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Traffic Light Control to Form Progressive Movements along an Arterial

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Abstract— A traffic light control protocol is proposed, which builds compact platoons of vehicles to make efficient use of green times. It forms short platoons at multiple links and times traffic lights to concatenate these to form a single dense platoon, in which vehicles mostly passes many intersections without stopping. The protocol minimizes the number of times vehicles stop; by simulation, it roughly halves that of greenwave under near saturated demand.

I. INTRODUCTION

Road traffic congestion in the US wasted 99 hours per person and cost $88 billion in lost time and excess fuel in 2019 [1]. To combat this, we developed a new paradigm to represent traffic dynamics and a new protocol using that paradigm for coordinating signals along an arterial road. The protocol extends the traditional green wave [2], in which a vehicle traveling along with the green wave will experience a cascade of green lights, and not stop at intersections.

Little [2] defined the bandwidth for a traffic direction of an arterial using a bi-directional green wave as the portion of a signal cycle that allows uninterrupted traffic flow in a direction along the entire arterial. This has been extended as follows. Chang et al. [3] include left-turn phase sequences in addition to green lights. Gartner et al. [4] allow different bandwidths for each segment of a single arterial, and Stamatiadis [5] and Gartner and Stamatiadis [6] consider a network of arterials. Tsay and Lin [7] and Lin et al. [8] relax the assumption that all queues at the traffic lights are dissipated within the cycle time. Recently, to save traffic energy consumption and network travel time, De Nunzio et al. [9] incorporate advisory speeds of links. However, if traffic load is too heavy or close to the intersection capacity, using such approaches can exacerbate congestion [10] and can become worse than using fixed cycles. This is because it causes the green light to start when the next platoon arrives, rather than causing the current queue to empty at that time.

Instead, a model-based control using a model that delineates the interactions between scheduling green lights and the response of traffic flow to that schedule is needed. Such models include [11], [12], [13], based on various model predictive control frameworks, equilibrium constraints. However, these models are for optimizing green time splits of a signal cycle, and lack the precision needed for determining when to start green lights in response to traffic situations.

To smooth traffic flow, Dotoli et al. [14], as an aside, suggest scheduling the green light at an intersection such that the front vehicle from the intersection upstream arrives at that intersection at the departure time of all vehicles previously in the link joining them. Their approach computes offsets of green lights based on estimated queue sizes, computed using dynamic equations that make the simplifying (but contradictory) assumption that all cycles at all intersections start simultaneously. Despite this simplification, simulations show such an approach can improve performance.

Formally generalizing and implementing the idea of Dotoli et al. [14] requires relatively sophisticated mathematical machinery. We develop a new paradigm of asynchronous systems to represent the traffic dynamics and implement it in real-time. The goal is to find times at which to start the green light of the traffic flow at every intersection to minimize the number of times vehicles are delayed. Due to space limitations, all proofs have been omitted; they are available from the authors.

II. Traffic Model

To align with the progressive movement, the model in this paper does not directly use the absolute time of a shared clock. Instead, each intersection has its proxy of time, which is its current cycle number.

A. General Notation

Bold letters denote vectors or tuples containing elements of the same type. Let $[t_1,t_2) := \{t : t_1 \le t < t_2\}$ and, given $I = [t_1,t_2)$, let $I+t = [t_1+t, t_2+t)$, for $t \in \mathbb{R}$.

Given value x that depends upon a large number of parameters, we write $x(u)$ with $u \subseteq s$, to denote the dependence of x on the given u, while keeping the parameters $s \setminus u$ unspecified. When we compare $x(u=u_1)$ and $x(u=u_2)$, we assume that those factors not included

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in u can be arbitrary but take the same value in a comparison.

Let $[n] = \{1, 2, \ldots, n\}$, and a_i be i^{th} element of vector $\mathbf{a}$. Let $|s|$ be the cardinality of set s.

B. Traffic Path

Consider a K-link traffic path along an arterial whose intersections with crossing streets are all signalized. Let

- $\boldsymbol{\pi}$ tuple $[\pi_i : i \in [K]]$ of links, with the path traversing $\pi_1, \pi_2, \ldots, \pi_K$ in this order;
- $\boldsymbol{\theta}$ tuple $[\theta_i : i \in [K] \cup \{0\}]$ of intersections, with θ_i joining π_i to π_{i+1}, and θ_0 being the source;
- l_i length of π_i including the road segment within θ_{i-1};
- w_i fixed number of traffic lanes of π_i;
- v_i fixed nominal speed of π_i;
- x_i distance from the source θ_0 to θ_i, $\sum_{k=1}^{i} l_k$;
- $\lambda_i(t)$ arrival rate of vehicles entering the arterial road at θ_i per unit time at absolution time t;
- $\gamma_i(t)$ turn ratio of θ_i at time t, defined as the fraction of the arrival rate of vehicles entering θ_i from π_i that continue on to link π_{i+1}, with $\gamma_0 = 1$. for all $0 \le j \le i \le K$.

C. Vehicle Mobility

We now define the vehicle dynamics. The position of a vehicle e at absolute time t is $x(e,t)$. If e is in the arterial at t, $x(e,t)$ is the distance from that vehicle to the origin of the path at absolute time t. If e is not in the arterial at t, we define $x(e,t) = \zeta$, where $\zeta \notin \mathbb{R}$, so both $x \le \zeta$ and $x > \zeta$ are false for all $x \in \mathbb{R}$. Consequently, the set of possible vehicle positions $A = [0, \sum_{k=1}^{K} l_k] \cup \{\zeta\}$. Given $x \in A \setminus \{\zeta\}$, define $\mu(x) = i$ if $x \in [\sum_{k=1}^{i-1} l_k, \sum_{k=1}^{i} l_k)$, in which $\sum_{k=1}^{0} l_k = 0$.

Given an arbitrary vehicle e and absolute time t, let $v(e,t)$ denote the velocity of e at t. Given an arbitrary vehicle e and absolute time t, define indicator $I^s(e,t)$, if e is stationary at time t, $I^s(e,t) = 1$; otherwise, $I^s(e,t) = 0$. With slight abuse of notation, let $\pi_{\mu(x)}$ be the link with $x_{\mu(x)-1} < x \le x_{\mu(x)}$.

Definition 1: Given the vehicle position $x(e,\tau)$, its future position at $t > \tau$ is $x(e,t) = x(e,\tau) + \int_{\tau}^{t} v(e,s)ds$ until it leaves the arterial after which $x(e,t) = \zeta$.

The following describe vehicle mobility.

- ι_i time compensation defined as the time for a vehicle to accelerate from v_i to v_{i+1} minus the time traveling the same distance with constant speed v_{i+1};
- $\tau_i^{\dagger}$ start-up lost time of θ_i defined as the travel time needed for a vehicle accelerating from rest to v_i minus that for traveling with constant speed v_i;
- $\mathsf{t}_i^{\dagger}$ $\iota_{i-1} + l_i/v_i$, i.e., the time for a vehicle with free-flow speed v_i passing π_i, given it entering π_i with speed v_{i-1}.

In this study, both ι_i and $\tau_i^{\dagger}$ are given. In addition, we assume vehicles come out from the source with speed v_1, which implies $\iota_0 = 0$.

Assumption 1: Vehicle mobility satisfies the following.

i. Overtaking never occurs.
ii. When the front vehicle is in free-flow, its speed is the nominal speed of the link it is in or it accelerates/decelerates due to entering the next link. In this state, it does not slow the vehicle behind.
iii. A vehicle e in some π_i does not stop unless there is a red signal in front of it, and all lanes from e to the stop line of that signal are occupied by vehicles with minimum head-to-head distance. $\square$

D. Road Capacity

Let r_i be the capacity of π_i defined as the maximum rate at which vehicles queued in π_i can leave at θ_i. Since the rate of vehicles in a queue in π_i leaving π_i is a strictly increasing function of their speed, this rate is r_i when vehicles leave π_i with a speed of v_i. We use r_i as the maximum rate of vehicles traveling in π_i to ensure stability.

It is assumed that $\tau_i^{\dagger}$ is also the difference between the actual time taken for m_i vehicles to pass and the time that would have been needed at rate r_i, where m_i is the number of vehicles queuing in the front passing the stop line of π_{i-1} before attaining rate r_i.

Assumption 2: Let $\gamma_i^{\uparrow}$ denote given maximum of $\gamma_i(t)$ over all t. For all $i \in [K-1]$, capacity r_i and maximum turning ratio $\gamma_i^{\uparrow}$ satisfies $r_{i+1} \ge \gamma_i^{\uparrow} r_i$. $\square$

That implies roads do not reduce capacity too quickly.

Lemma 1: Suppose in π_i there are mw_i vehicles queued in front of vehicle e at time t_G when the green signal starts. If e arrives at θ_j, for some $j > i$, without slowing down after it leaves π_i, it arrives at θ_j at

$$t_G + \tau_i^{\dagger} + \frac{mw_i}{r_i} + \sum_{k=i+1}^{j} \mathsf{t}_k^{\dagger} \qquad \square \tag{1}$$

Definition 2: Consider a vehicle e that queues in π_i at time G_i when the green signal for that queue starts that has mw_i vehicles queuing in front, and e leaves π_i in that cycle and does not leave the arterial before it leaves π_j, for some $j > i$.

We say that e is delayed (slows down) in π_j for the first time after it leaves π_i if it arrives at θ_j later than the time given by (1), but arrives at each θ_k with $i < k < j$ at the time computed by (1). Define the indicator function $I^h(e,(i,j)) = 1$ if this occurs, and 0 otherwise. In addition, we say that e is delayed (slows down) in π_k if either e stops in π_k for the first time or it leaves π_{k-1} with speed v_{k-1} and the time during which e travels in π_k is longer than $\mathsf{t}_k^{\dagger}$. $\square$

E. Traffic Signals

The optimal time for the green signal to start depends on both how many vehicles are queuing in the next link and when the green signal for that vehicle queue starts. We present a new method for indexing signal cycles along the considered path and use the cycle index as a proxy for time.

A cycle at θ_i, $i = 1, \ldots, K$, is defined as the interval between two consecutive times when θ_i turns green for the arterial path. Suppose that the green signals at θ_i are started at $t_i^1, t_i^2, t_i^3 \cdots$, then $c_i(u) := [t_i^u, t_i^{u+1}]$, $u = 1, 2, \ldots$, is a cycle. Let C_i denote the set of all cycles at θ_i. Let $|c_i(u)| = t_i^{u+1} - t_i^u$ denote the length of cycle u of θ_i.

Requirement 1: For all i, the duration of the green light for the path from π_i to π_{i+1} in each cycle is longer than the time needed for all vehicles in a queue in π_i to successfully leave π_i, given no downstream spillback occurring at θ_i. This requires that the traffic demand is below the capacity. □

Next, to simplify the coordination of events at different intersections, we present how to index cycles consistently between intersections and use the cycle index as a proxy for time. To serve this purpose, because vehicles keep on turning off the arterial, we define a moving particle p as one that does not leave the arterial until it reaches intersection θ_K.

Requirement 2: Particle p does not take space and does not pass the vehicle in front of it and is not passed by any vehicles. Except that it keeps on traveling in the arterial, it has the same mobility as a vehicle, as it starts moving in π_i and stops at a red light or at its place in the queue. □

This cycle indexing is slightly involved and a coordination between the indexing of different intersections is as follows.

Definition 3: Let $p_i(n)$ denote a particle that is in the front of the queue when cycle n of θ_i starts. Then, a cycle of θ_{i+1} also has index n if $p_i(n)$ leaves π_{i+1} within that cycle of θ_{i+1}. We use consecutive integers to index the cycles. For the cycle of θ_{i+1} having index n, $p_i(n)$ does not leave π_{i+1} before the light of n^{th} cycle of θ_{i+1} turned green but leaves π_{i+1} before the light of n^{th} cycle of θ_{i+1} turned red. We consider only signal policies under which, for any given intersection, consecutive cycles can be uniquely indexed by the consecutive integers. □

By indexing cycles of traffic lights, we specify which cycle at an intersection should coordinate with which other cycle at upstream or downstream intersections of that intersection.

We model finite acceleration by adjusting the "nominal" times that the signals change between red and green to be slightly offset from the "actual" times. Consider $\theta_i \in \boldsymbol{\theta}$ and a cycle indexed by $n \in \mathbb{Z}$. The next list describes symbols for specifying traffic lights.

- $G_i^-(n)$ actual start time of green light, which is also the start time of the corresponding cycle;
- $G_i(n)$ $= G_i^-(n) + \tau_i^\dagger$, nominal time that the signal at θ_i turns green, adjusted for acceleration time;
- $R_i(n)$ nominal time that the signal next turns red, which is the absolute time in the n^{th} cycle of θ_i after which no vehicle could leave π_i at nominal speed v_i and enter π_{i+1} before the $n+1^{st}$ cycle;
- $g_i(n)$ green duration $R_i(n) - G_i(n)$;
- $\mathbf{T}$ traffic light policy $(G_i(n), R_i(n)) : i = 1, \ldots, K, n \in \mathbb{Z}]$ that specifies the (nominal) start times for the signals along the path.

A vehicle e that is at the stop line of intersection θ_i just when the light turns red $(x(e, R_i(n)) = x_i)$, is not stopped by that red light, but no vehicle passes the stop line as soon as the signal turns to red. From now on, except in Section IV for simulation, we refer to nominal start time when we say start time of green light.

Lemma 2: Given $G_i(n)$, $R_{i+1}(n-1) < G_i(n) + \mathrm{t}_{i+1}^\dagger < R_{i+1}(n)$. □

We consider the case that the traffic from upstream of the corridor dominates the demand. Formally:

Requirement 3: Schedules under consideration satisfy

$$(\forall n)(\forall i \in [K-1]) R_{i+1}(n) \leq R_i(n) + \mathrm{t}_{i+1}^\dagger \tag{2}$$
$$< G_{i+1}^-(n+1). \tag{3}$$

F. State Variables with Respect to Indexed Cycles

It is convenient to study the traffic dynamics relative to the time specified by the cycle number, rather than the absolute time; note that the nth cycle at a downstream link may start after the $n+1$st cycle at an upstream link. This involves considerable complexity but simplifies the later analysis and the computation for controlling the signals.

We now define state variables for cycles of consecutive intersections over indices of cycles. The cycle number is a proxy for time at each intersection. Later in Section III, we present a dynamic equation for these state variables for increasing cycle indices. We will show later that these state variables are directly related to performance metrics, including the average stopping frequencies of those vehicles going through the arterial.

Definition 4: Given arbitrary time t, denote the set of vehicles in π_i at time t by $\mathscr{V}_i(t) = \{e : x_{i-1} < x(e,t) \leq x_i\}$. Let $f_i(t)$, $i \in [K] \cup \{0\}$, be the set of vehicles that enter π_{i+1} not before absolute time t, via θ_i from side streets, if $i \in [K]$, or the origin, if $i = 0$. Let $h_i(t)$, $i \in [K]$ be the set of vehicles that leave π_i and enter π_{i+1} not before t and, since there is no π_0, $h_0(\cdot) \equiv \emptyset$. Suppose that the second inequality in (2) of Requirement 3 is satisfied.

Let $G_i(n)$ and $G_i^-(n)$ be as defined before Lemma 2, and $G_0(n)$ and $G_0^-(n)$ with $G_0(n) = G_0^-(n)$ be variables that depend on $G_1(n)$ and the traffic situation in π_1 in cycle n. Define the set $\omega_i^n(G_i(n), G_{i-1}(n))$ of vehicles in π_i, $i \in [K]$, that corresponds to cycle index n of θ_i, as

$$\begin{aligned}&\omega_i^n(G_i(n), G_{i-1}(n)) \\ &= \mathscr{V}_i(G_i^-(n)) \setminus [f_{i-1}(G_i^-(n) - l_i/v_i) \sqcup h_{i-1}(G_{i-1}^-(n))]\end{aligned} \tag{4}$$

where "$\sqcup$" denotes disjoint union. For $i \in [K]$, the state variable N_i^n that corresponds to $\pi_i \in \boldsymbol{\pi}$ for cycle n is the number of vehicles in $\omega_i^n(G_i(n), G_{i-1}(n))$, that is $N_i^n = |\omega_i^n(G_i(n), G_{i-1}(n))|$.

Remark 1: For $i \in [K]$, $\omega_i^n(G_i(n), G_{i-1}(n))$ is the set of vehicles that are in π_i at the start of the n^{th} cycle of θ_i, at absolute time $G_i^-(n)$, excluding

i. those entering π_i via θ_{i-1} not before $G_i^-(n) - l_i/v_i$, which are considered as new arrivals during n^{th} cycle of θ_i, and

ii. those leaving π_{i-1} not before $G_{i-1}^-(n)$, which are already considered in the vehicle set of the n^{th} cycle of one of the upstream intersections. □

Those vehicles being excluded from $\omega_i^n(G_i(n), G_{i-1}(n))$ are considered in separate terms when the dynamic equation is derived. Definition 4 prevents a given vehicle from being counted multiple times at different links in a given cycle.

The relevant state variables are:

$\mathbf{N}^n$ tuple $[N_i^n : i \in [K]]$ of state variables of cycle n;

$N_i^n(\cdot)$ denotes the dependency of N_i^n on some inputs.

G. Traffic Demands

We consider the traffic demand on the considered path changing over time (dynamic) and define per-cycle traffic demands of different sources that correspond to state variables defined in Definition 4 as follows.

Given traffic light policy $\mathbf{T}$, the next list describe symbols for representing sets of vehicles passing a given location during given time intervals for each intersection θ_i, $i \in [K]$. Let t_1 and t_2 with $t_1 < t_2$ denote two arbitrary absolute time instances and $I = [t_1, t_2)$ be a time interval.

$\varepsilon_i(I)$ set of vehicles from side streets (or the source if $i = 1$) entering π_i via θ_{i-1} during $[t_1, t_2)$;

$\kappa_i(I)$ set of vehicles leaving π_i during $[t_1, t_2)$ including those leaving the arterial;

$\eta_i^-(I)$ set of vehicles going from π_i to π_{i+1} in $[t_1, t_2)$;

$\sigma_i(I)$ set of vehicles that leave π_{i-1} and either arrive at θ_i in $[t_1, t_2)$ without halting in π_i or would arrive at θ_i during $[t_1, t_2)$, if they did not stop in π_i;

$\eta_i(I)$ $\sigma_{i+1}(I - \mathsf{t}_{i+1}^\dagger)$.

Remark 2: For $I = [t_1, t_2)$ with $t_1 \geq t_2$,

- $\varepsilon_i(I)$, $\kappa_i(I)$, $\eta_i^-(I)$, $\sigma_i(I)$, and $\eta_{i-1}(I)$ are all $\emptyset$.
- η_i is introduced to account for start-up lost time while σ_i is introduced to specify η_i. □

From now on, we use the following shorthand.

$$\begin{aligned}
\varepsilon_i^n &:= \varepsilon_i([G_i^-(n), G_i^-(n+1))) \\
\varepsilon_{i,-}^n &:= \varepsilon_i([G_i^-(n), G_i^-(n+1)) - l_i/v_i) \\
\kappa_i^n &:= \kappa_i([G_i^-(n), G_i^-(n+1))) \\
(\eta_i^-)^n &:= \eta_i^-([G_i^-(n), G_i^-(n+1))) \\
\eta_i^n &:= \eta_i([G_i(n), G_i(n+1))) \\
(\eta_{i,\downarrow}^-)^n &:= \eta_i^-([G_i^-(n), G_i(n) + g_i(n))) \\
\eta_{i,\downarrow}^n &:= \eta_i([G_i(n), G_i(n) + g_i(n))).
\end{aligned}$$

Lemma 3 follows from Lemma 1 and Definition 1.

Lemma 3: Consider a particle at the front on the queue in π_i at $G_i^-(n)$ that leaves π_i at $G_i^-(n)$ and arrives at θ_{i+1}. If it does not stop in between, then it arrives at θ_{i+1} at $G_i(n) + \mathsf{t}_{i+1}^\dagger$ (note $G_i(n)$, not $G_i^-(n)$), and there exists a small $\tau \in (0, g_i(n))$ such that

i. a particle leaving π_i at some $t \in [G_i(n) + \tau, R_i(n))$ will arrive at θ_{i+1} at $t + \mathsf{t}_{i+1}^\dagger$ and

ii. $\eta_{i-1}^-([G_{i-1}^-(n), t)) = \sigma_i([G_{i-1}(n), t) + \mathsf{t}_i^\dagger) = \eta_{i-1}([G_{i-1}(n), t))$. □

Definition 5: Given policy $\mathbf{T}$, the demand of $\boldsymbol{\theta}$ over cycle n is described by tuple $[\varphi_i^n(G_i(n)) : i \in [K]]$, whose element

$$\varphi_i^n(G_i(n)) = \begin{cases} |\varepsilon_{1,-}^n| & \text{if } i = 1 \\ |\eta_{i-1,\downarrow}^n| + |\varepsilon_{i,-}^n| & \text{if } i = [K] \setminus \{1\} \end{cases} \tag{5}$$

where $\eta_{i-1,\downarrow}^n$ and $\varepsilon_{i,-}^n$ are sets of vehicles defined before. □

Suppose the queue in π_1 at $R_1(n-1)$ is empty. Then, to empty the queue again at $R_1(n)$, we set $R_1(n)$ to satisfy

$$\varepsilon_1([R_1(n-1), R_1(n)) - \tfrac{l_1}{v_1}) \leq r_1(R_1(n) - G_1(n)) \tag{6}$$

If no downstream spillback occurs, (6) is sufficient for Requirement 1 for π_1 to be satisfied.

The following simplifies analysis, but scarcely affects performance.

Assumption 3: For all $i \in [K]$, no vehicle from outside can enter the arterial via θ_i when the signal at θ_i is green. □

III. Dynamic Equations

We now present the dynamic equations for the foregoing model and then prove that, under a fixed demand, the traffic light policy under which platoons of individual queues in the consecutive links concatenate optimally reduces the frequency of vehicles slowing down or stopping.

A. Dynamic Equations

By Definition 4, N_i^n is the number of vehicles that would arrive at θ_i by $G_i^-(n)$ if they did not halt, but that actually stay in the queue of π_i at $G_i^-(n)$ due to a red light.

Theorem 1: Given $\mathbf{T}$, the dynamics of the state variables $\mathbf{N}^n$ by Definition 4 satisfy, for all $i > 1$,

$$N_1^{n+1} = |\varepsilon_1([R_1(n), G_1^-(n+1)) - l_1/v_1)| \tag{7}$$

$$N_i^{n+1} = N_i^n + |\varepsilon_{i,-}^n| + |\eta_{i-1,\downarrow}^n| - |\kappa_i^n| \quad \square \tag{8}$$

B. Good Points in Time for Starting Green Signals

We want to have a small number of vehicles slowing down, defined in Definition 2, in each link in each cycle.

We now introduce the main decision variable of this paper, $\mathscr{G}$, and define a threshold $\tau_i^n(\cdot)$ that will be shown to separate departure times from π_i that cause vehicles to slowing down and those that do not.

Definition 6: Given $i \in [K]$ and functions η_{i-1} and ε_i, fix $G_{i-1}(n)$. Let decision variable $\mathscr{G}_i(n)$ be the nominal start time of a green signal. Define the earliest non-delaying time $\tau_i^n(\mathscr{G}_i(n))$ as

$$\begin{aligned}
\tau_i^n(\mathscr{G}_i(n)) = \min\{&t \in [\mathscr{G}_i(n), \mathscr{G}_i(n) + g_i) : N_i^n(\mathscr{G}_i(n)) \\
&+ |\varepsilon_i([\mathscr{G}_i^-(n) - \frac{l_i}{v_i}, t - \frac{l_i}{v_i}))| + |\eta_{i-1}([G_{i-1}(n), t - \mathsf{t}_i^\dagger))| \\
&\leq r_i(t - \mathscr{G}_i(n))\} \quad \square
\end{aligned} \tag{9}$$

The term "earliest non-delaying" comes from the following lemma.

Lemma 4: Given $\mathscr{G}_i(n)$ and $\tau_i^n(\mathscr{G}_i(n))$ from Definition 6, for all vehicles leaving π_{i+1} during $[\mathscr{G}_i^-(n), \mathscr{G}_i(n) + g_i)$, those leaving π_{i+1} during $I = [\mathscr{G}_i^-(n), \tau_i^n(\mathscr{G}_i(n)))$ are delayed in π_{i+1} and no others are delayed in π_{i+1}. Correspondingly, there are $r_i(\tau_i^n(\mathscr{G}_i(n)) - \mathscr{G}_i(n))$ vehicles that are delayed in π_i that leave π_i during $[\mathscr{G}_i^-(n), \mathscr{G}_i(n) + g_i)$. $\square$

Vehicle platooning helps to use transportation networks more efficiently. The stability of platooning on a freeway under varying inter-vehicle communication settings has been well studied [18], [19]. Instead, we control the start times of green signals of consecutive intersections so that a lasting platoon can be formed by concatenating platoons of individual queues in consecutive links without inter-vehicle communication.

Definition 7: We say that a time $G_i^{*-}(n)$ is a prime point in time for starting the green signal of n^{th} cycle of θ_i if it satisfies

$$\begin{aligned} N_i^n(G_i^*(n)) + |\varepsilon_i([G_i^{*-}(n) - \frac{l_i}{v_i}, G_{i-1}^-(n)))| \\ = [(G_{i-1}(n) + \mathrm{t}_i^\dagger) - G_i^*(n)]r_i \qquad (10) \end{aligned}$$

where $N_i^n(G_i^*(n))$ is the state variable of cycle n of θ_i given that the corresponding green signal starts at $G_i^{*-}(n)$. $\square$

Given a traffic path with signals turning green at prime points, the vehicle at the head of a platoon discharged from a link will catch up to a vehicle at the tail of a platoon formed by vehicles slowed in the downstream link at the exit of that downstream link. This way, a platoon passing through consecutive intersections can be formed by concatenating smaller platoons of vehicles queued multiple links.

Next, we show that starting the green time at a prime point, $\mathscr{G}_i(n) = G_i^*(n)$, minimizes ψ_i^n, the number of vehicles that are slowed down in a cycle, except in two special cases.

Theorem 2: Given $i \in [K]$ and functions η_{i-1} and ε_i, fix $G_{i-1}(n)$. Recall that $\mathscr{G}$ is the nominal start time of a green signal. Fix $|\kappa_i^n(\mathscr{G}_i(n))| \equiv |\kappa_i(n)|$ and $g_i(n)$ independent of $\mathscr{G}_i(n)$. Given interval I, let $\alpha_i(I)$ be the number of vehicles arriving at θ_i during I if they travel free-flow in π_i. Let $\psi_i^n(|\kappa_i(n)|, \mathscr{G}_i(n))$ be the number of vehicles that leave π_i during n^{th} cycle of θ_i and are delayed in π_i when the nominal green start time is (decision variable) $\mathscr{G}_i(n)$. Let $G_i^{*-}(n)$ be the prime point of Definition 7, and $u = \mathscr{G}_i(n) - G_i^*(n)$. Then $\psi_i^n(|\kappa_i(n)|, G_i^*(n)) \le \psi_i^n(|\kappa_i(n)|, \mathscr{G}_i(n))$ if and only if either $u = 0$ or $(\alpha_i(I_1) - \alpha_i(I_2))u > 0$ where $I_1 = [\,[t_0, t_0 + u))$, where $[\,[t_1, t_2))$ denotes the interval $[\min(t_1, t_2), \max(t_1, t_2))$ and $t_0 = \tau_i^n(G_i^*(n))$ is the earliest non-delaying time for $G^*(n)$, and $I_2 = [\,[G_i^*(n), \mathscr{G}_i(n) + g_i))$.

Then $\psi_i^n(|\kappa_i(n)|, G_i^*(n)) \le \psi_i^n(|\kappa_i(n)|, \mathscr{G}_i(n))$, for prime point $G_i^{*-}(n)$ of Definition 7, iff the following implication holds: If $\mathscr{G}_i(n) = G_i^*(n) + u$ for some $u \ne 0$, then $(\alpha_i(I_1) - \alpha_i(I_2))u > 0$ where $I_1 = [\,[t_0, t_0 + u))$, where $t_0 = \tau_i^n(G_i^*(n))$, and $I_2 = [\,[G_i^*(n), \mathscr{G}_i(n) + g_i))$, and $[\,[t_1, t_2))$ denotes the interval $[\min(t_1, t_2), \max(t_1, t_2))$. $\square$
The condition with $u > 0$ says the density of traffic flow from π_{i-1} gradually decreases; $u < 0$ says the density of traffic from streets crossing θ_{i-1} is lower than that of the traffic from π_{i-1}.

IV. Simulations

To numerically evaluate techniques, we built a simulation testbed using PTV Vissim for a region in Hangzhou, China, including three arterial roads: Yuhangtang Rd (9 km, 12 intersections), Wenyi West Rd (11 km, 17), and Wener West Rd (9.5 km, 20), on the east side beginning at the overpasses of Changshen Expressway.

We simulated traffic of all streets in the area, and traffic demand was controlled by adjusting the actual demand relative to capacity. The cycle time and green time of the most upstream intersection of each arterial road are 100 seconds and 30 second, respectively. We divide each simulation into warm-up, peak, and cool-down. The 2000 second warm-up stage initializes the traffic, with up to 70% of the per-cycle capacity. In the peak stage, the load varies between 80% and 90% for 4000 s. The 1000 s cool-down has 70% load.

The compared control protocols are as follows. The new developed progressive movement control (PMC) and progressive movement control with responsive signal duration (PMC-RSD) in which the green light of the forward movement at each intersection starts at the prime point of Definition 7, are used to against the conventional fixed time control (FTC) and green-wave control (GWC). For all protocols used in the comparison, the traffic demands and their variations throughout the entire simulation process do not vary from one protocol to another. Moreover, we keep parameters other than the starting points of green light among protocols FTC, GMC, and PMC to be the same, while in PMC-RSD, an additional difference is that the duration of green lights could be slightly prolonged when the traffic buildups due to fluctuations of demand are detected.

Consider Yuhangtang Rd. Fig. 1 shows that at the later stage of the simulations using PMC vehicles stop on average around 20 times and PMC-RSD around 12 times. Under FTC and GWC this fluctuates around 40 times. Fig. 2 shows that the average vehicle delays of PMC and PMC-RSD are respectively about two thirds and one third of the corresponding delays of both FTC and GWC. Figures 3 and 4 show the similar comparisons for Wener Rd.

Due to space limitations, other details of the simulations have been omitted; they are available from the authors. To understand the benefits, it is useful to view the videos of 3D-simulations of three roads at https://youtu.be/ePqFQOk6fVc, https://youtu.be/JYzPXJwwJuU, and https://youtu.be/Vz29u3mGcfo. These videos form part of the results section. In each of these, driver-view simulations of FTC, GWC, PMC, and PMC-RSD that appear in the same frame in this order are presented, followed sequentially by aerial views of simulations of protocols in the same order.

V. Conclusion

A new paradigm for representing traffic dynamics has been presented, along with a new protocol to coordinate

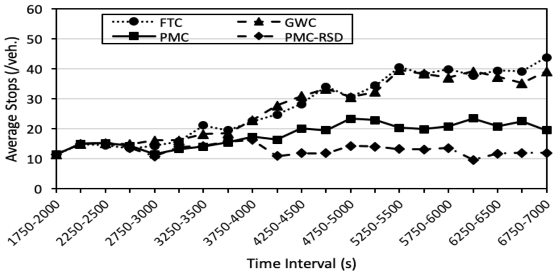

Fig. 1. Stop frequencies of vehicles traveling along Yuhangtang Rd. for four control mechanisms: fixed-time control (FTC), green-wave control (GWT), progressive movement control (PMC, proposed), and progressive movement control with responsive signal duration (PMC-RSD, proposed).

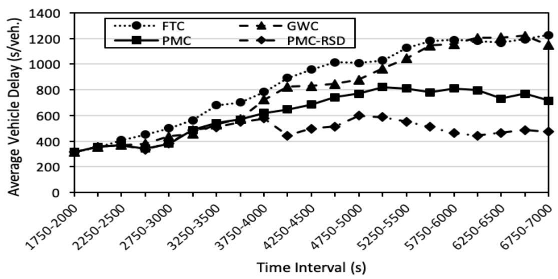

Fig. 2. Delays of vehicles traveling along Yuhangtang Rd. for four control mechanisms. For Legend, see Fig. 1.

traffic lights using that paradigm to structure progressive movements along an arterial road. The protocol actively concatenates vehicle platoons starting at multiple links to form a single contiguous platoon to pass downstream junctions. This was proved to minimize the number of times vehicles stop, and simulations show the significant improvements achieved over greenwave under near saturated traffic demand.

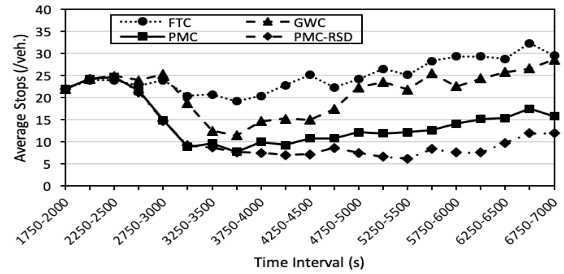

Fig. 3. Stop frequencies of vehicles traveling along Wener West Rd. for four control mechanisms. For Legend, see Fig. 1.

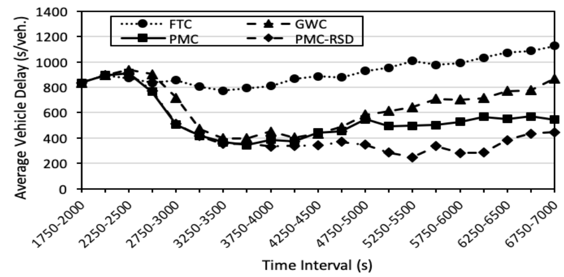

Fig. 4. Delays of vehicles traveling along Wener West Rd. for four control mechanisms. For Legend, see Fig. 1.

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