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Event-Triggered Prescribed Performance Control for SISO Uncertain Nonlinear Systems in Brunovsky Canonical Form

Thomais A. Aforozi, George A. Rovithakis

Abstract— **In this work, we consider the problem of designing tracking controllers for SISO uncertain high relative degree systems in Brunovsky canonical form in the presence of non-periodic communication. The proposed control scheme is static, and requires no hard calculations, analytic or numerical, to produce the control signal. Event-triggered mechanisms are considered in both sensor-to-controller and controller-to-actuator channels, yet the enforcement of prescribed performance bounds in terms of steady-state accuracy and convergence rate is ensured. No prior knowledge or estimation structure regarding system nonlinearities are required and no high-order derivatives of the desired output trajectories are incorporated in the controller design. Simulation results clarify and verify the theoretical findings.**

I. INTRODUCTION

Networked control systems (NCSs) are comprised of spatially distributed components that communicate over a digital network. Despite their significant advantages that include modularity, easy of maintenance, reduced cabling and cost, the presence of the underlying communication network introduces several technological barriers, mainly attributed to time delays, packet losses and restrictions on the available bandwidth [1], [2]. As an effect, issues related to performance degradation, or even instability, typically appear. The problems become even more demanding when the controlled system is nonlinear. In the literature, an effective way to tackle bandwidth limitations is to incorporate non-periodic communication, leading to the emergence of event-triggered control (ETC) [3].

The majority of works in the area of controlling nonlinear systems using ETC consider the presence of event-triggered communication either in the controller-to-actuator (CtA) [4]-[7] or in the sensor-to-controller (StC) [8], [9] channels and study closed-loop system stability. Surprisingly, the general configuration where event-triggered communication is present in both channels is rarely considered. This is mainly attributed to the fact that this intermittent communication at the StC channel formulates an impulsive nonlinear system making control design even more difficult, which escalates further when events satisfaction enables information exchange in the CtA channel as well. In this direction, global output regulation is achieved for relative degree one nonlinear systems in [10], while in [11] an event-triggered controller equipped with a parameter estimator was utilized to address the stabilization problem for a class of strict-feedback systems. In [12], dynamic filtering was employed to enable the use of backstepping, developing a novel control scheme to guarantee the boundedness of all signals in the closed loop, while introducing adjustable transient performance, in the mean square error sense, via appropriate selection of design parameters.

Nevertheless, the problem of enforcing prescribed performance characteristics in the sense of maximum overshoot, minimum convergence rate and maximum steady-state error when tracking an output reference trajectory, despite the presence of event-triggered communication, has been typically overlooked. Works addressing this highly significant in applications issue consider event-triggered communication only at the CtA channel and the proposed solutions utilize the prescribed performance control (PPC) methodology. PPC was pioneered in [13] and since then has been evolved to design controllers for more complex system structures [14], [15], see also [16] and references therein. Event-triggered PPC designs include [6], where the tracking problem was considered for a class of SISO strict-feedback systems having unknown control directions. To reduce further the demanded bandwidth, the proposed solution was further extended to the case of binary information transmission in the CtA channel. An asymmetric barrier Lyapunov function dynamic surface controller was designed in [17]. For a class of uncertain pure-feedback systems an adaptive fuzzy ETC was developed in [18], employing state observers to estimate the unmeasured state variables. A robust adaptive fuzzy event-triggered PPC was proposed in [19], for a class of strict-feedback systems. In [20], an adaptive event-triggered PPC was designed to handle a class of uncertain strict-feedback systems in the presence of input saturation.

Motivated by the aforementioned observations, and utilizing the PPC design methodology, we address, in this work, the problem of establishing not only the boundedness of all signals in the closed-loop but additionally of enforcing prescribed transient and steady-state error bounds on the output tracking error. The class of nonlinear systems we consider are high relative degree in Brunovsky canonical form. For the system nonlinearities we assume they are locally Lipschitz. However, their analytic expressions are considered unknown. No approximation structures (i.e., neural networks, fuzzy systems) and no adaptive techniques are incorporated to acquire such knowledge. No derivatives of the desired output trajectory are utilized. Information exchange in both CtA and StC channels is subject to event-triggered communication. The proposed control scheme is static involving no hard

T. A. Aforozi and G. A. Rovithakis are with the Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, 54124, Thessaloniki, Greece. Emails: taforozi@ece.auth.gr, rovithak@ece.auth.gr

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calculations, either analytic or numerical, thus resulting in a low complexity controller.

II. Problem Statement

Consider single-input single-output systems of n-th order described as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, ..., n-1 \\ \dot{x}_n &= f(x) + g(x)\upsilon \\ y &= x_1 \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}$ for $i = 1, ..., n$ are the states of the system, with $x = [x_1 \dots x_n]^T \in \mathbb{R}^n$. Moreover, $\upsilon \in \mathbb{R}$ is the control input that results from an event-triggered mechanism applied on the actual controller output u, $y \in \mathbb{R}$ is the output of the system and $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}$ are locally Lipschitz nonlinear functions with unknown analytical expressions.

Assumption 1: The function $g(\cdot)$ is either strictly positive or strictly negative and its sign denoted with $sgn(g)$ is considered known.

Assumption 2: The desired trajectory $y_d(t) : \mathbb{R}^+ \to \mathbb{R}$ is known, continuously differentiable and bounded function of time with bounded, yet unknown, first derivative.

Remark 1: Assumption 1 imposes a sufficient global controllability condition on (1). Furthermore, from Assumption 2, we only require knowledge of the desired trajectory y_d and none of its high order derivatives. Therefore, we can also consider applications where the desired trajectory is not a priori known but it is measured.

In this paper we consider event-triggered system-controller communication. Specifically, for all $i = 1, ..., n$ and $k, l \in \mathbb{N}$ we define the event conditions:

$$t_{s_{k+1},i} = inf\{t > t_{s_k,i} : |x_i(t) - x_i(t_{s_k,i})| \geq \delta_{x,i}\}, \tag{2a}$$

$$t_{u_{l+1}} = inf\{t > t_{u_l} : |u(t) - u(t_{u_l})| \geq \delta_u\}, \tag{2b}$$

where $t_{s_{k+1},i}$ and $t_{u_{l+1}}$ denote the time instants (2a) and (2b) are satisfied respectively, and $\delta_{x,i}, \delta_u > 0$ are constant step-sizes. Event-triggered mechanism (2a) is applied on the system state, while (2b) is applied on the controller output. The corresponding state signal remains constant for all $t \in [t_{s_k,i}, t_{s_{k+1},i})$ at the value acquired at $t = t_{s_k,i}$ and, similarly, the controller output signal remains constant for all $t \in [t_{u_l}, t_{u_{l+1}})$ at the value acquired at $t = t_{u_l}$. Owing to (2) it holds:

$$|x_i(t) - x_i(t_{s_k,i})| < \delta_{x,i}, \quad \forall t \in [t_{s_k,i}, t_{s_{k+1},i}), \tag{3a}$$

$$|u(t) - u(t_{u_l})| < \delta_u, \qquad \forall t \in [t_{u_l}, t_{u_{l+1}}). \tag{3b}$$

Hence, there exist $\lambda_{x,i}(t)$ and $\lambda_u(t)$ satisfying the properties:

$$\lambda_{x,i}(t_{s_k,i}) = 0,\ \lambda_{x,i}(t_{s_{k+1},i}) = \pm 1,\ |\lambda_{x,i}(t)| \leq 1, \quad \forall t \in [t_{s_k,i}, t_{s_{k+1},i}), \tag{4a}$$

$$\lambda_u(t_{u_l}) = 0,\ \lambda_u(t_{u_{l+1}}) = \pm 1,\ |\lambda_u(t)| \leq 1, \quad \forall t \in [t_{u_l}, t_{u_{l+1}}), \tag{4b}$$

such that

$$x_i(t) = x_i(t_{s_k,i}) + \lambda_{x,i}(t)\delta_{x,i},\ \forall t \in [t_{s_k,i}, t_{s_{k+1},i}), \tag{5a}$$

$$u(t) = u(t_{u_l}) + \lambda_u(t)\delta_u, \qquad \forall t \in [t_{u_l}, t_{u_{l+1}}). \tag{5b}$$

Remark 2: The presence of event-triggered communication formulates a discontinuous closed-loop system; thus raising questions related to the appearance of chattering and the existence of solution. However, the incorporation of the proposed event-triggered mechanisms (2) efficiently avoids both, as they introduce a strictly positive dwell time between any two consequent transitions.

The problem addressed in this paper reads as follows.

Problem (Event-triggered Prescribed Performance Control, ETPPC): Consider system (1), satisfying Assumption 1, with system state and controller output being transmitted only at discrete time instants issued by the event-triggered mechanisms (2). Consider also desired output trajectories $y_d(t)$ satisfying Assumption 2 and define the output tracking error $e(t) = y(t) - y_d(t)$. Given any initial condition $x(0) \in \mathbb{R}^n$, construct a continuously differentiable, strictly positive, decreasing and bounded time-function $\rho(t)$, and design a state-feedback controller such that all signals in the closed-loop remain bounded and $|e(t)| < \rho(t)$ for all $t \geq 0$.

Remark 3: Enforcing $|e(t)| < \rho(t)$ for all $t \geq 0$ practically introduces prescribed transient and steady-state performance attributes on the output tracking error. This can be straightforwardly verified when considering exponentially decaying time-functions $\rho(t) = (\rho^0 - \rho^\infty)e^{-\mu t} + \rho^\infty$. As it was first explained in [13], where PPC was pioneered, the parameter $\rho^\infty > 0$ determines the maximum allowable error at steady-state, while $\mu \geq 0$ introduces the required minimum convergence rate.

III. Control Design

Define the auxiliary, strictly increasing function $T : (-1, 1) \to \mathbb{R}$ satisfying $\lim_{\zeta \to 1^-} T(\zeta) = +\infty$ and $\lim_{\zeta \to -1^+} T(\zeta) = -\infty$ of the form $T(\zeta) = ln\big((1+\zeta)/(1-\zeta)\big)$. Given the desired trajectory y_d satisfying Assumption 2 and any initial condition $x(0) \in \mathbb{R}^n$, we propose the following recursive control design procedure. For $t \geqslant 0$:

$$\xi_i(t) = \frac{x_i(t_{s_k,i}) - a_{i-1}(t)}{\rho_i(t) - \delta_{x,i}}, \quad i = 1, ..., n, \tag{6a}$$

$$\varepsilon_i(t) = T(\xi_i(t)), \quad i = 1, ..., n, \tag{6b}$$

$$a_i(t) = -k_i \varepsilon_i(t), \quad i = 1, ..., n-1, \tag{6c}$$

$$a_n(t) = u(t) = -sgn(g)k_n\varepsilon_n(t),\ \upsilon(t) = u(t_{u_l}), \tag{6d}$$

where $a_0 \equiv y_d$. In (6) $k_i > 0$, $i = 1, ..., n$ are control gains and $\rho_i(t) = (\rho_i^0 - \rho_i^\infty)e^{-\mu_i t} + \rho_i^\infty$ with parameters $\mu_i \geq 0$ and $\rho_i^0, \rho_i^\infty > 0$ satisfying:

$$\rho_i^0 > |x_i(0) - \lambda_{x,i}(0)\delta_{x,i} - a_{i-1}(0)| + \delta_{x,i}, \tag{7a}$$

$$\rho_i^\infty \geq \big((2 + M_i)\delta_{x,i} + \bar{a}_{i-1}\big)/M_i. \tag{7b}$$

Moreover, $\bar{a}_0 = 0$, $\bar{a}_{i-1}$, $i = 2, ..., n$, are positive constants and $M_i \in (0, 1)$, $i = 1, ..., n$,.

Remark 4: As clarified in Remark 3, only $\rho_1(t)$ is responsible for enforcing prescribed performance attributes on the output tracking error. The rest $\rho_i(t)$, $i = 2..., n$, are not directly connected to the aforementioned task. Therefore, they can be arbitrarily selected provided their parameters satisfy (7). The control gains k_i, $i = 1, ..., n$, are also freely chosen. However, extensive simulation studies have revealed that these selections may be influential to the overall closed-loop system performance. Specifically, large μ_i and high-valued control gains can lead to significant or excessively large control effort in the transient, while exhibiting satisfactory performance at steady-state. On the other hand, opposite selection usually results in proper transient behavior and oscillatory steady-state evolution. Therefore, careful selection of all design parameters may positively influence the quality of the output tracking error, and the demanded control effort.

***Remark** 5:* Sparse communication of the system state has significant impact on the desired steady-state accuracy. As (7b) indicates, loose communication, i.e., larger step-size values $\delta_{x,i}$, leads to a more relaxed performance bound on the steady state. In contrast, frequent communication results in a steady state error that is bounded closer to zero. Notice that no such issue appears on the controller output side, thus making the selection of the step-size δ_u more robust.

***Remark** 6:* As Remark 3 underlines, prescribed performance at the output is enforced if we establish $|e(t)| < \rho_1(t)$, for all $t \geq 0$, or stated otherwise if $|x_1(t) - a_0(t)| < \rho_1(t)$, for all $t \geq 0$. In that direction, notice that if $|\xi_i(t)| < 1$, for all $t \geq 0$ then owing to (6a) it holds $-\rho_i(t) + \delta_{x,i} < x_i(t_{s_k,i}) - a_{i-1} < \rho_i(t) - \delta_{x,i}$, for all $t \geq 0$, $i = 1, ..., n$. However, owing to (4a) we straightforwardly conclude that $\rho_i(t) - \delta_{x,i} < \rho_i(t) - \lambda_{x,i}(t)\delta_{x,i}$ and $-\rho_i(t) - \lambda_{x,i}(t)\delta_{x,i} < -\rho_i(t) + \delta_{x,i}$, for all $t \geq 0$ and $i = 1, ..., n$. Hence, $-\rho_i(t) < x_i(t_{s_k,i}) + \lambda_{x,i}(t)\delta_{x,i} - a_{i-1}(t) < \rho_i(t)$, which owing to (5a) yields $|x_i(t) - a_{i-1}(t)| < \rho_i(t)$, for all $t \geq 0$, $i = 1, ..., n$. Therefore, to establish prescribed performance, despite the presence of event-triggered communication, it suffices to guarantee $|\xi_i(t)| < 1$, for all $t \geq 0$, $i = 1, ..., n$. A thorough examination of the T-function reveals that the latter is achieved if ε_i are proven bounded. As a consequence, by (6b), we interpret the problem as minimizing the quadratic and positive definite functions $\frac{1}{2}\varepsilon_i^2$ with respect to $|\xi_i| < 1$ for all $t \geq 0$. Hence, the proposed controller operates similarly to barrier functions in constrained optimization, admitting high negative or positive values depending on whether ξ_i approaches 1 or -1, thus restricting the evolution of $\xi_i(t)$ inside its constrained region.

IV. Main Results

For all $i = 1, ..., n-1$ let us define

$$\xi_i^{min}(t) = \big(x_i(t) - \delta_{x,i} - a_{i-1}^{min}(t)\big) / \big(\rho_i(t) - \delta_{x,i}\big), \quad (8a)$$

$$\xi_i^{max}(t) = \big(x_i(t) + \delta_{x,i} - a_{i-1}^{max}(t)\big) / \big(\rho_i(t) - \delta_{x,i}\big). \quad (8b)$$

Moreover,

$$\xi_n^{min}(t) = \begin{cases} \frac{x_n(t) - \delta_{x,n} - a_{n-1}^{min}(t)}{\rho_n(t) - \delta_{x,n}}, sgn(g) = 1 \\ \frac{x_n(t) - \delta_{x,n} - a_{n-1}^{max}(t)}{\rho_n(t) - \delta_{x,n}}, sgn(g) = -1 \end{cases} \quad (9a)$$

$$\xi_n^{max}(t) = \begin{cases} \frac{x_n(t) + \delta_{x,n} - a_{n-1}^{max}(t)}{\rho_n(t) - \delta_{x,n}}, sgn(g) = 1 \\ \frac{x_n(t) + \delta_{x,n} - a_{n-1}^{min}(t)}{\rho_n(t) - \delta_{x,n}}, sgn(g) = -1 \end{cases} \quad (9b)$$

where $a_0^{min}(t) = a_0^{max}(t) \equiv y_d(t)$ and

$$a_i^{min}(t) = -k_i \varepsilon_i^{min}(t), \;\; a_i^{max}(t) = -k_i \varepsilon_i^{max}(t), \quad (10a)$$

$$a_n^{min}(t) = -sgn(g) k_n \varepsilon_n^{min}(t), \quad (10b)$$

$$a_n^{max}(t) = -sgn(g) k_n \varepsilon_n^{max}(t), \quad (10c)$$

$$\varepsilon_i^{min}(t) = T(\xi_i^{min}(t)), \; i = 1, ..., n, \quad (10d)$$

$$\varepsilon_i^{max}(t) = T(\xi_i^{max}(t)), \; i = 1, ..., n. \quad (10e)$$

A closer look to (5a), (6a), (8) and (9) reveals that:

$$\xi_i^{min}(t) \leq \xi_i(t) \leq \xi_i^{max}(t), \; i = 1, ..., n, \; \forall t \geq 0. \quad (11)$$

Differentiating ξ_i^{min}, ξ_i^{max}, $i = 1, ..., n-2$, we obtain:

$$\dot{\xi}_i^{min} := h_i^{min} = \frac{1}{\rho_i(t) - \delta_{x,i}} [\xi_{i+1}^{min}(\rho_{i+1}(t) - \delta_{x,i+1}) + \delta_{x,i+1} + a_i^{min} - \dot{a}_{i-1}^{min} - \xi_i^{min}\dot{\rho}_i], \quad (12a)$$

$$\dot{\xi}_i^{max} := h_i^{max} = \frac{1}{\rho_i(t) - \delta_{x,i}} [\xi_{i+1}^{max}(\rho_{i+1}(t) - \delta_{x,i+1}) - \delta_{x,i+1} + a_i^{max} - \dot{a}_{i-1}^{max} - \xi_i^{max}\dot{\rho}_i]. \quad (12b)$$

Similarly,

$$\dot{\xi}_{n-1}^{min} := h_{n-1}^{min} = \frac{1}{\rho_{n-1}(t) - \delta_{x,n-1}} [\xi_n^{min}(\rho_n(t) - \delta_{x,n}) + \delta_{x,n} + a_{n-1}^{\gamma_1} - \dot{a}_{n-2}^{min} - \xi_{n-1}^{min}\dot{\rho}_{n-1}(t)], \quad (13a)$$

$$\dot{\xi}_{n-1}^{max} := h_{n-1}^{max} = \frac{1}{\rho_{n-1}(t) - \delta_{x,n-1}} [\xi_n^{max}(\rho_n(t) - \delta_{x,n}) - \delta_{x,n} + a_{n-1}^{\gamma_2} - \dot{a}_{n-2}^{max} - \xi_{n-1}^{max}\dot{\rho}_{n-1}(t)], \quad (13b)$$

$$\dot{\xi}_n^{min} := h_n^{min} = \frac{f(x) + g(x)\upsilon - \dot{a}_{n-1}^{\gamma_1} - \xi_n^{min}\dot{\rho}_n}{\rho_n(t) - \delta_{x,n}}, \quad (13c)$$

$$\dot{\xi}_n^{max} := h_n^{max} = \frac{f(x) + g(x)\upsilon - \dot{a}_{n-1}^{\gamma_2} - \xi_n^{max}\dot{\rho}_n}{\rho_n(t) - \delta_{x,n}} \quad (13d)$$

where $\gamma_1, \gamma_2 \in \{min, max\}$ with $\gamma_1 = min, \gamma_2 = max$ for $sgn(g) = 1$ and $\gamma_1 = max, \gamma_2 = min$ for $sgn(g) = -1$. Define $\xi = [\xi_1^{min} \dots \xi_n^{min} \; \xi_1^{max} \dots \xi_n^{max}]^T \in \mathbb{R}^{2n}$. Therefore, the closed-loop system can be written as:

$$\dot{\xi} = h(\xi, t) = [h_1^{min} \dots h_n^{min} \; h_1^{max} \dots h_n^{max}] \in \mathbb{R}^{2n}. \quad (14)$$

Let $\Omega_\xi = (-1, 1)^{2n} \subset \mathbb{R}^{2n}$. The main results of this work are summarized in the following theorem.

***Theorem** 1:* Consider system (1), a desired trajectory $y_d(t)$ and Assumptions 1,2. Consider also the non-periodic communication mechanisms (2). The controller (6),(7) guarantees the solution of the *ETPPC* problem.

Proof: The proof of Theorem 1 consists of two parts. In Part A, the existence and uniqueness of a maximal solution $\xi(t) : [0, \tau_{max}) \to \Omega_\xi$ of (14) for some $\tau_{max} \in (0, +\infty]$ is ensured. Subsequently, in Part B, a recursive procedure is followed to prove that ξ evolves strictly within a compact subset of Ω_ξ for all $t \in [0, \tau_{max})$ and, eventually, following standard arguments, we extend τ_{max} to $+\infty$.

Part A: The set Ω_ξ is open and nonempty. Owing to (7a), it is obtained that $\xi(0) \in \Omega_\xi$. The existence of the piecewise continuous signal υ in the right-side of (13c) and (13d) results in the discontinuous closed loop system (14). However, deploying the reasoning of Remark 2, we deduce the existence of a unique maximally extended solution of (14) for any initial condition in a time interval $[0, \tau_{max})$ for some $\tau_{max} \in (0, +\infty]$. Therefore, the signals (8)-(10) are well-defined for all $t \in [0, \tau_{max})$.

Part B: The analysis follows n-steps.

Step 1 ($i = 1, t \in [0, \tau_{max})$): Consider the following positive definite and radially unbounded Lyapunov function $V_1^{min} = \frac{1}{2}\varepsilon_1^{min^2}$. By (10d) and (12a) the time derivative of V_1^{min} yields $\dot{V}_1^{min} = \frac{2\varepsilon_1^{min}}{(1-\xi_1^{min^2})(\rho_1(t)-\delta_{x,1})}[\xi_2^{min}(\rho_2(t) - \delta_{x,2}) + \delta_{x,2} + a_1^{min} - \dot{y}_d - \xi_1^{min}\dot{\rho}_1]$. Let us define $F_1(t) = \xi_2^{min}(\rho_2(t) - \delta_{x,2}) + \delta_{x,2} - \dot{y}_d - \xi_1^{min}\dot{\rho}_1$. Notice that $\xi_1^{min}, \xi_2^{min} \in (-1, 1)$ for all $t \in [0, \tau_{max})$ and the signals $\rho_2, \dot{\rho}_1, \dot{y}_d, \delta_{x,2}$ are bounded by construction. Thus, through the application of the Extreme Value Theorem, there exists a constant $\bar{F}_1 > 0$ such that $|F_1(t)| \le \bar{F}_1$, for all $t \in [0, \tau_{max})$. Thus, owing to (10a), we obtain

$$\dot{V}_1^{min} = \frac{2\varepsilon_1^{min}}{(1-\xi_1^{min^2})(\rho_1(t)-\delta_{x,1})}[F_1(t) - k_1\varepsilon_1^{min}] \le \frac{2|\varepsilon_1^{min}|}{(1-\xi_1^{min^2})(\rho_1(t)-\delta_{x,1})}[\bar{F}_1 - k_1|\varepsilon_1^{min}|],$$

where owing to (7b) it holds $\rho_1(t) > \delta_{x,1}$ for all $t \in [0, \tau_{max})$. At this point, notice that ρ_1^∞ can be chosen in a proper way, such that

$$\xi_1^{max} - \xi_1^{min} \le M_1,\ M_1 \in (0, 1). \tag{16}$$

Thus, by substituting (8) on (16), we obtain that $\rho_1(t)$ should satisfy

$$\big((2 + M_1)\delta_{x,1}\big)/M_1 \le \rho_1^\infty \le \rho_1(t). \tag{17}$$

Observe that $\dot{V}_1^{min} \le 0$ when $|\varepsilon_1^{min}(t)| \ge \bar{F}_1/k_1$. Hence, there exists a constant $\bar{\varepsilon}_1^{min} > 0$ such that:

$$|\varepsilon_1^{min}(t)| \le \bar{\varepsilon}_1^{min} := max\{|\varepsilon_1^{min}(0)|, \bar{F}_1/k_1\}, \tag{18}$$

for all $t \in [0, \tau_{max})$. Taking the inverse of the T-function we deduce

$$-1 < T^{-1}(-\bar{\varepsilon}_1^{min}) \le \xi_1^{min}(t) \le T^{-1}(\bar{\varepsilon}_1^{min}) < 1, \tag{19}$$

for all $t \in [0, \tau_{max})$. Owing to (16), we conclude the existence of a positive constant d_1 such that

$$\varepsilon_1^{max}(t) - \varepsilon_1^{min}(t) \le d_1. \tag{20}$$

From (18), (20) and the utilization of the inverse T-function, it is straightforwardly obtained that there exists a positive constant $\bar{\varepsilon}_1^{max}$ such that

$$-1 < T^{-1}(-\bar{\varepsilon}_1^{max}) \le \xi_1^{max}(t) \le T^{-1}(\bar{\varepsilon}_1^{max}) < 1, \tag{21}$$

for all $t \in [0, \tau_{max})$. From (11), (19) and (21) we obtain

$$-1 < T^{-1}(-\bar{\varepsilon}_1^{min}) \le \xi_1(t) \le T^{-1}(\bar{\varepsilon}_1^{max}) < 1. \tag{22}$$

By (10a) we conclude the boundedness of a_1^{min} and a_1^{max}. Moreover, owing to the continuity of h_1^{min} in (12a) and the Extreme Value Theorem, we deduce the existence of a constant $\bar{h}_1 > 0$ such that $|h_1^{min}| < \bar{h}_1$. Thus, by differentiating (10) we deduce the boundedness of $\dot{a}_1^{min}$. Similarly, we can conclude the boundedness of $\dot{a}_1^{max}$.

Step i ($i = 2, ..., n-2, t \in [0, \tau_{max})$): Defining $V_i^{min} = \frac{1}{2}\varepsilon_i^{min^2}$, selecting ρ_i^∞ such that

$$\big((2 + M_i)\delta_{x,i} - a_{i-1}^{max}(t) + a_{i-1}^{min}(t)\big)/M_i \le \rho_i^\infty, \tag{23}$$

with $M_i \in (0, 1)$ and following the same line of analysis as in *Step 1*, we can directly conclude the existence of positive constants $\bar{\varepsilon}_i^{min}$ and $\bar{\varepsilon}_i^{max}$ so that, for all $t \in [0, \tau_{max})$,

$$-1 < T^{-1}(-\bar{\varepsilon}_i^{min}) \le \xi_i(t) \le T^{-1}(\bar{\varepsilon}_i^{max}) < 1 \tag{24}$$

holds true, and further that a_i^{min}, a_i^{max}, $\dot{a}_i^{min}$ and $\dot{a}_i^{max}$ are bounded for all $t \in [0, \tau_{max})$. Notice that in (23) the term $-a_{i-1}^{max}(t) + a_{i-1}^{min}(t)$ is positive owing to (10), (11) and the strictly increasing property of T-function. Moreover, it has been proven bounded in *Step i-1* and thus there exists a positive constant $\bar{a}_{i-1}$ such that $-a_{i-1}^{max}(t) + a_{i-1}^{min}(t) < \bar{a}_{i-1}$.

Step n-1 ($i = n-1, t \in [0, \tau_{max})$): We define $V_{n-1}^{min} = \frac{1}{2}\varepsilon_{n-1}^{min^2}$ and we select ρ_{n-1}^∞ such that $\big((2 + M_{n-1})\delta_{x,n-1} - a_{n-2}^{max}(t) + a_{n-2}^{min}(t)\big)/M_{n-1} \le \rho_{n-1}^\infty$.

Let $sgn(g) = 1$. Utilizing (13a) and following the same line of analysis as in previous steps, we deduce the existence of positive constants $\bar{\varepsilon}_{n-1}^{min}, \bar{\varepsilon}_{n-1}^{max}$ such that, for all $t \in [0, \tau_{max})$,

$$-1 < T^{-1}(-\bar{\varepsilon}_{n-1}^{min}) \le \xi_{n-1}(t) \le T^{-1}(\bar{\varepsilon}_{n-1}^{max}) < 1. \tag{25}$$

Now let $sgn(g) = -1$. The substitution of (13a) and (10a) leads to $\dot{V}_{n-1}^{min} = \frac{2\varepsilon_{n-1}^{min}}{(1-\xi_{n-1}^{min^2})(\rho_{n-1}(t)-\delta_{x,n-1})}[F_{n-1}^{min}(t) - k_{n-1}\varepsilon_{n-1}^{max}]$, where $F_{n-1}^{min}(t) = \xi_n^{min}(\rho_n(t) - \delta_{x,n}) + \delta_{x,n} - \dot{a}_{n-2}^{min} - \xi_{n-1}^{min}\dot{\rho}_{n-1}(t)$, which can be proven bounded. At this point, we distinguish the following scenarios: *a*) $-1 < \xi_{n-1}^{min} < \xi_{n-1}^{max} < 0$, *b*) $0 < \xi_{n-1}^{min} < \xi_{n-1}^{max} < 1$ and *c*) $-1 < \xi_{n-1}^{min} < 0 < \xi_{n-1}^{max} < 1$.

Scenario a) Notice that in this case, owing to the strictly increasing property of T-function, there exists a time function $c_1(t) \in [1, +\infty)$ such that $\varepsilon_{n-1}^{min} = c_1(t)\varepsilon_{n-1}^{max}$. Therefore, following similar analysis as in previous steps, ε_{n-1}^{max} is proven bounded and, due to the selection of ρ_{n-1}^∞, it holds that there exists a positive constant $\bar{\varepsilon}_{n-1}^{min}$ such that:

$$-1 < T^{-1}(-\bar{\varepsilon}_{n-1}^{min}) \le \xi_{n-1}^{min}(t),\ \forall t \in [0, \tau_{max}). \tag{26}$$

Scenario b) Similarly, there exists a time function $c_1(t) \in [1, +\infty)$ such that $\varepsilon_{n-1}^{max} = c_1(t)\varepsilon_{n-1}^{min}$. Thus, ε_{n-1}^{min} is, firstly, proven bounded and, owing to the selection of ρ_{n-1}^∞, we conclude the existence of a positive constant $\bar{\varepsilon}_{n-1}^{max}$ such that:

$$\xi_{n-1}^{max}(t) \le T^{-1}(\bar{\varepsilon}_{n-1}^{max}) < 1,\ \forall t \in [0, \tau_{max}). \tag{27}$$

Scenario c) Owing to the selection of ρ_{n-1}^∞ and the utilization of (11), we directly deduce, for all $t \in [0, \tau_{max})$,

$$-1 < -M_{n-1} < \xi_{n-1}(t) < M_{n-1} < 1. \tag{28}$$

Therefore, each scenario leads also to the boundedness of a_{n-1}^{min}, a_{n-1}^{max}, $\dot{a}_{n-1}^{min}$ and $\dot{a}_{n-1}^{max}$ for all $t \in [0, \tau_{max})$.

Step n ($i = n, t \in [0, \tau_{max})$): We consider the positive definite and radially unbounded function $V_n^{min} = \frac{1}{2}\varepsilon_n^{min^2}$. Let $sgn(g) = 1$. Utilizing (5b), (10d) and (13c), the time derivative of V_n^{min} yields $\dot{V}_n^{min} =$

$\frac{2\varepsilon_n^{min}}{(1-\xi_n^{min^2})(\rho_n(t)-\delta_{x,n})}[f(x) + g(x)u - g(x)\lambda_u(t)\delta_u - \dot{a}_{n-1}^{min} - \xi_n^{min}\dot{\rho}_n]$, where, owing to (7b), it holds $\rho_n(t) > \delta_{x,n}$, for all $t \in [0, \tau_{max})$. Define $F_n(t) = f(x) - g(x)\lambda_u(t)\delta_u - \dot{a}_{n-1}^{min} - \xi_n^{min}\dot{\rho}_n$. Notice that $\xi_n^{min} \in \Omega_\xi$ for all $t \in [0, \tau_{max})$, δ_u is constant and $\lambda_u(t)$, $\dot{\rho}_n$ are bounded by construction. Therefore, by the Extreme Value Theorem, we can deduce the existence of positive constants f^*, g^*, g_* such that $|f(\cdot)| \leq f^*$ and $0 < g_* < g(\cdot) < g^*$. Furthermore, $\dot{a}_{n-1}^{min}$ is bounded from *Step n-1*. Thus, there exists a constant $\bar{F}_n > 0$ such that $|F_n(t)| \leq \bar{F}_n$, for all $t \in [0, \tau_{max})$ and owing to (6d) we obtain

$$\dot{V}_n^{min} = \frac{2\varepsilon_n^{min}[F_n(t) - g(x)sgn(g)k_n\varepsilon_n]}{(1 - \xi_n^{min^2})(\rho_n(t) - \delta_{x,n})} \leq \frac{2[\bar{F}_n|\varepsilon_n^{min}| - |g(x)|k_n\varepsilon_n\varepsilon_n^{min}]}{(1 - \xi_n^{min^2})(\rho_n(t) - \delta_{x,n})}. \quad (29)$$

To enforce

$$\xi_n^{max} - \xi_n^{min} \leq M_n,\ M_n \in (0, 1), \quad (30)$$

notice that utilizing (9a) and (9b), we conclude:

$$\big(2\delta_{x,n} - a_{n-1}^{max}(t) + a_{n-1}^{min}(t)\big)/\big(\rho_n(t) - \delta_{x,n}\big) \leq M_n.$$

Thus, $\rho_n(t)$ should satisfy:

$$\big((2 + M_n)\delta_{x,n} - a_{n-1}^{max}(t) + a_{n-1}^{min}(t)\big)/M_n \leq \rho_n(t). \quad (31)$$

Hence, (31) is satisfied and therefore (30) is enforced if

$$\rho_n^\infty \geq \big((2 + M_n)\delta_{x,n} + \bar{a}_{n-1}\big)/M_n. \quad (32)$$

As in *Step i*, we deduce that $-a_{n-1}^{max}(t) + a_{n-1}^{min}(t) > 0$, for which $-a_{n-1}^{max}(t) + a_{n-1}^{min}(t) < \bar{a}_{n-1}$ with $\bar{a}_{n-1} > 0$ also holds.

To proceed, we distinguish the following scenarios: $a)\ -1 < \xi_n^{min} < \xi_n < \xi_n^{max} < 0$, $b)\ 0 < \xi_n^{min} < \xi_n < \xi_n^{max} < 1$ and $c)\ -1 < \xi_n^{min} < 0 < \xi_n^{max} < 1$.

Scenario a) In this case, what needs to be proven is that ξ_n^{min} does not approach -1. In that direction notice that owing to the strictly increasing property of T-function and (6b), (10d), (10e) there exist time-functions $c_1(t), c_2(t) \in [1, +\infty)$ such that $\varepsilon_n^{min} = c_1(t)\varepsilon_n$ and $\varepsilon_n = c_2(t)\varepsilon_n^{max}$. Thus, (29) becomes $\dot{V}_n^{min} \leq \frac{2c_1(t)c_2(t)|\varepsilon_n^{max}|\left[\bar{F}_n - |g(x)|k_n c_2(t)|\varepsilon_n^{max}|\right]}{(1-\xi_n^{min^2})(\rho_n(t)-\delta_{x,n})}$. Hence, $\dot{V}_n^{min} \leq 0$ when $|\varepsilon_n^{max}(t)| \geq \bar{F}_n/(g_*k_n)$. Therefore, there exists a constant $\bar{\varepsilon}_n^{max} > 0$ such that:

$$|\varepsilon_n^{max}(t)| \leq \bar{\varepsilon}_n^{max} := max\{|\varepsilon_n^{max}(0)|, \bar{F}_n/(g_*k_n)\}. \quad (33)$$

Owing to (30), it is straightforwardly obtained that there exists a positive constant d_n such that:

$$\varepsilon_n^{max}(t) - \varepsilon_n^{min}(t) \leq d_n,\ \forall t \in [0, \tau_{max}). \quad (34)$$

From (33) and (34), we can deduce the existence of a positive constant $\bar{\varepsilon}_n^{min}$ such that

$$-\infty < -\bar{\varepsilon}_n^{min} \leq \varepsilon_n^{min}(t),\ \forall t \in [0, \tau_{max}) \quad (35)$$

and by applying the inverse T-function we conclude that

$$-1 < T(-\bar{\varepsilon}_n^{min}) \leq \xi_n^{min}(t),\ \forall t \in [0, \tau_{max}). \quad (36)$$

By (10b) and (10c), a_n^{min} and a_n^{max} can be proven bounded as well.

Scenario b) In this case we have to guarantee that ξ_n^{max} evolves away from 1. Similarly, by utilizing the fact that in this case $\varepsilon_n^{max} = c_1(t)\varepsilon_n$ and $\varepsilon_n = c_2(t)\varepsilon_n^{min}$ with $c_1(t), c_2(t) \in [1, +\infty)$, we can deduce from (29) that $\dot{V}_n^{min} \leq \frac{2|\varepsilon_n^{min}|\left[\bar{F}_n - |g(x)|k_n c_2(t)|\varepsilon_n^{min}|\right]}{(1-\xi_n^{min^2})(\rho_n(t)-\delta_{x,n})}$, which reveals $\dot{V}_n^{min} \leq 0$ when $|\varepsilon_n^{min}(t)| \geq \bar{F}_n/(g_*k_n)$. Thus, there exists a constant $\bar{\varepsilon}_n^{min} > 0$ such that:

$$|\varepsilon_n^{min}(t)| \leq \bar{\varepsilon}_n^{min} := max\{|\varepsilon_n^{min}(0)|, \bar{F}_n/(g_*k_n)\}.$$

At this point, owing to (30) and the boundedness of ε_n^{min}, we can straightforwardly conclude the existence of a positive constant $\bar{\varepsilon}_n^{max}$ such that

$$\varepsilon_n^{max}(t) \leq \bar{\varepsilon}_n^{max} < +\infty,\ \forall t \in [0, \tau_{max}) \quad (37)$$

holds. Taking the inverse T-function we deduce

$$\xi_n^{max}(t) \leq T^{-1}(\bar{\varepsilon}_n^{max}) < 1,\ \forall t \in [0, \tau_{max}). \quad (38)$$

By (10b) and (10c), a_n^{min} and a_n^{max} remain bounded as well in the aforementioned time interval.

Scenario c) Owing to (30), we directly deduce $\xi_n^{max} \leq M_n + \xi_n^{min} < M_n < 1$ and $-1 < -M_n < \xi_n^{max} - M_n \leq \xi_n^{min}$. Hence, deploying (11), we conclude

$$-1 < -M_n < \xi_n(t) < M_n < 1. \quad (39)$$

Similarly for $sgn(g) = -1$, utilizing (13c), we define $F_n(t) = f(x) - g(x)\lambda_u(t)\delta_u - \dot{a}_{n-1}^{max} - \xi_n^{min}\dot{\rho}_n$. Furthermore, with the use of (9a) and (9b), we can obtain

$$\big((2 + M_n)\delta_{x,n} + |a_{n-1}^{max}(t) - a_{n-1}^{min}(t)|\big)/M_n \leq \rho_n^\infty. \quad (40)$$

Following the same line of analysis as in case of positive $sgn(g)$, we can prove that

$$-1 < -M_n < \xi_n(t) < M_n < 1, \quad (41)$$

which reveals that $\xi_n(t)$ evolves strictly within $(-1, 1)$.

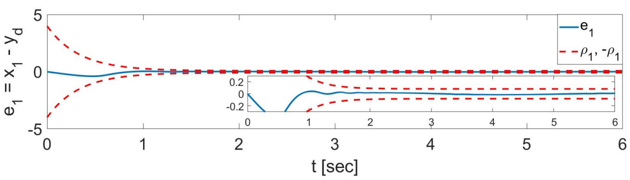
At this point, what remains to be shown is that $\tau_{max} = +\infty$. Owing to (22), (24)-(28), (36), (38), (39) and (41) we conclude that all ξ_i, $i = 1, ..., n$, evolves strictly within $(-1, 1)$. Therefore, following standard arguments ([21],Theorem 3.3), we can extend the solution to $+\infty$. The presented analysis implies that all signals in the closed-loop remain bounded and the output tracks the desired trajectory with prescribed performance, thus completing the proof of Theorem 1.

V. Simulation Results

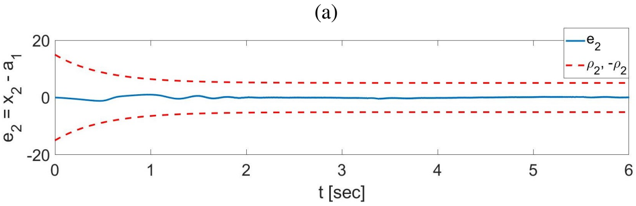
To illustrate the effectiveness of the proposed control, we perform simulation studies on the nonlinear system given by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_2(x_1 + sin(x_1)) + (1 + x_2^2)\upsilon \\ y &= x_1 \end{aligned} \quad (42)$$

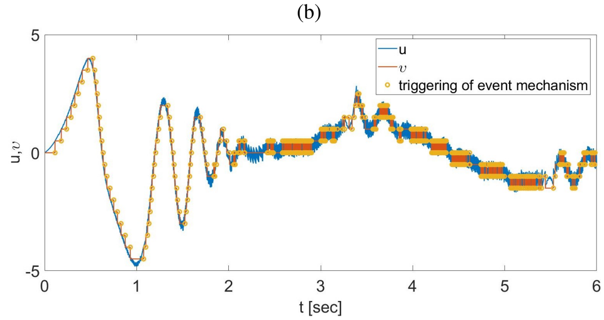
System (42) is in Brunovsky canonical form and satisfies Assumption 1. The system initially begins at $x_1(0) = x_2(0) = 0$. For $M_1 = 0.05$ and given the step-size of the event-triggered mechanism applied on x_1 as $\delta_{x,1} = 0.002$, the control target is the system output y to track the desired trajectory $y_d(t) = \frac{\pi}{8}sin(0.5\pi t) + \frac{\pi}{8}sin(0.4\pi t)$ for all $t \geq 0$, with steady-state error no more than 0.082 and minimum convergence rate as dictated by the exponential e^{-3t}. Thus, we select $\rho_1(t) = (4 - 0.082)e^{-3t} + 0.082$. We choose the design elements $\delta_{x,2} = 0.02$, $\delta_u = 0.5$, $M_2 = 0.5$, $\bar{a}_1 = 5$ and $\rho_2(t) = (15 - 10.1)e^{-2t} + 10.1$. The control gains are selected $k_1 = 2.5$ and $k_2 = 15$. The proposed control scheme (6) is applied on (42). The output tracking error $y - y_d$ as well as the intermediate error $x_2 - a_1$ alongside their corresponding performance bounds are presented in Fig. 1a and 1b, while the required control effort is illustrated in Fig. 1c. Hence, all signals of the closed-loop remain bounded and prescribed performance is achieved.



(a)



(b)



(c)

Fig. 1: (a) The tracking error along with its performance bounds; (b) The intermediate error alongside its corresponding performance bounds; (c) The required control effort.

VI. Conclusions

In this work we proposed a state-feedback controller to guarantee predefined bounds on the maximum steady-state error and minimum convergence rate of the output tracking error, for the class SISO uncertain systems in Brunovsky canonical form under non-periodic communication. The proposed control scheme is considered a low-complexity solution as it is static, it does not incorporate any prior knowledge of the system's nonlinearities and it does not utilize any approximation structures to obtain such information. No hard calculations, analytic or numerical, are required to produce the control signal. The theoretical findings are clarified and verified through simulation studies.

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