_{w,2} \geq 0$ and with δ as in Proposition 2, then the system (5) is $\mathcal{L}_p$ stable from w to y with gain $\gamma = \frac{\gamma_{w,1} + \gamma_x\gamma_{w,2}}{1 - \gamma_x\gamma_e}$ and $t_{j+1} - t_j \geq \delta$. Moreover, if the state x of subsystem (5a) is $\mathcal{L}_p$ to $\mathcal{L}_p$ detectable from (y, w) with gain γ_d and the state e of subsystem (5b) is $\mathcal{L}_p$ to $\mathcal{L}_p$ detectable from $(W(e), w)$ with gain γ_d, then the system (5) is $\mathcal{L}_p$ stable from w to (x, e) with gain $\gamma = \gamma_d\frac{\gamma_{w,1}(1+\gamma_e) + \gamma_{w,2}(1+\gamma_x)}{1 - \gamma_x\gamma_e}$.

Proof: The lower bound on $t_{j+1} - t_j$ follows for all j immediately from (11). If $t_{j+1} = t_j + \delta$ for some $j \in \mathbb{N}_0$, then (8) holds for $t \in [t_j, t_{j+1}]$ due to Proposition 2 and Assumption 2. Otherwise, (9) and thus (8) hold for $t \in [t_j, t_{j+1}]$. Hence (8) holds for all $j \in \mathbb{N}_0$ for $t \in [t_j, t_{j+1}]$. Summing it up for all j, we can conclude that (7) holds for all $t \geq t_0$. The statement of the theorem then follows immediately due to Proposition 1 and Assumption 1. ∎

Some remarks are in order.

Remark 5: The way how w enters in Assumptions 1 and 2 may potentially be restrictive. It can however be relaxed

[3] All except for a set of Lebesque measure 0.

[4] To make δ large, κ should typically be chosen close to 1.

by considering $\alpha(|w|)$ instead of $|w|$ for some $\alpha \in \mathcal{K}$ in (6) and (12). Then, one verifies

$$\|W(e)\|_{\mathcal{L}_p[t_0,t]} \leq K_e(|e(t_0)|) + \gamma_e \|y\|_{\mathcal{L}_p[t_j,t]} + \gamma_{w,2}\|\alpha(|w|)\|_{\mathcal{L}_p[t_j,t]},$$

instead of (7). In this case, the conclusions of Theorem 1 regarding $\mathcal{L}_p$ stability still hold for the $\mathcal{L}_p$ gain from $\alpha(|w|)$ to y instead from w to y.

Remark 6: Note that in Theorem 1, $\lim_{\gamma_e,\gamma_{w,2}\to 0} \gamma = \gamma_{w,1}$. Thus $\gamma_e > 0$ and $\gamma_{w,2} \geq 0$ can be chosen such that the $\mathcal{L}_p$ gain γ is arbitrarily close to the $\mathcal{L}_p$ gain of the original system. However, it has in general to be expected that smaller values for γ_e and $\gamma_{w,2}$ also lead to smaller sampling intervals and more sampling instants. The resulting trade-off is studied for a particular example in Section VI.

Remark 7: In (9), the norms of $W(e)$ and y are only considered over the recent sampling interval. Note that instead, for $p \in [1,\infty)$, it is also possible to consider $\|W(e)\|_{\mathcal{L}_p[t_0,t]}$ and $\|y\|_{\mathcal{L}_p[t_0,t]}$ in (9) to determine sampling intervals, which also implies that (7) is satisfied. This may be advantageous to increase future sampling intervals if sampling instants are triggered earlier than prescribed by t^{**}_{j+1}.

For $p = \infty$, it is alternatively possible to consider $\|W(e)\|_{\mathcal{L}_p[t_j,t]}$ and $\|y\|_{\mathcal{L}_p[t_0,t]}$ in (9), i.e., to compare the essential supremum of $W(e)$ since the last sampling time to the essential supremum of y since t_0. For $p = \infty$, this also implies that (7) is satisfied, but may be less restrictive than considering $\|y\|_{\mathcal{L}_p[t_j,t]}$ in (9).

Remark 8: For $p \in [1,\infty)$, we note that $\|y\|_{\mathcal{L}_p} < \infty$ implies that $y(t)$ converges to 0 as $t \to \infty$ (cf. [16, Lemma 8.2]). In particular, if additionally, $y = \alpha(|x|)$ for some $\alpha \in \mathcal{K}$, then this means that our proposed framework can also be used to obtain guarantees for asymptotic stability.

V. Relationship with Existing Triggering Rules

In this section, we illustrate how ETC triggering rules from the literature are covered by the proposed framework. In particular, we discuss how guarantees for $\mathcal{L}_p$-stability for static ETC triggering rules in the form from, e.g., [4], [5] and for the dynamic ETC triggering rules from [9], [10] can be obtained by using the proposed framework. As in the references [4], [5], [9], [10], we mainly focus in this section on the disturbance-free case, i.e., we assume $w = 0$. First, we focus on the relation of the proposed framework to static ETC. In the second subsection, the relation of the proposed framework to dynamic ETC is discussed.

A. Static ETC

In this subsection, we illustrate how the static ETC triggering rule from [4], [5] is covered by the proposed framework. We consider here static ETC triggering rules of the form

$$t_{j+1} = \inf\{t > t_j : \gamma_s(|e(t)|) \geq \alpha_s(|x(t)|)\}. \tag{13}$$

for $\alpha_s, \gamma_s \in \mathcal{K}$. Such triggering rules were, e.g., used in [4], [5]. In [5], an ISS condition in the "max" form

$$\|x\|_{\mathcal{L}_\infty} \leq \max\left\{\beta_{\max}(|x(t_0)|), \|\gamma_{\max}(|e|)\|_{\mathcal{L}_\infty}\right\} \tag{14}$$

for $\beta_{\max}, \gamma_{\max} \in \mathcal{K}$ is considered, leading in [5] to the triggering rule

$$t_{j+1} = \inf\{t > t_j : \rho(|x(t)|) - |e(t)| \leq 0\}, \tag{15}$$

where ρ can be any $\mathcal{K}$-function satisfying

$$\rho \circ \gamma_{\max} < \mathrm{Id}. \tag{16}$$

Note that (15) is equivalent to (13) for $\alpha_s = \rho$ and $\gamma_s = 1$.

To relate the approach from [5] to our proposed framework, first note that the ISS condition in the "max" form from (14) implies[5] the ISS condition in "plus" form given by the $\mathcal{L}_\infty$-gain condition

$$\|x\|_{\mathcal{L}_\infty} \leq \beta_{\max}(|x(t_0)|) + \gamma_{\max}(\|e\|_{\mathcal{L}_\infty}), \tag{17}$$

and thus

$$\left\|\gamma_{\max}^{-1}(|x|)\right\|_{\mathcal{L}_\infty} \leq \gamma_{\max}^{-1}(\beta_{\max}(|x(t_0)|)) + \|e\|_{\mathcal{L}_\infty}, \tag{18}$$

which corresponds to (6) with $\gamma_x = 1, W(e) = |e|$ and $H(x,e,w) = \gamma_{\max}^{-1}(|x|)$.

Using our proposed framework for the $\mathcal{L}_\infty$-norm case[6], we thus obtain

$$t^{**}_{j+1} = \inf\left\{t > t_j : \|e\|_{\mathcal{L}_\infty[t_j,t]} \geq \gamma_e \left\|\gamma_{\max}^{-1}(|x|)\right\|_{\mathcal{L}_\infty[t_0,t]}\right\} \tag{19}$$

for any $\gamma_e < 1$ as equivalent for (9). Note that

$$\inf\left\{t > t_j : \|e\|_{\mathcal{L}_\infty[t_j,t]} \geq \gamma_e \left\|\gamma_{\max}^{-1}(|x|)\right\|_{\mathcal{L}_\infty[t_0,t]}\right\} \geq \inf\left\{t > t_j : |e(t)| \geq \gamma_e \gamma_{\max}^{-1}(|x(t)|)\right\}$$

which implies that

$$t^{**}_{j+1} \geq \inf\left\{t > t_j : |e(t)| \geq \gamma_e \gamma_{\max}^{-1}(|x(t)|)\right\}. \tag{20}$$

Hence, for any ρ satisfying $\rho(s) \leq \gamma_e \gamma_{\max}^{-1}(s)$ for all s and arbitrary $\gamma_e \in (0,1)$, and thus for all ρ such that $\rho \circ \gamma_{\max} \leq (1-\epsilon)\,\mathrm{Id}$ for arbitrary small $\epsilon > 0$, our proposed approach also implies a guarantee for a finite $\mathcal{L}_\infty$-gain for the triggering rule from [5]. Note that this condition is equivalent to (16) on compact sets. Thus, our proposed framework can be used almost for the same set of triggering rules as the approach from [5]. We only need to exclude here the rather constructed case that $\rho(s)$ converges to $\gamma_{\max}^{-1}(s)$ as $s \to \infty$. In turn, our proposed framework may lead to less restrictive triggering rules when the ISS condition in "plus" form allows different $\mathcal{K}$-functions as the ISS condition in "max" form. Moreover, it naturally allows under the same assumptions as in [5] the potentially less restrictive triggering rule (19) based on the $\mathcal{L}_\infty$ norms of signals instead of their current values. In [4], an ISS condition in the dissipation form

$$\frac{\partial V(x)}{\partial x} f(x,e) \leq -\alpha_{\text{diss}}(|x|) + \gamma_{\text{diss}}(|e|) \tag{21}$$

[5]Since $\max\{a,b\} \leq a+b \leq \max\{2a,2b\}$, if the ISS condition in the "max" form holds, then the ISS condition in the "plus" form holds with the same $\mathcal{K}$ functions. In turn, if the condition in the "plus" form holds, then the condition in the "max" form holds as well, but potentially with larger (and thus for the triggering rule more restrictive) $\mathcal{K}$ functions.

[6]As discussed in Remark 7, we consider $\|W(e)\|_{\mathcal{L}_p[t_j,t]}$ and $\|y\|_{\mathcal{L}_p[t_0,t]}$.

for a positive definite function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ and class $\mathcal{K}$ functions α_{diss} and γ_{diss} is considered, leading to a triggering rule of the form (13) with $\alpha_s = \alpha_{\text{diss}}$ and $\gamma_s = \gamma_{\text{diss}}$. Note that according to [17], the ISS condition in dissipation form (21) implies an ISS condition in "plus" form of (17). Thus, whenever the approach from [4] can be used to derive a triggering rule of the form (13), then the proposed framework can be used to derive one as well. In the general case, the resulting functions α_s and γ_s may however be different. Moreover, our proposed approach can be viewed as a trajectory based alternative to the results from [4] with the potential advantage that it does not require knowledge of a Lyapunov function.

Furthermore, as analyzed in [18], the approaches from [4], [5] may lead, even for arbitrarily small disturbances w, to Zeno behavior. In [18], [5], it is proposed to add a constant term to the right hand side of the triggering rule (13), leading to $t_{j+1} = \inf\{t > t_j : \gamma_s(|e(t)|) \geq \alpha_s(|x(t)|) + d\}$ for some $d > 0$. Note that our proposed framework does not suffer from the same shortcoming when using (19) directly, since the $\gamma_e \|\gamma_{\max}^{-1}(|x|)\|_{\mathcal{L}_\infty[t_0,t]}$ term is, e.g., lower bounded by $\gamma_e \gamma_{\max}^{-1}(|x(0)|)$ and does thus not vanish even if $x(t)$ goes to 0. Using this fact, one can straightforwardly modify the results for Zeno-freeness from [18], [5] to conclude Zeno-freeness for (19) under the same assumptions as in [18], [5] if arbitrary large but essentially bounded disturbances are present. As an alternative, the proposed time regularization can be used which yields in our proposed framework a natural way to guarantee Zeno-freeness and a tunable bound on the resulting $\mathcal{L}_\infty$-gain.

B. Dynamic ETC

In this subsection, we study the relation of the proposed framework to dynamic ETC triggering rules and study how they can be captured by it. We consider here dynamic ETC triggering rules of the form

$$t_{j+1} := \inf\{t > t_j : \eta(t) \geq 0\} \tag{22}$$

for η satisfying $\dot{\eta} = -\beta(\eta) + \sigma\alpha_d(|x|) - \gamma_d(|e|)$, for[7] $\eta(0) = 0$, where $\alpha_d, \gamma_d \in \mathcal{K}$, $\beta \in \mathcal{K} \cup \{0\}$ and $\sigma \in (0, 1)$. Such approaches have, e.g., been proposed in [9], [10]. In both papers, the authors consider an ISS condition in the dissipation form (21) and derive a triggering rule of the form (22) with $\alpha_d = \alpha_{\text{diss}}$ and $\gamma_d = \gamma_{\text{diss}}$ (however stated in [10] in an integral form).

Note that due to the comparison Lemma [16, p. 102], it holds for any $\beta \in \mathcal{K} \cup \{0\}$ and $\alpha_d = \alpha_{\text{diss}}$ and $\gamma_d = \gamma_{\text{diss}}$ that $\eta \leq \tilde{\eta}$, where $\tilde{\eta}$ is the solution of $\dot{\tilde{\eta}} = \sigma\alpha_{\text{diss}}(|x|) - \gamma_{\text{diss}}(|e|)$ for $t \in [t_j, t_{j+1}]$ with $\tilde{\eta}(t_j) = \eta(t_j)$. It holds for $t \in [t_j, t_{j+1}]$ that

$$\begin{aligned}\tilde{\eta}(t) &= \int_{t_j}^{t} \sigma\alpha_{\text{diss}}(|x(\tau)|) - \gamma_{\text{diss}}(|e(\tau)|)\mathrm{d}\tau \\ &= \sigma\|\alpha_{\text{diss}}(|x|)\|_{\mathcal{L}_1[t_j,t]} - \|\gamma_{\text{diss}}(|e|)\|_{\mathcal{L}_1[t_j,t]}.\end{aligned}$$

[7] To simplify notation, we restrict ourselves here w.l.o.g. to $\eta(0) = 0$ instead of considering arbitrary $\eta(0) \geq 0$.

Hence, triggering rule (22) ensures that $\sigma\|\alpha_{\text{diss}}(|x|)\|_{\mathcal{L}_1[t_j,t]} - \|\gamma_{\text{diss}}(|e|)\|_{\mathcal{L}_1[t_j,t]} = \tilde{\eta}(t) \geq \eta(t) \geq 0$ and thus that $t_{j+1} \leq t_{j+1}^{**}$ for t_{j+1}^{**} according to (9) with $\gamma_e = \sigma$. Furthermore, the ISS condition in dissipation form (21) implies the $\mathcal{L}_1$-gain condition $\|\alpha_{\text{diss}}(|x|)\|_{\mathcal{L}_1} \leq K_x(|x(0)|) + \|\gamma_{\text{diss}}(|e|)\|_{\mathcal{L}_1}$ for some $K_x \in \mathcal{K}$ and thus that (6) holds with $\gamma_x = 1$. Thus, the dynamic ETC triggering rules according to [9], [10] are captured by our proposed framework that can be used as an alternative approach to obtain a guarantee for $\mathcal{L}_1$-stability and, since the above choice corresponds to $y = \alpha_{\text{diss}}(|x|)$ for $\alpha_{\text{diss}} \in \mathcal{K}$, also for asymptotic stability.

Besides that, the guarantee for positive inter-event times for the triggering rules from [10] can be adopted for our proposed framework for the disturbance free case if $W(e)$ and $H(x, e, w)$ are chosen in a suitable way and can then serve as an alternative for the time regularization in the proposed framework.

Moreover, our proposed framework thus yields a natural way to extend the dynamic ETC triggering rules from [9], [10] to handle $\mathcal{L}_1$-disturbances and to derive dynamic ETC schemes for different $p \in [2, \infty)$. It thus yields an approach to guarantee a certain performance in terms of a guaranteed $\mathcal{L}_p$ gain. Furthermore, it also allows different choices for the functions $W(e)$ and $H(x, e, w)$ that are not restricted to $\mathcal{K}$-functions, leading also potentially to more flexible triggering rules for dynamic ETC compared to those from [9], [10], that were restricted to $\mathcal{K}$ functions of $|x|$ and $|e|$.

Remark 9: In [9], also hybrid triggering rules using the sum of a static and a dynamic triggering rule are proposed. Note that such triggering rules can also be derived for our proposed framework by considering the sum of an $\mathcal{L}_1$ and an $\mathcal{L}_\infty$ triggering rule. Then, one obtains a bound on either the $\mathcal{L}_1$ or the $\mathcal{L}_\infty$ norm of the output.

VI. Example

In this section, we illustrate the proposed ETC framework with a numerical example. We consider the same example that was used in [19]. The example system is a single-link robot arm described by

$$\begin{aligned}\dot{x}_1 &= x_2 + w \\ \dot{x}_2 &= -\sin(x_1) + \hat{u}\end{aligned}$$

with the static state feedback controller $u = \sin(x_1) - 3x_1 - 2x_2$. We define $x = (x_1, x_2)$ and $e = (e_1, e_2)$ and chose $W(e) = [|e_1|, |e_2|]^\top$ and $H(x, e, w) = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} x + 0.6 \left(\begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin(x_1) - \sin(x_1 + e_1) \end{bmatrix} \right)$. Moreover we select $p = 2$ for which we can use the same approach as in [20] to compute γ_x and $\gamma_{w,1}$ and to verify Assumption 2.

Similar as in [19], we first consider the disturbance free case (i.e. $w(t) = 0$ for all $t \geq t_0$). In this case, Assumption 1 holds with $\gamma_x = 3.008$ and Assumption 2 holds with $L = 1.6971$ leading for $\kappa = 0.999$ to $\delta = 0.2636\,\text{s}$.

We conducted simulations for the same set of 10 different initial conditions for $x(0)$, which are uniformly distributed

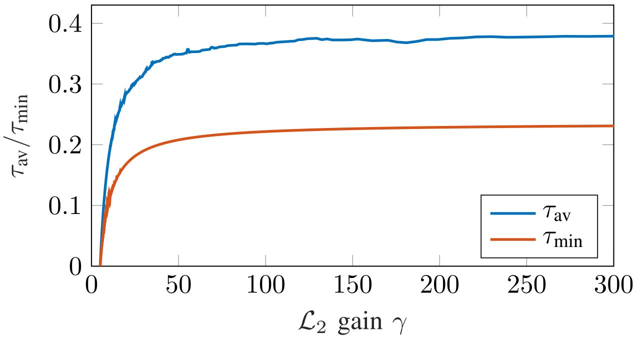

Fig. 1. Trade-off curve between guaranteed $\mathcal{L}_2$ gain and minimum (τ_{min}) and average (τ_{avg}) sampling intervals.

over a circle centered at the origin with radius 20, that was used in [19]. As in [19], we have always selected $e(0) = 0$. Additionally, we chose $\gamma_e = 0.3291 = \frac{0.99}{\gamma_x}$. We obtained $\tau_{\text{avg}} = 0.5380$ as average inter-sampling time, $\tau_{\text{min}} = 0.4374$ as minimum inter-sampling time and $\tau_{\text{max}} = 0.5989$ as maximum inter-sampling time.

Next we illustrate the case with disturbances, i.e., with arbitrary w. In this case, Assumption 1 holds, e.g.[8], with $\gamma_x = 3.041$ and $\gamma_{w,1} = 5$. Assumption 2 still holds with $L = 1.6971$. To illustrate the trade-off between inter-sampling times and the guaranteed $\mathcal{L}_2$ gain γ, we conducted simulations with the same 10 initial conditions as above but with the disturbance $w(t) = \sin(t)$. To vary the guaranteed $\mathcal{L}_2$ gain, we sampled different values of γ_e uniformly in the interval $[0.01, 0.99]\frac{1}{\gamma_x}$ and $\gamma_{w,3}$ uniformly in the interval $2 \cdot [0.01, 0.99]$. Figure 1 shows the resulting minimal and average inter-sampling times that were observed in the simulations as a function of the chosen $\mathcal{L}_2$ gain. For values of γ_e and $\gamma_{w,3}$ that lead to a small guaranteed $\mathcal{L}_2$ gain, the minimum and average inter-sampling times are small but increase as a larger overall $\mathcal{L}_2$ gain γ is allowed.

VII. Conclusion

We have proposed a novel framework for dynamic ETC based on measured $\mathcal{L}_p$ norms. Transmissions are triggered such that the $\mathcal{L}_p$ gain of the subsystem that describes the sampling induced error is sufficiently small so stability can be concluded from a simple small-gain condition. We have also presented a time-regularization that can be used to guarantee a lower bound on the inter-sampling times. The framework that is proposed in this paper is based on different technical concepts than the known approaches in the literature. Still, it covers common dynamic and static triggering rules from the literature. Besides providing an alternative way to interpret static and dynamic trigger rules, it further provides an approach to obtain $\mathcal{L}_p$ stability guarantees for the setups from [4], [5], [9], [10]. The proposed strategy was illustrated with a numerical example highlighting the trade-off between guaranteed $\mathcal{L}_p$ gain and average inter-sampling times.

[8]Note that there is a trade-off between γ_x and $\gamma_{w,1}$ for the considered example.

Our future research will focus on extending the framework by finding different conditions to guarantee a lower bound on the inter-sampling times, further investigation of the connections to different ETC approaches and on more realistic network setups, e.g., by considering decentralized systems.

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