2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Discrete-time Prescribed Performance Control and Maximum Allowable Transmission Interval

Lampros N. Bikas[1] and George A. Rovithakis[1]

Abstract— **In this paper, we consider a discrete-time implementation of prescribed performance control (PPC), focusing on its robustness and operability. Specifically, given a prescribed performance controller that guarantees prescribed performance attributes, in terms of maximum overshoot, minimum convergence rate and maximum steady-state error, when operating in continuous-time, the task is to derive sufficient conditions on the maximum allowable transmission interval to enable PPC to preserve its performance characteristics. Interestingly, the maximum allowable transmission interval is directly related with the performance achieved at steady-state. Simulations clarify and verify the theoretical findings.**

I. Introduction

In recent years, the continuous development of communication networks together with the expansion of the application fields of control systems, have led to the emergence of networked control systems (NCSs) [1]-[3], which are met in many areas such as industrial control, remote control, and distributed systems among others. In NCSs, a digital communication channel is intervened between the remote parts of the closed-loop system to transmit the information. Along with the advantages that this setting offers to the overall operation, such as reliability, flexibility, easy installation/maintenance, and low cost, at the same time manifests many challenges that led to important research questions, as the network inevitably introduces limitations to the transmitted information, such as signal quantization, time delays, and loss of information during transmission.

In addition to the aforementioned problems, another dominant constraint in NCSs, is the transmission of feedback information in discrete-time, meaning that both the state measurements and the control input are available to the controller and to the system, respectively, only at some discrete time instants, which are typically determined by the sampling period. Early research on NCSs reveals that the limitation resulting from the sampling of feedback information may lead to serious performance degradation or even to instability of the closed-loop system [4]-[6]. Significant progress has been made on the derivation of the maximum allowable transmission interval [7]-[10], such that continuous-time controllers preserve the stability of the closed-loop system regardless of operating in discrete-time. Recently, notable efforts have been reported towards extending the maximum allowable transmission interval [11]-[13], as typically its derivation is carried out via a worst case analysis, rendering the obtained conditions sufficient though not necessary, and therefore, quite restrictive. Another approach of control design within the sampled-data framework is to incorporate event-triggered mechanisms [14]-[17], according to which, the sampling time instants are generated only when certain state-dependent conditions are satisfied. In this case, however, it is infeasible to implement the proposed solutions whenever the sampling period is pre-fixed and therefore it is not considered as a control element to be designed. A common characteristic of all aforementioned works is that they establish only stability conditions and they are incapable of imposing a priori performance characteristics on the output tracking error, such as maximum overshoot, minimum convergence rate, and maximum steady-state error.

In the literature, prescribed performance control (PPC) has been developed to guarantee, when operating in continuous-time, predetermined transient and steady-state performance bounds, thus enforcing maximum overshoot, minimum convergence rate and maximum steady-state error performance characteristics on the output tracking error of the closed-loop system. It was originally proposed in [18] and consequently utilized for various nonlinear system classes (see [19]-[21] and references therein). The PPC methodology was utilized in NCSs environments to address signal quantization in [22], [23]. Recently, in [24]-[26], event-triggered mechanisms were introduced within the PPC framework; relying, however, on the assumption that the states of the system are continuously measured by the controller and only the produced control input is subject to discrete-time transmission.

In this paper, we aim at extending the continuous-time framework of PPC, to address its robustness and operability under discrete-time transmission of information in the closed-loop. Specifically, given a nominal PPC controller that guarantees the prescribed performance attributes when considering continuous-time operation, the objective is to derive sufficient conditions on the maximum allowable transmission interval, such that the controller preserves its functionality and guarantees the required performance despite being subject to discrete-time implementation. A key characteristic of PPC controllers is that the control procedure employs barrier-like functions, which are well-defined only when the error evolves strictly within a constructed performance envelope. Therefore, it is crucial to enforce a sufficiently small transmission interval to maintain the evolution of the error strictly inside the envelope. We further show that the

[1]The authors are with the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece. lnmpikas@ece.auth.gr, rovithak@ece.auth.gr

The research leading to this result has received funding by the EU Horizon Europe Research and Innovation Programme, under grant agreement No 101120823, project MANiBOT.

Copyright ©2024 EUCA

maximum allowable transmission interval is directly related with the guaranteed performance level at steady-state.

The rest of the paper is organized as follows. In Section II the problem addressed is formulated and in Section III the main results are presented. In Section IV, simulation results are provided. Finally, we conclude in Section V.

II. Problem Formulation

Consider nonlinear systems in strict-feedback form

$$\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1},\ i = 1, \ldots, n-1, \tag{1a}$$
$$\dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u, \tag{1b}$$

where $\bar{x}_i = [x_1 \ldots x_i]^T \in \mathbb{R}^i$, and $\bar{x}_n = [x_1 \ldots x_n]^T \in \mathbb{R}^n$ is the system state. Moreover, $x_1 \in \mathbb{R}$ is the system output, $u \in \mathbb{R}$ is the control input, and $f_i(\bar{x}_i),\ g_i(\bar{x}_i) : \mathbb{R}^i \to \mathbb{R}$, $i = 1, \ldots, n$, are nonlinear functions locally Lipschitz in $\bar{x}_i$. Consider further a reference output tracking trajectory denoted by $y_d(t) \in \mathbb{R}$, and define the output tracking error

$$e_1(t) = x_1(t) - y_d(t),\ \forall\ t \geq 0. \tag{2}$$

Assumption 1. The functions $g_i(\bar{x}_i)$, $i = 1, \ldots, n$, are either strictly positive or strictly negative for all $\bar{x}_i$, and their signs are known and are denoted by $\text{sgn}(g_i)$, $i = 1, \ldots, n$.

Assumption 2. The reference trajectory satisfies $|y_d(t)| \leq \bar{y}_d$ and $|\dot{y}_d(t)| \leq \bar{\dot{y}}_d$, with $\bar{y}_d > 0$, $\bar{\dot{y}}_d > 0$ some known constants.

In this paper we consider the scenario where the controller is receiving state information only at some discrete time instants determined by a sampling period. Furthermore, the control input is transmitted to the system at these sampling time instants. Therefore, between two consequent transmissions, the controller has no longer access to the state measurements and the system is forced to operate with the control input produced at the latest sampling time instant. The aforementioned mode of operation is formulated in the following assumption.

Assumption 3. There exist some time instants t_k, $k \in \mathbb{N}$, with $t_0 = 0$ and $t_{k+1} > t_k$, for all $k \in \mathbb{N}$, representing the sampling time instants. Therefore, $u(t) = u(t_k)$, for all $t \in [t_k, t_{k+1}),\ k \in \mathbb{N}$.

The control objective is to establish pre-defined bounds on $e_1(t)$ with respect to transient and steady-state behavior, in the presence of sampled-data control implementation as specified in Assumption 3. More specifically, we aim to derive sufficient conditions on the maximum allowable transmission interval, such that a nominal controller, capable of guaranteeing the aforementioned performance characteristics when operating in continuous-time, preserves its functionality and the performance guarantees, despite being subject to sampled-data operation.

Remark 1. To enforce the performance requirements on the output tracking error the prescribed performance control (PPC) methodology is utilized. According to PPC [18], when considering continuous-time operation, the error is guaranteed to evolve strictly within a constructed performance envelope, whose selection directly introduces the pre-specified performance characteristics on the output error in terms of transient and steady-state behavior. A key property of this type of controllers is that the control solution adopts barrier-like functions, which are well-defined only as the error evolves strictly within the aforementioned performance envelope. Therefore, in the presence of discrete-time control implementation, it is crucial to guarantee that the transmission intervals are small enough such that, at each sampling time instant, the evolution of the error strictly within the envelope is preserved.

III. Main Results

In the following theorem, the main results of this paper are summarized.

Theorem 1. *Consider system* (1)*, any initial condition $x_i(0)$, $i = 1, \ldots, n$, any reference trajectory $y_d(t)$, and Assumptions 1-3. Consider further the controller, for all $i = 1, \ldots, n$,*

$$a_0(t) = y_d(t), \tag{3a}$$
$$\xi_i(t) = \frac{x_i(t) - a_{i-1}(t)}{\rho_i(t)}, \tag{3b}$$
$$\epsilon_i(t) = T(\xi_i(t)), \tag{3c}$$
$$a_i(t) = -sgn(g_i)c_i\epsilon_i(t), \tag{3d}$$
$$u(t) = a_n(t), \tag{3e}$$

where $\rho_i(t) = \left(\rho_i^0 - \rho_i^\infty\right)e^{-\lambda_i t} + \rho_i^\infty$, with $\rho_i^\infty > 0$, $\lambda_i \geq 0$, $\rho_i^0 > |x_i(0) - a_{i-1}(0)|$, $T(\xi) = \ln\left(\frac{1+\xi}{1-\xi}\right)$, and $c_i > 0$ some freely selected control gains. If the sampling time instants t_k satisfy for all $k \in \mathbb{N}$,

$$t_{k+1} - t_k \leq \tau_{mati} \triangleq \min_{i=1,\ldots,n}\left\{\frac{\rho_i^\infty\left(\bar{\xi}_i - T^{-1}\left(\frac{\bar{F}_i}{\underline{g}_i c_i}\right)\right)}{\bar{F}_i + \bar{g}_i c_i T\left(\bar{\xi}_i\right)}\right\}, \tag{4}$$

where the constants $\bar{\xi}_i > 0$, $i = 1, \ldots, n$, satisfy:

$$\max\left\{|\xi_i(0)|, T^{-1}\left(\frac{\bar{F}_i}{\underline{g}_i c_i}\right)\right\} < \bar{\xi}_i < 1, \tag{5}$$

and

$$\bar{F}_i = \bar{f}_i + \bar{g}_i\rho_{i+1}^0 + \bar{\dot{\rho}}_i + \bar{\dot{a}}_{i-1},\ i = 1, \ldots, n-1, \tag{6a}$$
$$\bar{F}_n = \bar{f}_n + \bar{\dot{\rho}}_n + \bar{\dot{a}}_{n-1}, \tag{6b}$$
$$\bar{f}_i = \sup_{|x_j| < \rho_j^0 + \bar{a}_{j-1},\ j=1,\ldots,i.} |f_i(\bar{x}_i)|,\ i = 1, \ldots, n, \tag{6c}$$
$$\bar{g}_i = \sup_{|x_j| < \rho_j^0 + \bar{a}_{j-1},\ j=1,\ldots,i.} |g_i(\bar{x}_i)|,\ i = 1, \ldots, n, \tag{6d}$$
$$\underline{g}_i = \inf_{|x_j| < \rho_j^0 + \bar{a}_{j-1},\ j=1,\ldots,i.} |g_i(\bar{x}_i)|,\ i = 1, \ldots, n, \tag{6e}$$
$$\bar{\dot{a}}_0 = \bar{\dot{y}}_d,\ \bar{\dot{a}}_i = \frac{2c_i\left(\bar{F}_i + \bar{g}_i\bar{a}_i\right)}{(1 - \bar{\xi}_i^2)\rho_i^\infty},\ i = 1, \ldots, n-1, \tag{6f}$$
$$\bar{a}_0 = \bar{y}_d,\ \bar{a}_i = c_i T\left(\bar{\xi}_i\right),\ i = 1, \ldots, n, \tag{6g}$$
$$\bar{\dot{\rho}}_i = \lambda_i\rho_i^0,\ i = 1, \ldots, n, \tag{6h}$$

then controller (3) *with $u(t) = u(t_k)$, $i = 1, \ldots, n$, for all $t \in [t_k, t_{k+1})$, guarantees that*

- *all signals in the closed-loop are bounded,*
- *$|e_1(t)| < \rho_1(t)$, for all $t \geq 0$.*

Remark 2. Introducing sampling within the PPC methodology is a key attribute of this work, as it violates the standard assumption of having continuous accessibility of the sate measurements as well as continuous implementation of the control input; a crucial assumption when aiming to impose continuous performance bounds on the output tracking error. In Theorem 1, $\tau_{\text{mati}} > 0$ represents the maximum allowable transmission interval, providing a sufficient condition on the sampling time instants such that all signals in the closed-loop remain bounded and the output tracking error satisfies the required performance, i.e., $|e_1(t)| < \rho_1(t)$.

Remark 3. Establishing $|e_1(t)| < \rho_1(t)$ implies that $e_1(t)$ converges strictly within $(-\rho_1^\infty, \rho_1^\infty)$, with convergence rate no less than $e^{-\lambda_1 t}$. Hence, the selection of ρ_1^∞ and λ_1 directly introduce performance attributes on $e_1(t)$ in terms of maximum steady-state error and minimum convergence rate, respectively. In this work, we omitted performance specifications on the maximum overshoot to simplify notation, without, however, loss of generality. Further, the performance functions $\rho_i(t)$, $i = 2, \ldots, n$, are not related directly with any performance index, and therefore, they can be freely chosen. However, their careful selection may positively influence the evolution of the errors $x_i(t) - a_{i-1}(t)$, $i = 2, \ldots, n$, within the corresponding performance envelopes. The same observation holds also for the control gains c_i, $i = 1, \ldots, n$.

Remark 4. Careful inspection of (4) reveals a trade-off between the maximum allowable transmission interval $\tau_{\text{mati}} > 0$ and the guaranteed output tracking accuracy at steady-state given by $\rho_1^\infty > 0$. In that respect, for any fixed sampling period $\tau_s > 0$, one can preserve that $\tau_{\text{mati}} \geq \tau_s$, by appropriately enlarging ρ_i^∞, for all $i = 1, \ldots, n$; degrading, however, the derived performance with respect to the maximum output tracking error at steady-state.

Proof of Theorem 1: Define the non-empty and open set $\Omega_\xi = (-1, 1) \subset \mathbb{R}$. Owing to $\rho_i^0 > |x_i(0) - a_{i-1}(0)|$, $i = 1, \ldots, n$, it holds that $\xi_i(0) \in \Omega_\xi$, $i = 1, \ldots, n$, which further concludes that $\epsilon_i(0)$, $i = 1, \ldots, n$, are well-defined. Further, notice that owing to (5) it holds that $\xi_i(0) \in \left(-\bar{\xi}_i, \bar{\xi}_i\right) \subset \Omega_\xi$. Let $t_1 > 0$, be the next sampling time instant. Owing to (3b) we deduce the continuity of $\xi_i(t)$, $i = 1, \ldots, n$, and therefore, the existence of a maximal time interval $[0, t_{\max}^1)$ such that $\xi_i(t) \in \Omega_\xi$ for all $t \in [0, t_{\max}^1)$. Moreover, as $\left[-\bar{\xi}_i, \bar{\xi}_i\right] \subset \Omega_\xi$ it holds $t_{\max}^1 > t_1$. Taking the derivative of (3b) for all $t \in [0, t_1)$, we deduce that for $i = 1, \ldots, n-1$,

$$\dot{\xi}_i = \frac{1}{\rho_i}\left(f_i + g_i \xi_{i+1}\rho_{i+1} - \dot{a}_{i-1} - \xi_i \dot{\rho}_i + g_i a_i\right), \quad \text{(7a)}$$

$$\dot{\xi}_n = \frac{1}{\rho_n}\left(f_n - \dot{a}_{n-1} - \xi_n \dot{\rho}_n + g_n a_n(0)\right). \quad \text{(7b)}$$

To proceed, the analysis follows a recursive procedure.

Step 1. ($i = 1$): Let us define the positive-definite and radially unbounded Lyapunov function $V_1 = \frac{1}{2}\epsilon_1^2$ for all $t \in [0, t_1)$. Differentiating V_1 with respect to time in view of (7a) we deduce for all $t \in [0, t_1)$,

$$\dot{V}_1 = \frac{2\epsilon_1}{(1-\xi_1^2)\,\rho_1}\Big(f_1 + g_1\xi_2\rho_2 - \dot{y}_d - \xi_1\dot{\rho}_1 - |g_1| c_1 \epsilon_1\Big). \quad (8)$$

To continue, notice that as $\xi_1(t) \in \Omega_\xi$ for all $t \in [0, t_1)$, and owing to (3b) and Assumption 2, we derive $|x_1(t)| < \rho_1^0 + \bar{y}_d$ for all $t \in [0, t_1)$. Hence, by the latter and the Extreme Value Theorem, we conclude the existence of strictly positive constants $\bar{f}_1$, $\underline{g}_1$ and $\bar{g}_1$, such that $|f_1(x_1)| \leq \bar{f}_1$, and $\underline{g}_1 \leq |g_1(x_1)| \leq \bar{g}_1$. Further, $\xi_2(t) \in \Omega_\xi$ for all $t \in [0, t_1)$, $|\dot{y}_d(t)| \leq \bar{\dot{y}}_d$ by Assumption 2, $\rho_2(t) \leq \rho_2^0$, and $|\dot{\rho}_1(t)| \leq \lambda_1 \left|\rho_1^0 - \rho_1^\infty\right| < \lambda_1 \rho_1^0$. Moreover, $1 - \xi_1^2(t) > 0$ for all $t \in [0, t_1)$. Owing to the aforementioned analysis, and by recalling (6a), (6c)-(6e), (6h), we conclude that for all $t \in [0, t_1)$, $\dot{V}_1$ becomes

$$\dot{V}_1 \leq \frac{2\,|\epsilon_1|\,|g_1|\,c_1}{(1-\xi_1^2)\rho_1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1} - |\epsilon_1|\right). \quad (9)$$

Thus, by employing (9) and the inverse of the logarithmic function in (3c), we conclude that for all $t \in [0, t_1)$,

$$\dot{V}_1 < 0, \text{ if } |\xi_1(t)| > T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right). \quad (10)$$

Further, by employing (6a), (7a), and for $\xi_1(t) \in \left[-\bar{\xi}_1, \bar{\xi}_1\right] \subset \Omega_\xi$, we derive that for all $t \in [0, t_1)$,

$$\begin{aligned} |\dot{\xi}_1(t)| &\leq \frac{1}{\rho_1^\infty}\left(\bar{f}_1 + \bar{g}_1\rho_2^0 + \bar{\dot{y}}_d + \bar{\dot{\rho}}_1 + \bar{g}_1|a_1|\right) \\ &\leq \frac{\bar{F}_1 + \bar{g}_1 c_1 T\left(\bar{\xi}_1\right)}{\rho_1^\infty} \triangleq \bar{\dot{\xi}}_1. \end{aligned} \quad (11)$$

We proceed by investigating separately the following cases: i) $|\xi_1(t)| > T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$, ii) $|\xi_1(t)| \leq T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$.

Case i): Owing to the continuity of $\xi_1(t)$, we conclude the existence of a time instant $t_1^* > 0$ such that $|\xi_1(t)| \geq T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$ for all $t \in [0, t_1^*)$. Further, by (10) $|\xi_1(t)|$ will be strictly decreasing for all $t \in [0, t_1^*)$, implying also that $|\xi_1(t)| < |\xi_1(0)| < \bar{\xi}_1$ for all $t \in [0, t_1^*)$. Therefore, as we aim to derive the maximum time instant t_1 such that $\xi_1(t) \in [-\bar{\xi}_1, \bar{\xi}_1]$ for all $t \in [0, t_1)$, we conclude that $t_1 > t_1^*$. Furthermore, the time instant t_1^* satisfies $t_1^* \geq \frac{|\xi_1(0)| - T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)}{\bar{\dot{\xi}}_1}$ and $\xi_1(t_1^*) = T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$. Moreover, by (10), we conclude that for all $t \in \left[t_1^*, t_1\right)$ it holds that

$$|\xi_1(t)| < |\xi_1(t_1^*)| + (t_1 - t_1^*)\bar{\dot{\xi}}_1. \quad (12)$$

Therefore, owing to the aforementioned analysis and (12), we deduce that if the following inequality holds true

$$t_1 \bar{\dot{\xi}}_1 - |\xi_1(0)| + 2T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right) \leq \bar{\xi}_1, \quad (13)$$

then $\xi_1(t) \in \left(-\bar{\xi}_1, \bar{\xi}_1\right)$ for all $t \in [0, t_1)$. Hence, by substituting (11), and owing to the continuity of $\xi_1(t)$ we conclude that $\xi_1(t) \in \left[-\bar{\xi}_1, \bar{\xi}_1\right]$ for all $t \in [0, t_1]$ if

$$t_1 \leq \frac{\rho_1^\infty\left(\bar{\xi}_1 + |\xi_1(0)| - 2T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)\right)}{\bar{F}_1 + \bar{g}_1 c_1 T\left(\bar{\xi}_1\right)}. \quad (14)$$

Case ii): In this case, we directly conclude that for all $t \in [0, t_1)$ it holds that

$$|\xi_1(t)| < |\xi_1(0)| + t_1\bar{\dot{\xi}}_1. \tag{15}$$

Hence, by employing (15), we obtain that if it holds

$$|\xi_1(0)| + t_1\bar{\dot{\xi}}_1 \leq \bar{\xi}_1, \tag{16}$$

then $\xi_1(t) \in \left(-\bar{\xi}_1, \bar{\xi}_1\right) \subset \Omega_\xi$, for all $t \in [0, t_1)$. Consequently, owing to the fact that $|\xi_1(0)| \leq T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$, to (11), and to the continuity of $\xi_1(t)$, we conclude that $\xi_1(t) \in \left[-\bar{\xi}_1, \bar{\xi}_1\right]$ for all $t \in [0, t_1]$ if

$$t_1 \leq \frac{\rho_1^\infty\left(\bar{\xi}_1 - T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)\right)}{\bar{F}_1 + \bar{g}_1 c_1 T\left(\bar{\xi}_1\right)}. \tag{17}$$

Aggregating (14) and (17), and owing to the fact that $\bar{\xi}_1 + |\xi_1(0)| - 2T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right) \geq \bar{\xi}_1 - T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$ when $|\xi_1(t)| > T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)$, we conclude that if the time instant $t_1 > 0$ satisfies

$$t_1 \leq \frac{\rho_1^\infty\left(\bar{\xi}_1 - T^{-1}\left(\frac{\bar{F}_1}{\underline{g}_1 c_1}\right)\right)}{\bar{F}_1 + \bar{g}_1 c_1 T\left(\bar{\xi}_1\right)}, \tag{18}$$

then $\xi_1(t) \in \left[-\bar{\xi}_1, \bar{\xi}_1\right] \subset \Omega_\xi$, for all $t \in [0, t_1]$. Moreover, owing to (3a) and (3b), we conclude that $|e_1(t)| < \rho_1(t)$ for all $t \in [0, t_1]$. Further, we establish that for all $t \in [0, t_1]$,

$$|a_1(t)| \leq \bar{a}_1 = c_1 T\left(\bar{\xi}_1\right), \tag{19a}$$

$$|\dot{a}_1(t)| \leq \bar{\dot{a}}_1 = \frac{2c_1\left(\bar{F}_1 + \bar{g}_1\bar{a}_1\right)}{\left(1 - \bar{\xi}_1^2\right)\rho_1^\infty}. \tag{19b}$$

Step i. ($i = 2, \ldots, n-1$): Let us define the positive-definite and radially unbounded Lyapunov function $V_i = \frac{1}{2}\epsilon_i^2$ for all $t \in [0, t_1)$. Differentiating V_i with respect to time in view of (7a), we deduce that for all $t \in [0, t_1)$,

$$\dot{V}_i = \frac{2\epsilon_i}{\left(1 - \xi_i^2\right)\rho_i}\Big(f_i + g_i\xi_{i+1}\rho_{i+1} - \dot{a}_{i-1} - \xi_i\dot{\rho}_i - |g_i|c_i\epsilon_i\Big), \tag{20}$$

To continue, notice that as $\xi_i(t) \in \Omega_\xi$ for all $t \in [0, t_1]$, and owing to (3b) and (19a), we derive $|x_i(t)| < \rho_i^0 + \bar{a}_{i-1}$ for all $t \in [0, t_1)$. Hence, by employing the latter and the Extreme Value Theorem, we conclude the existence of strictly positive constants $\bar{f}_i$, $\underline{g}_i$ and $\bar{g}_i$, such that $|f_i(\bar{x}_i)| \leq \bar{f}_i$, and $\bar{g}_i \leq |g_i(\bar{x}_i)| \leq \bar{g}_i$. Therefore, by repeating the line of analysis of Step 1, we conclude that if the time instant $t_1 > 0$ satisfies

$$t_1 \leq \frac{\rho_i^\infty\left(\bar{\xi}_i - T^{-1}\left(\frac{\bar{F}_i}{\underline{g}_i c_i}\right)\right)}{\bar{F}_i + \bar{g}_i c_i T\left(\bar{\xi}_i\right)}, \tag{21}$$

then $\xi_i(t) \in \left[-\bar{\xi}_i, \bar{\xi}_i\right] \subset \Omega_\xi$ for all $t \in [0, t_1]$. Moreover, by (3a) and (3b), we establish that $|x_i(t) - a_{i-1}(t)| < \rho_i(t)$ for all $t \in [0, t_1]$, and $|a_i(t)| \leq \bar{a}_i = c_i T\left(\bar{\xi}_i\right)$, $|\dot{a}_i(t)| \leq \bar{\dot{a}}_i = \frac{2c_i\left(\bar{F}_i + \bar{g}_i\bar{a}_i\right)}{\left(1 - \bar{\xi}_i^2\right)\rho_i^\infty}$, for all $t \in [0, t_1]$.

Step n. ($i = n$): Let us define the positive-definite and radially unbounded Lyapunov function $V_n = \frac{1}{2}\epsilon_n^2$ for all $t \in [0, t_1)$. Differentiating V_n with respect to time in view of (7b), we deduce that for all $t \in [0, t_1)$,

$$\dot{V}_n = \frac{2\epsilon_n}{\left(1 - \xi_n^2\right)\rho_n}\Big(f_n - \dot{a}_{n-1} - \xi_n\dot{\rho}_n - |g_n|c_n\epsilon_n(0)\Big), \tag{22}$$

By repeating the line of analysis of Step $n-1$, we conclude that for all $t \in [0, t_1)$, if $\text{sgn}(\epsilon_n(t)) = \text{sgn}(\epsilon_n(0))$, then (22) becomes

$$\dot{V}_n \leq \frac{2\,|\epsilon_n|\,|g_n|\,c_n}{(1 - \xi_n^2)\rho_n}\left(\frac{\bar{F}_n}{\underline{g}_n c_n} - |\epsilon_n(0)|\right). \tag{23}$$

Hence, by taking the inverse of the logarithmic function in (3c), and employing the fact that $\text{sgn}(\xi_n(t)) = \text{sgn}(\epsilon_n(t))$ for all $t \geq 0$, we conclude that for all $t \in [0, t_1)$,

$$\dot{V}_n < 0, \text{ if } \begin{cases} \text{sgn}(\xi_n(t)) = \text{sgn}(\xi_n(0)), \\ \text{and} \\ |\xi_n(0)| > T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right). \end{cases} \tag{24}$$

Furthermore, as in Steps $1, \ldots, n-1$, we derive for all $t \in [0, t_1)$,

$$|\dot{\xi}_n(t)| \leq \frac{\bar{F}_n + \bar{g}_n c_n T\left(\bar{\xi}_n\right)}{\rho_n^\infty} \triangleq \bar{\dot{\xi}}_n. \tag{25}$$

Hence, similarly to Steps $1, \ldots, n-1$, We proceed by investigating separately the following cases: i) $|\xi_n(0)| > T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)$, ii) $|\xi_n(0)| \leq T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)$.

Case i): Owing to the continuity of $\xi_n(t)$, we conclude the existence of a time instant $t_1^* > 0$ such that $\text{sgn}(\xi_n(t)) = \text{sgn}(\xi_n(0))$ for all $t \in [0, t_1^*)$. Further, by (24) $|\xi_n(t)|$ will be strictly decreasing for all $t \in [0, t_1^*)$, implying also that $|\xi_n(t)| < |\xi_n(0)| < \bar{\xi}_n$ for all $t \in [0, t_1^*)$. Therefore, as we aim to derive the maximum time instant t_1 such that $\xi_n(t) \in [-\bar{\xi}_n, \bar{\xi}_n]$ for all $t \in [0, t_1)$, we conclude that $t_1 > t_1^*$. Furthermore, the time instant t_1^* satisfies $t_1^* \geq \frac{|\xi_n(0)|}{\bar{\dot{\xi}}_n}$ and $\xi_n(t_1^*) = 0$, which also results in $\text{sgn}(\xi_n(t_1^*)) \neq \text{sgn}(\xi_n(0))$. Consequently, owing to the latter and to (10), we conclude that for all $t \in [t_1^*, t_1)$ it holds that $|\xi_n(t)| < |\xi_n(t_1^*)| + (t_1 - t_1^*)\bar{\dot{\xi}}_n$. Therefore, owing to the latter analysis we deduce that if $t_1\bar{\dot{\xi}}_n - |\xi_n(0)| \leq \bar{\xi}_n$ then $\xi_n(t) \in \left(-\bar{\xi}_n, \bar{\xi}_n\right)$ for all $t \in [0, t_1)$. Hence, by substituting (11), and owing to the continuity of $\xi_n(t)$ we conclude that $\xi_n(t) \in \left[-\bar{\xi}_n, \bar{\xi}_n\right]$ for all $t \in [0, t_1]$ if

$$t_1 \leq \frac{\rho_n^\infty\left(\bar{\xi}_n + |\xi_n(0)|\right)}{\bar{F}_n + \bar{g}_n c_n T\left(\bar{\xi}_n\right)}. \tag{26}$$

Case ii): In this case, similarly with the previous Steps, we conclude that $\xi_n(t) \in \left[-\bar{\xi}_n, \bar{\xi}_n\right]$ for all $t \in [0, t_1]$ if

$$t_1 \leq \frac{\rho_n^\infty\left(\bar{\xi}_n - T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)\right)}{\bar{F}_n + \bar{g}_n c_n T\left(\bar{\xi}_n\right)}. \tag{27}$$

Aggregating (26) and (27), and owing to the fact that $\bar{\xi}_n + |\xi_n(0)| \geq \bar{\xi}_n - T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)$, we conclude that if the time instant $t_1 > 0$ satisfies

$$t_1 \leq \frac{\rho_n^{\infty}\left(\bar{\xi}_n - T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)\right)}{\bar{F}_n + \bar{g}_n c_n T\left(\bar{\xi}_n\right)}, \tag{28}$$

then $\xi_n(t) \in \left[-\bar{\xi}_n, \bar{\xi}_n\right] \subset \Omega_{\xi}$, for all $t \in [0, t_1]$. Moreover, by (3a) and (3b), we establish that $|x_n(t) - a_{n-1}(t)| < \rho_n(t)$ for all $t \in [0, t_1]$.

Consequently, by combining (28) and (21), and the analysis presented in Steps $1, \ldots, n$, we derive that if the time instant $t_1 > 0$ satisfies

$$t_1 \leq \min_{i=1,\ldots,n}\left\{\frac{\rho_n^{\infty}\left(\bar{\xi}_n - T^{-1}\left(\frac{\bar{F}_n}{\underline{g}_n c_n}\right)\right)}{\bar{F}_n + \bar{g}_n c_n T\left(\bar{\xi}_n\right)}\right\}, \tag{29}$$

then $\xi_i(t) \in \left[-\bar{\xi}_i, \bar{\xi}_i\right] \subset \Omega_{\xi}$, $i = 1, \ldots, n$, for all $t \in [0, t_1)$.

Identically, the line of analysis is straightforwardly extended to all time intervals $[t_k, t_{k+1})$, $k \in \mathbb{N}$, with t_{k+1} satisfying $t_k < t_{k+1} < t_k + t_{\text{max}}^k$, $k \in \mathbb{N}$, with t_{max}^k being the corresponding maximal time. Therefore, if

$$t_{k+1} - t_k \leq \tau_{\text{mati}} = \min_{i=1,\ldots,n}\left\{\frac{\rho_i^{\infty}\left(\bar{\xi}_i - T^{-1}\left(\frac{\bar{F}_i}{\underline{g}_i c_i}\right)\right)}{\bar{F}_i + \bar{g}_i c_i T\left(\bar{\xi}_i\right)}\right\}, \tag{30}$$

then for all $i = 1, \ldots, n$,

$$\xi_i(t) \in \left[-\bar{\xi}_i, \bar{\xi}_i\right] \subset \Omega_{\xi}, \forall\, t \in [t_k, t_{k+1}),\ k \in \mathbb{N}. \tag{31}$$

The latter implies that the solution of $\xi_i(t)$, $i = 1, \ldots, n$, evolves strictly inside Ω_{ξ} for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$; consequently extending the solution to $+\infty$. Hence, all signals in the closed-loop are bounded and in addition $|e_1(t)| < \rho_1(t)$ for all $t \geq 0$; thus preserving the prescribed performance characteristics of the output tracking error, and concluding the proof.

IV. Simulation Results

To verify the theoretical results we conducted simulation studies on a single-link robotic manipulator, the dynamics of which are given by

$$\dot{x}_1 = x_2, \tag{32a}$$

$$\dot{x}_2 = -\frac{\mathrm{g}}{\mathrm{m}}\sin(x_1) - \frac{\mathrm{c}}{\mathrm{m}}x_2 + \frac{1}{\mathrm{m}}u, \tag{32b}$$

with x_1 [rad], x_2 [rad/s], representing the angular position and velocity, respectively, and system parameters $\mathrm{g} = 10$ [N m], $\mathrm{c} = 1$ [N m s/rad] and $\mathrm{m} = 2$ [N m s^2/rad]. The desired trajectory is chosen as $y_d(t) = \frac{\pi}{10}\sin(0.2\pi t)$. Hence, we have $\bar{y}_d = \frac{\pi}{10}$ and $\bar{\dot{y}}_d = 0.02\pi^2$. Further, we consider initial conditions $x_1(0) = \frac{\pi}{10}$ [rad] and $x_2(0) = 0$ [rad/s].

In the first simulation scenario, the control objective was to achieve tracking performance with output error at steady-state no greater than 0.01 [rad] and convergence rate no less than e^{-t}. Hence, we selected $\rho_1^{\infty} = 0.01$ and $\lambda_1 = 1$. Further, we chose $\rho_1^0 = 0.6283$, $c_1 = 1$, $\rho_2^{\infty} = 0.5$, $\lambda_2 = 0.5$, $\rho_2^0 = 4.3944$, $c_2 = 5$, $\bar{\xi}_1 = 0.9943$ and $\bar{\xi}_2 = 0.9920$. Moreover, for system (32) we obtain $\bar{f}_1 = 0$, $\bar{g}_1 = \underline{g}_1 = 1$, and $\bar{f}_2 = 10.1302$, $\bar{g}_2 = \underline{g}_2 = 0.5$. Therefore, employing (4), we obtain $\tau_{\text{mati}} = 4.8858 \times 10^{-6}$ [sec]. In Fig. 1 we illustrate simulation results with sampling period $\tau_s = 4.8858 \times 10^{-6}$ [sec]. As it is clearly shown, the desired performance is achieved as all errors evolve strictly within the corresponding performance envelopes, while all signals in the closed-loop remained bounded.

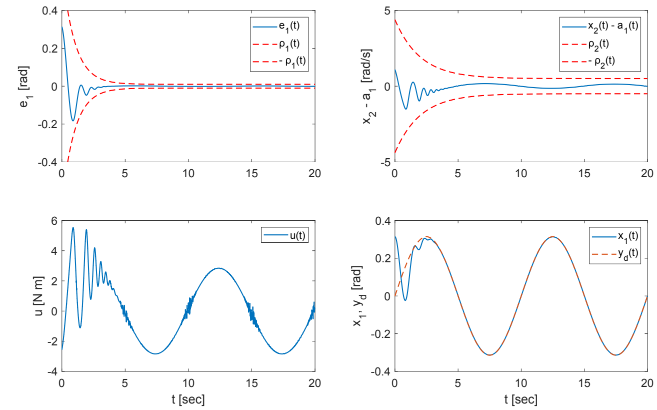

Fig. 1. Tracking performance and control input with $\rho_1^{\infty} = 0.01$ and sampling period $\tau_s = \tau_{\text{mati}} = 4.8858 \times 10^{-6}$ [sec].

Consequently we increased ρ_1^{∞} to 0.1 and kept intact all remaining controller parameters to highlight the relation between the output tracking error accuracy that can be guaranteed at steady-state with respect to the maximum allowable transmission interval. Given this enlarged selection for ρ_1^{∞} we obtain $\tau_{\text{mati}} = 5.4310 \times 10^{-5}$ [sec]. Hence we selected $\tau_s = 5.4310 \times 10^{-5}$ [sec]. As shown in Fig. 2, in this scenario where the control is implemented with larger sampling period, the derived performance was degraded, as the guaranteed performance envelope was enlarged. The evolution of the output tracking error, however, attained fairly small values, evolving significantly below the performance bounds.

Finally, we kept intact all control parameters of the previous scenario and we increased the sampling period to $\tau_s = 8.1464 \times 10^{-2}$ [sec]. By the latter scenario we highlight that the derived condition (4) on the transmission intervals is sufficient though not necessary, as it is derived via a worst case analysis. Therefore, the control objectives may be preserved even if the sampling period is significantly larger than τ_{mati}. The results are shown in Fig. 3. Notice that the enlarged sampling period clearly has a negative impact on the overall performance as the errors exhibit increased oscillations within the performance envelope and the produced control input has been degraded with respect to magnitude and slew-rate.

V. Conclusion

In an attempt to robustly discretize prescribed performance controllers, we derived the maximum allowable transmission interval to preserve the prescribed performance attributes guaranteed in continuous-time operation. We showed that

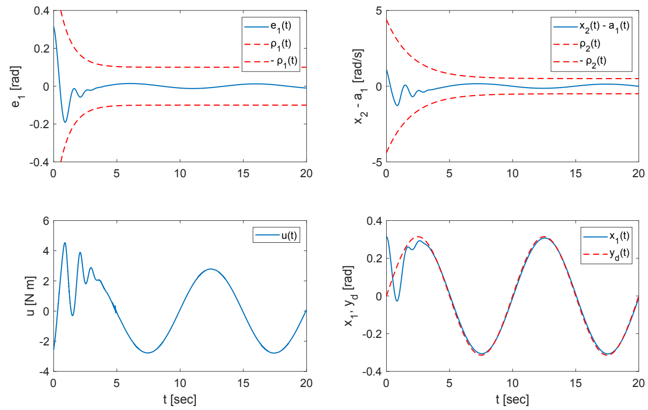

Fig. 2. Tracking performance and control input with $\rho_1^\infty = 0.1$ and sampling period $\tau_s = \tau_{\text{mati}} = 5.4310 \times 10^{-5}$ [sec].

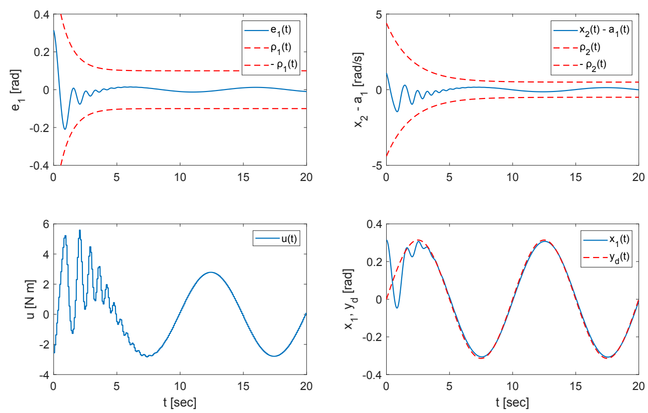

Fig. 3. Tracking performance and control input with $\rho_1^\infty = 0.1$ and sampling period $\tau_s = 8.1464 \times 10^{-2}$ [sec].

the maximum allowable interval is directly related with the performance guaranteed at steady-state. The theoretical results were verified and clarified via simulation studies.

References

[1] M. Cloosterman, L. Hetel, N. van de Wouw, W. Heemels, J. Daafouz and H. Nijmeijer, "Controller synthesis for networked control systems," *Automatica*, vol. 46, no. 10, pp. 1584-1594, 2010.

[2] Y. Xia, Y. Gao, L. Yan and M. Fu, "Recent progress in networked control systems: a survey," *International Journal of Automation and Computing*, vol. 12, no. 4, pp. 343-367, 2015.

[3] D. Zhang, P. Shi, Q. Wang and L. Yu, "Analysis and synthesis of networked control systems: a survey of recent advances and challenges," *ISA Transactions*, vol. 66, no. 1, pp. 376-392, 2017.

[4] D. Nesic and A. Teel, "Input-output stability properties of networked control systems," *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1650-1667, 2004.

[5] D. Nesic and A. Teel, "Input-to-state stability of networked control systems," *Automatica*, vol. 40, no. 12, pp. 2121-2128, 2004.

[6] M. Tabbara, D. Nesic and A. Teel, "Stability of wireless and wireline networked control systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1615-1630, 2007.

[7] L. Montestruque and P. Antsaklis, "Stability of model-based networked control systems with time-varying transmission times," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1562-1572, 2004.

[8] W. Heemels, A. Teel, N. van de Wouw and D. Nesic, "Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance," *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1781-1796, 2010.

[9] D. Tolic and S. Hirche, "Stabilizing transmission intervals for nonlinear delayed networked control systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 1, pp. 488-494, 2017.

[10] S. Heijmans, R. Postoyan, D. Nesic and W. Heemels, "Computing minimal and maximal allowable transmission intervals for networked control systems using the hybrid systems approach," *IEEE Control Systems Letters*, vol. 1, no. 1, pp. 56-61, 2017.

[11] D. Carnevale, A. Teel and D. Nesic, "A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 892-897, 2007.

[12] S. Heijmans, R. Postoyan, D. Nesic and W. Heemels, "An average allowable transmission interval condition for the stability of networked control systems," *IEEE Transactions on Automatic Control*, vol. 66, no. 6, pp. 2526-2541, 2021.

[13] M. Hertneck and F. Allgower, "A simple approach to increase the maximum allowable transmission interval," *IFAC-PapersOnLine*, vol. 54, no. 14, pp. 418-423, 2021.

[14] W. Wu, S. Reimann and S. Liu, "Event-triggered control for linear systems subject to actuator saturation," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 9492-9497, 2014.

[15] R. Postoyan, P. Tabuada, D. Nesic and A. Anta, "A framework for the event-triggered stabilization of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 982-996, 2015.

[16] V. Dolk and W. Heemels, "Event-triggered control systems under packet losses," *Automatica*, vol. 80, no. 6, pp. 143-155, 2017.

[17] T. Liu and Z. Jiang, "Event-triggered control of nonlinear systems with state quantization," *IEEE Transactions on Automatic Control*, vol. 64, no. 2, pp. 797-803, 2019.

[18] C. Bechlioulis and G. Rovithakis, "Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2090-2099, 2008.

[19] C. Bechlioulis and G. Rovithakis, "A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems," *Automatica*, vol. 50, no. 4, pp. 1217-1226, 2014.

[20] I. Dimanidis, C. Bechlioulis and G. Rovithakis, "Output feedback approximation-free prescribed performance tracking control for uncertain MIMO nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5058-5069, 2020.

[21] I. Katsoukis and G. Rovithakis, "A low complexity robust output synchronization protocol with prescribed performance for high-order heterogeneous uncertain MIMO nonlinear multiagent systems," *IEEE Transactions on Automatic Control*, vol. 67, no. 6, pp. 3128-5069, 3133, 2021.

[22] L. Bikas and G. Rovithakis, "Combining prescribed tracking performance and controller simplicity for a class of uncertain MIMO nonlinear systems with input quantization," *IEEE Transactions on Automatic Control*, vol. 66, no. 7, pp. 3311-3316, 2018.

[23] L. Bikas and G. Rovithakis, "Tracking performance guarantees in the presence of quantization for uncertain nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 3, pp. 1228-1235, 2021.

[24] Y. Cao, C. Wen and Y. Song, "Prescribed performance control of strict-feedback systems under actuation saturation and output constraint via event-triggered approach," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 1, 2019.

[25] N. Zhao, X. Ouyang, L. Wu and F. Shi, "Event-triggered adaptive prescribed performance control of uncertain nonlinear systems with unknown control directions," *ISA Transactions*, vol. 108, no. 2, pp. 121-130, 2021.

[26] P. Ning, C. Hua, K. Li and R. Meng, "Event-triggered adaptive prescribed-time control for nonlinear systems with uncertain time-varying parameters," *Automatica*, vol. 157, no. 111229, 2023.