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Time-Optimal Motion Generation and Load-Sway Suppression for Rotary Cranes with a Two-Stage S-Curve Trajectory Based on Skilled Operation Analysis

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Abstract—**This paper presents an angular velocity trajectory generation method for a rotary crane boom based on the analysis of skilled operation, which is able to suppress the two-dimensional load-sway consisting of radial and tangential components in a short time by only boom horizontal rotational motion. The proposed trajectory consists of two-stage S-curve profiles for both acceleration and deceleration periods so that the natural period of the load-sway can be changed effectively to achieve the short motion time with load-sway suppression. The proposed trajectory is generated by solving the problem to find minimal motion time considering crane dynamics and constraints on load-sway and machine limitation. The motion performance by the proposed trajectory is compared with the conventional trajectory in simulation considering rope length change and boom-twist typical in a large crane. The proposed trajectory successfully suppresses the load-sway while satisfying the crane constraints in a short time.**

I. Introduction

Rotary cranes transport heavy loads from one point to another in various environments, such as factories, construction sites, and ports. Crane operations require accurate positioning and residual load-sway suppression. If crane operators fail to control the load, not only less productivity but also severe accident may occur. Indeed, the load-sway may cause collision with workers and walls.

Since rotary cranes generate two-dimensional (tangential and radial) load-sway due to centrifugal force, it is more difficult for operators to control than overhead cranes.

There have been many studies on control system design for load-sway suppression: Optimal control [1], [2], Trajectory planning method [3], [4] Regulator control [5], [6], Adaptive control [7], [8], Input shaping control [9], [10], Disturbance observer-based control [11], Sliding mode control [12], [13], Fuzzy control [14]. Although two-dimensional load-sway is typically suppressed by using both boom hoisting and horizontal motions, a few studies have proposed controller designs that suppress load-sway using only rotational horizontal motion [15]-[19]. Uchiyama et al. and Takahashi et al. have shown that a simple S-shaped angular velocity based on a cycloid function can suppress load-sway without sensors [15]-[17]. Abe have proposed a control technique for suppressing the load-sway using a combination of polynomial and cycloid functions [18]. Ohto et al. analyzed a control strategy of a skilled operator for suppressing the load-sway in the tangential and radial directions using only rotational motion, and proposed an anti-sway control method of the rotary crane [19]. However, except for [17], little attention has been given to high-speed operations to improve the productivity. In addition, most studies do not consider the machine performance and applicability.

This study proposes the two-stage S-curve trajectory of angular velocity for both acceleration/deceleration periods in a nonlinear rotary crane to suppress the two-dimensional load-sway in the tangential and radial directions using only the horizontal motion of the boom. The optimization problem generates the two-stage S-curve trajectory considering constraints including the machine performance. Simulation results compare it with the conventional trajectory and show the effectiveness to the rope length change and the boom-twist typical in a large crane. The proposed method successfully generates optimal motion time results with sufficient load-sway suppression.

II. Crane Dynamics and Motion Trajectory

A. Crane dynamics

This section explains the dynamics of a rotary crane and the two-stage S-curve trajectory design. Fig. 1 shows schematic of a rotary crane, and its equations of motion are

$$
\begin{aligned}
&ml^2(1+\theta_1^2)\ddot{\theta}_1 + ml^2\theta_1\theta_2\ddot{\theta}_2 - ml^2\theta_2\ddot{\theta}_4 + ml^2\theta_1\left(\dot{\theta}_1^2+\dot{\theta}_2^2\right) \\
&- ml(l\theta_1 + L\sin\theta_3)\dot{\theta}_4^2 - 2ml^2\dot{\theta}_2\dot{\theta}_4 + gml\theta_1 = 0 \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&ml^2\theta_1\theta_2\ddot{\theta}_1 + ml^2(1+\theta_2^2)\ddot{\theta}_2 + ml(L\sin\theta_3 + l\theta_1)\ddot{\theta}_4 \\
&+ ml^2\theta_2\left(\dot{\theta}_1^2+\dot{\theta}_2^2\right) - ml^2\theta_2\dot{\theta}_4^2 - ml^2\theta_2\dot{\theta}_4^2 + 2ml^2\dot{\theta}_1\dot{\theta}_4 \\
&+ mgl\theta_2 = 0 \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&-ml^2\theta_2\ddot{\theta}_1 + ml(L\sin\theta_3 + l\theta_1)\ddot{\theta}_2 + \left\{(mL^2 + I_x - I_z)\sin^2\theta_3\right. \\
&\left. + ml^2(\theta_1^2+\theta_2^2) + 2mLl\theta_1\sin\theta_3 + I_z\right\}\ddot{\theta}_4 + 2mLl\dot{\theta}_4\dot{\theta}_1\sin\theta_3 \\
&+ 2ml^2\left(\theta_1\dot{\theta}_1 + \theta_2\dot{\theta}_2\right)\dot{\theta}_4 + K(\theta_4-\theta_5) + C\left(\dot{\theta}_4 - \dot{\theta}_5\right) = 0 \qquad (3)
\end{aligned}
$$

where L denotes boom length, l rope length, m mass of load, m_b boom mass, I_x, I_y, I_z each axial moment of inertia around the center of gravity, K the rotational spring constant of the boom, C its viscous friction coefficient. θ_1 denotes the

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load-sway angle in radial direction, θ_2 the load-sway angle in tangential direction, θ_3 the hoisting angle of boom, θ_4 horizontal rotational angle of boom, θ_5 rotational angle of an actuator, g gravitational force. Note that the dynamics in this study is the same as in previous studies (see details in [17]). The spring-damper model is used to consider boom-twist in a large crane.

B. Design of two-stage S-curve trajectory

This study considers motion control that the rotary crane rotates to the desired angle based the proposed trajectory in a short time. Then, the trajectory suppresses the residual load-sway at the desired angle. We first analyze the behavior of actual rotary crane's operation by a skilled operator in Fig. 2. The angular velocity is characterized by constant velocity or acceleration/deceleration. The crane operator reduces the acceleration/deceleration one time during the acceleration and deceleration sections highlighted by circles. Hence, this study proposes the two-stage acceleration/deceleration trajectory based on the cycloid motion in Fig. 3. Eq. (4) shows the equation for the trajectory in Fig. 3, and Eq. (5) is the maximum angular velocity of the trajectory.

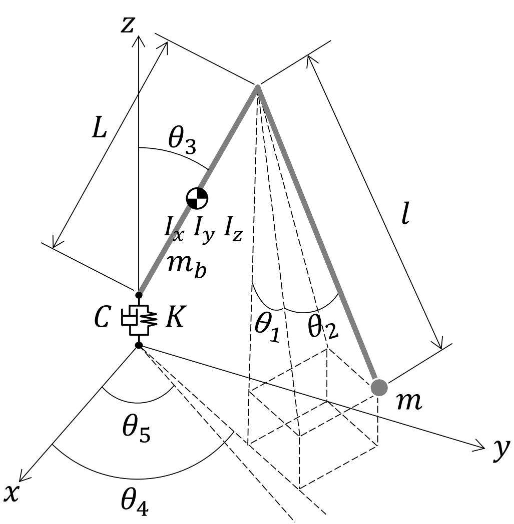


Fig. 1 Schematic of a rotary crane

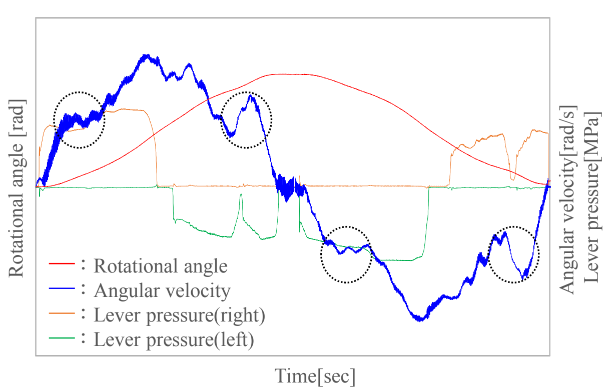


Fig. 2 Result of actual crane operation

$$\dot{\theta}_5 = \begin{cases} r_1\dot{\theta}_{5max}\left(1-\cos\pi\frac{t}{t_1}\right) & t\in[0,t_1] \\ 2r_1\dot{\theta}_{5max} & t\in\left[t_1,\sum_{i=1}^{2}t_i\right] \\ (1-r_1)\dot{\theta}_{5max}\left(1-\cos\pi\frac{t}{t_3}\right) & t\in\left[\sum_{i=1}^{2}t_i,\sum_{i=1}^{3}t_i\right] \\ 2\dot{\theta}_{5max} & t\in\left[\sum_{i=1}^{3}t_i,\sum_{i=1}^{4}t_i\right] \\ (1-r_2)\dot{\theta}_{5max}\left(1-\cos\pi\frac{t}{t_5}\right) & t\in\left[\sum_{i=1}^{4}t_i,\sum_{i=1}^{5}t_i\right] \\ 2r_2\dot{\theta}_{5max} & t\in\left[\sum_{i=1}^{5}t_i,\sum_{i=1}^{6}t_i\right] \\ r_2\dot{\theta}_{5max}\left(1-\cos\pi\frac{t}{t_7}\right) & t\in\left[\sum_{i=1}^{6}t_i,T\right] \end{cases} \quad (4)$$

$$\frac{\theta_{4d}}{\dot{\theta}_{5max}} = r_1\left(t_1-\frac{\pi}{t_1}\sin\pi\right)+2r_1t_2+(1-r_1)\left(t_3-\frac{\pi}{t_3}\sin\pi\right)+2t_4 +(1-r_2)\left(t_5-\frac{\pi}{t_5}\sin\pi\right)+2r_2t_6+r_2\left(t_7-\frac{\pi}{t_7}\sin\pi\right) \quad (5)$$

where total time T consists of acceleration sections $t_1 \sim t_3$, constant speed section t_4, and deceleration sections $t_5 \sim t_7$. The maximum rotational velocity is $\dot{\theta}_{5max}$, θ_{4d} is the desired final angle of θ_4, and ratios of steps in the angular velocity trajectory are r_1 and r_2 in the acceleration and deceleration sections, respectively.

The horizontal motion in section t_1 is used to generate the load vibration intentionally for changing the natural period shorter. The second part of the trajectory t_2 suppresses the load-sway by the acceleration. After that, the crane increases the centrifugal force by further accelerating in section t_3 and generates radial directional load-sway intentionally. The crane suppresses this radial directional load-sway by deceleration in section t_5. Finally, the crane suppresses only the remaining tangential load-sway actively in section t_7. The constant speed period in t_2, t_4, t_6 adjusts the load-sway and the natural period.

Uchiyama et al. set the acceleration/deceleration time with a few multiples of load-sway's natural period using a cycloid trajectory below and Fig. 4 [15]. Eq. (6) shows the condition of residual load-sway suppression given in [15].

$$t_1 = t_3 = i_1\frac{\pi}{\omega}, i_1 = 3,4,\ldots$$
$$t_2 = 2i_2\frac{\pi}{\omega}, i_2 = 0,1,\ldots \quad (6)$$

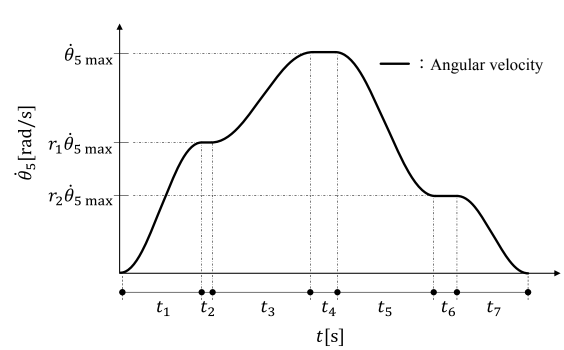


Fig. 3 Two-stage angular velocity trajectory

where $\omega = \sqrt{g/l}$. In this method, the transportation time depends on the natural period of the load-sway, and it is difficult to transport the load at high speed for large cranes with a long natural period. The proposed two-stage S-curve trajectory enables to change the natural period because the trajectory increases the number of parameters from three to nine.

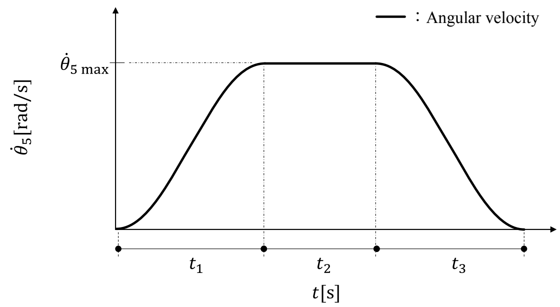


Fig. 4 Cycloid trajectory

C. *Optimization of two-stage S-curve trajectory*

The following optimization problem is considered in this study by assuming the symmetricity of the trajectory to reduce the computation time. The constraints of the maximum angular acceleration and the residual load-sway are considered. The parameter t_i and r_i in Eq. (4) are variables for optimization.

【Constraints】

Crane dynamics in Eqs.(1)-(3) and Two-stage S-curve in Eq. (4).

$$\begin{aligned} &t_{1\min} < t_1 = t_7 < t_{1\max} \quad t_{2\min} < t_2 = t_6 < t_{2\max} \\ &t_{3\min} < t_3 = t_5 < t_{3\max} \quad t_{4\min} < t_4 < t_{4\max} \\ &r_{1\min} < r_1 = r_2 < r_{1\max} \end{aligned} \tag{7}$$

Allowable residual load-sway:

$$\begin{aligned} |\theta_1| < \theta_{1\max} \qquad t \in [T + T_p, T + 2T_p] \\ |\theta_2| < \theta_{2\max} \qquad t \in [T + T_p, T + 2T_p] \end{aligned} \tag{8}$$

where $T_p = 2\pi\sqrt{l/g}$.

Limitation of Angular acceleration:

$$|\ddot{\theta}_5| < \ddot{\theta}_{5\max} \qquad t \in [0, T] \tag{9}$$

III. SIMULATION

A. *Simulation condition*

We use the sequential quadratic programming method in the optimization toolbox in MATLAB® for optimization and verify the following cases:

case 1: Without angular acceleration constraint in Eq. (9)
case 2: With Eq. (9)
case 3: Cycloid trajectory in [17]
case 4: For short rope length (l=1.0m)
case 5: With boom-twist due to lower stiffness of the large crane (K=400 Nm/rad, C=55 Nms/rad)

Table 1 shows the simulation condition. The computational time was about 7~14 min on a PC (Intel(R) Core (TM) i7-9700F CPU@ 3.00GHz)

Table 1 Simulation condition

Parameter	[unit]	case No.		
		1 ~ 3	4	5
L	[m]		2.0	
l	[m]	2.0	1.0	2.0
m	[kg]		2.0	
m_b	[kg]		1.61	
I_x	[kgm^2]		54.95	
I_y	[kgm^2]		54.95	
I_z	[kgm^2]		2.19	
K	[Nm/rad]	40000		400
C	[Nms/rad]	5500		55
$t_{1\min} / t_{1\max}$	[sec]		0.5 / 5.0	
$t_{2\min} / t_{2\max}$	[sec]		0.0 / 5.0	
$t_{3\min} / t_{3\max}$	[sec]		0.5 / 5.0	
$t_{4\min} / t_{4\max}$	[sec]		0.0 / 2.0	
$r_{1\min} / r_{1\max}$	[-]		0.3 / 0.7	
$\theta_{1\max}$	[rad]		0.005	
$\theta_{2\max}$	[rad]		0.005	

B. *Simulation results*

(case 1) Without angular acceleration constraint

Fig. 5 shows the result without the constraints of the crane acceleration in the proposed trajectory. The trajectory sufficiently suppresses the residual load-sway in a short time. This result shows that the effectiveness for suppressing the two-dimensional load-sway in the tangential and radial directions to boom rotation.

(case 2) With constraints of angular acceleration

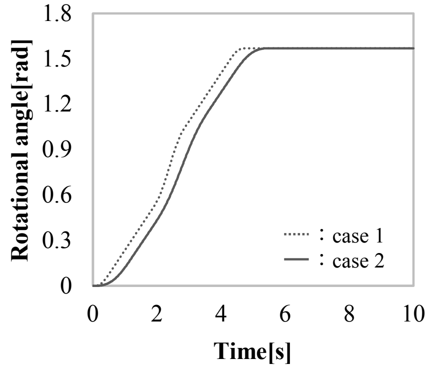
The effectiveness of proposed trajectory with the acceleration constraint of the horizontal motion The constraint is set as follows:

$$|\ddot{\theta}_5| < 0.4 \text{ rad/s}^2 \qquad t \in [0, T] \tag{10}$$

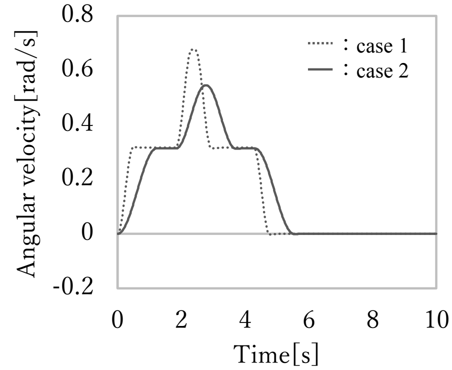
Fig. 5 also shows the result of case 2 where the proposed trajectory satisfies the constraints of acceleration and suppresses the load-sway.

(case 3) Comparison with conventional trajectory

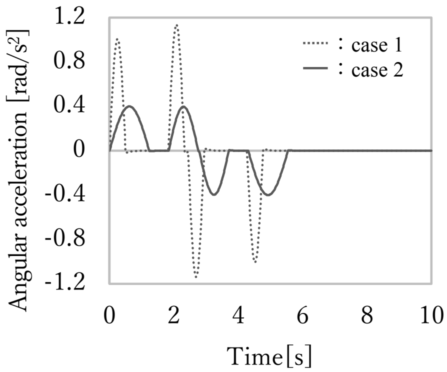
The proposed trajectory is compared with the trajectory proposed in [15] ~ [17]. Fig. 6 shows the result of case 3 with case 2. Both trajectories sufficiently suppress the residual load-sway. The proposed trajectory case 2 requires in approximately 25% shorter motion time than case 3. In addition, case 3 does not reach the acceleration limit, which means the crane performance is not fully utilized. The proposed trajectory fully utilizes machine performance with load-sway suppression.



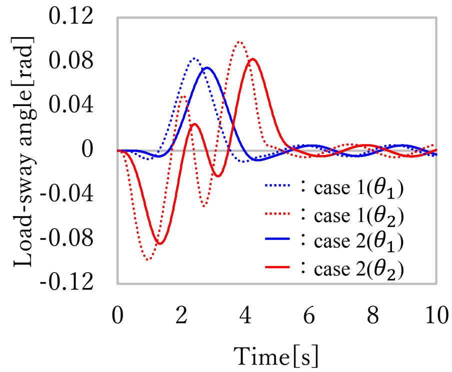
（a） Rotational angle θ_5



（b） Angular velocity $\dot{\theta}_5$

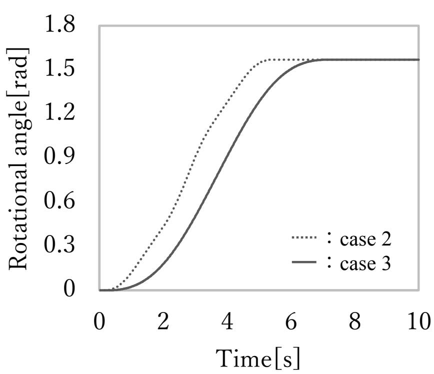


（c） Angular acceleration $\ddot{\theta}_5$

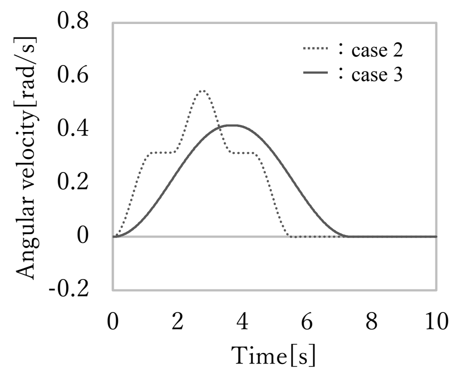


（d） load-sway angle θ_1, θ_2

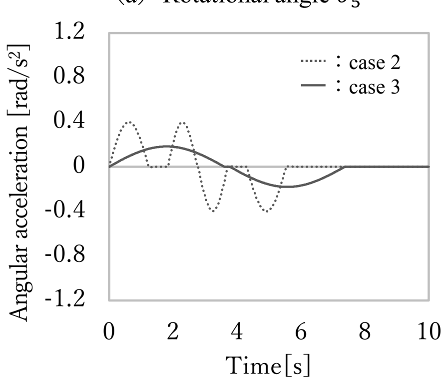
Fig. 5 Simulation results in case 1 and case 2



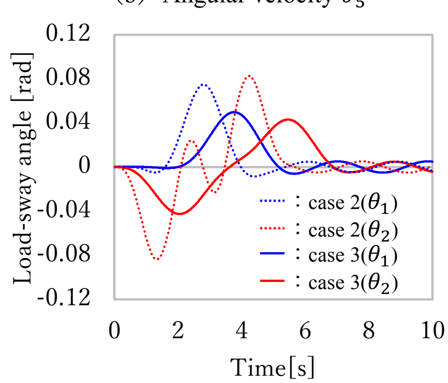
（a） Rotational angle θ_5



（b） Angular velocity $\dot{\theta}_5$



（c） Angular acceleration $\ddot{\theta}_5$



（d） load-sway angle θ_1, θ_2

Fig. 6 Simulation results in case 3

(case 4) Short rope length

We examine the case that rope length is shorter than case 2, where the rope length is 1.0m , half of the previous length. Fig. 7 shows the result where the trajectory in case 4 also suppresses the residual load-sway. The positioning time is shorter approximately 9% than case 2. The natural period of load-sway changes the required motion time.

(case 5) Considering boom-twist

We examine the case that boom has twist typical in a large crane. The properties of the spring-damper model are K=400Nm/rad and C=55Nms/rad. Fig. 8 shows the result where the trajectory in case 5 also suppresses the load-sway, and the proposed trajectory is effective for boom-twist typical in a large crane.

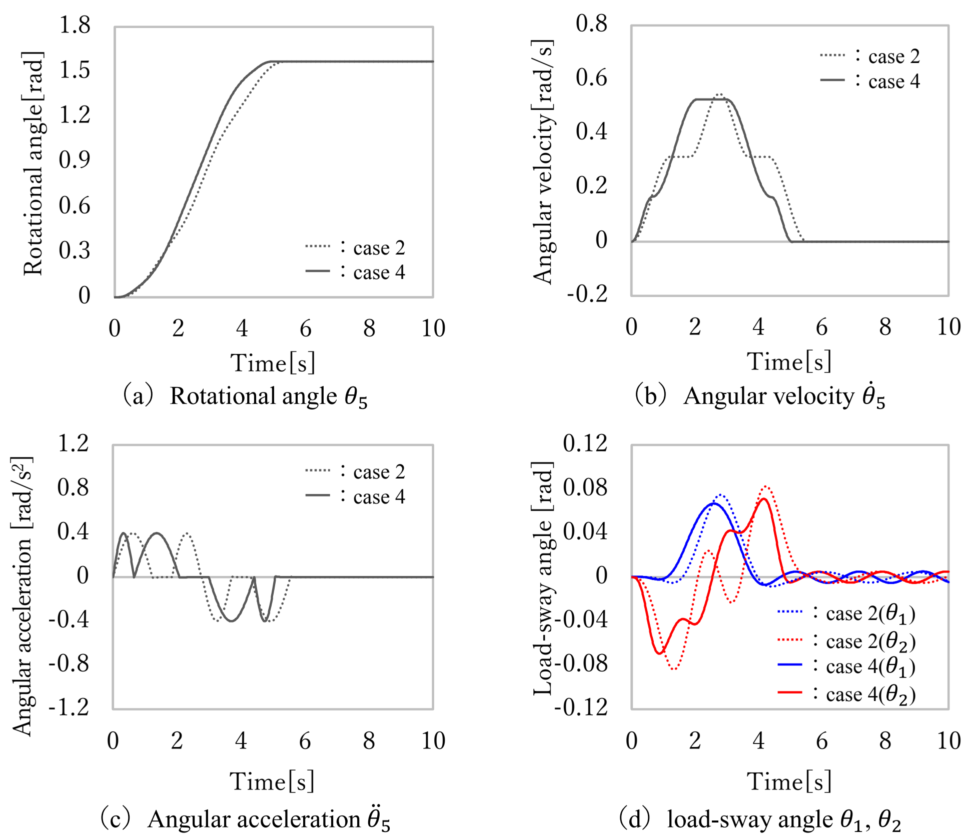
C. Summary of simulation results

From the simulation results, the two-stage S-curve angular velocity trajectory for the horizontal motion is effective in suppressing the load-sway in a short time. It is generally difficult to control load-sway of a rotary crane because of not only the tangential vibration caused by rotational movement but also the radial vibration caused by centrifugal force. We confirm that the proposed trajectory in which the angular velocity changes in two stages is effective in suppressing two-directional load-sway. The rotational trajectory is based on a cycloid function with less stress in the machine during acceleration/deceleration, and can be applied to actual machines practically.

IV. CONCLUSION

This paper presents a two-stage S-curve angular velocity trajectory based on the actual rotary crane operation to transport the load. Not only tangential load-sway but also radial one caused by centrifugal force to the boom horizontal motion can be controlled effectively to increase the work productivity.

Furthermore, we examined the effectiveness of the proposed method by simulation including cases that a crane has short rope length and boom-twist. The simulation results showed the effectiveness in suppressing the load-sway successfully while considering the crane constraints. Besides, the proposed trajectory obtained the result of faster approximately 25% than the conventional trajectory to improve the productivity. The load-sway is suppressed for different rope length and boom-twist. Experimental verification is left for future work.



(a) Rotational angle θ_5

(b) Angular velocity $\dot{\theta}_5$

(c) Angular acceleration $\ddot{\theta}_5$

(d) load-sway angle θ_1, θ_2

Fig. 7 Simulation results in case 4

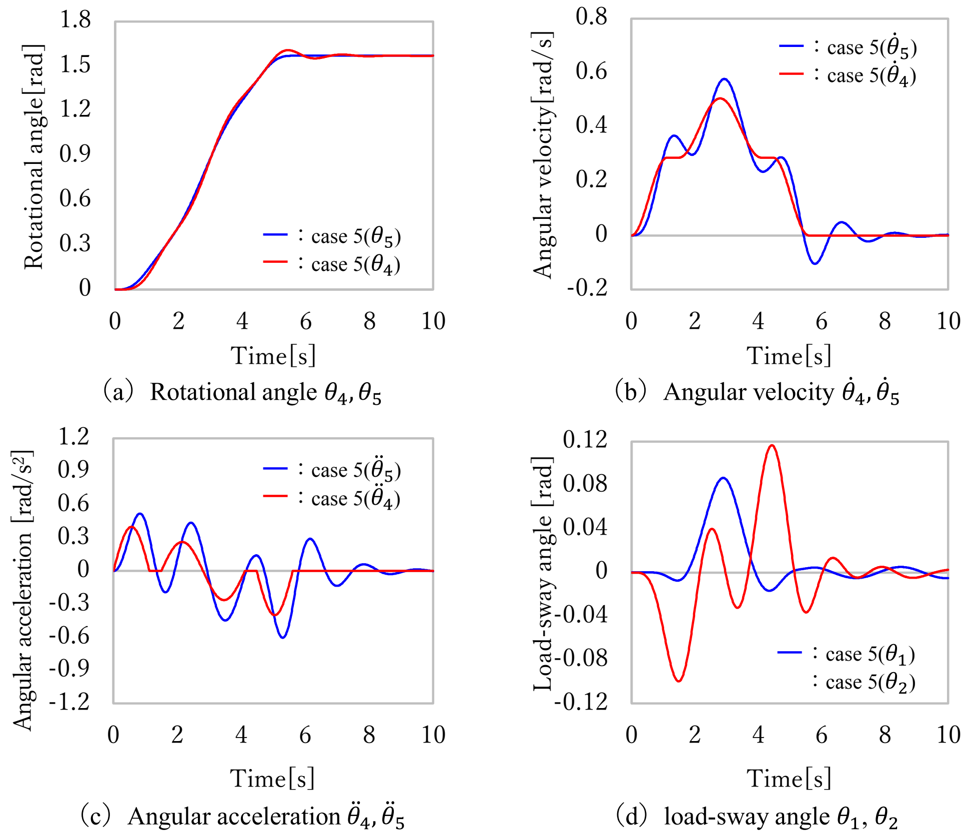


Fig. 8 Simulation results in case 5

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