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Position Control of Single-Link Flexible Manipulator: A Functional Observer Based Sliding Mode Approach

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***Abstract*— This paper proposes a functional observer-based sliding mode control for position control of a Single-Link Flexible Manipulator (SLFM). The proposed control considers the unmodelled system dynamics as uncertainty and aims to achieve position control. The proposed control scheme is designed considering the reduced order dynamics. A functional observer is used to directly compute a sliding function and the control signal, which guarantees the system's robustness and stability. The proposed control scheme is validated for large-order ordinary differential equation (ODE) model of the SLFM using numerical simulations.**

I. Introduction

In recent years, robotic manipulators have been explored for a wide range of applications, including industrial production [1], hostile environments (nuclear sites, deep sea, etc.) [2], space exploration [3], health care equipment [4], building construction [5]. It is required that the robotic manipulators provide faster, cost-effective, and accurate operation [6]. The rigid robot manipulators are made up of rigid links, which makes them bulkier. The industries need an upgrade to the existing classical robots in order to reduce construction costs, minimize energy consumption brought on by big actuator sizes, and increased production.

Thus, in applications where there is a requirement that the weight-to-volume ratio of a manipulator is low, inevitably, manipulators tend to be flexible. There are applications where large and lighter manipulators are required [7], and as a consequence of larger and lighter arms, flexibility comes into the picture. Further, as the payload-to-weight ratio increases, the tendency of flexible modes to get excited increases. The flexibility in the manipulator can be modelled as the link deformation [8]. Hence, analysis of such systems can not be performed as rigid manipulators. If we consider the flexibility, the system formed will be infinite-dimensional, i.e., the dynamic model of the flexible link robot manipulator is described as a distributed parameter system. This makes the dynamics of a flexible link robot manipulator depend on both space and time. Hence, the mathematical analysis of a flexible link manipulator would involve partial differential equations (PDE) rather than the ordinary differential equation (ODE). From a control viewpoint, finding the direct analytical solution to PDEs may only sometimes be possible, and the solutions obtained may only sometimes be realizable. So, we need to approximate the PDE-based mathematical model of the flexible link manipulator to an ODE-based model. There are various approximation methods available in the literature [9] including finite element method (FEM) [10], [11], assumed mode method (AMM) [12]–[15]. In this paper, the assumed mode method is chosen over the finite element method [16]. This is because the computational complexity of the assumed mode method is better than the finite element approach.

The study of the SLFM was the starting point for flexible robot research. There are various methods of modelling (SLFM) available in the literature, Lumped parameter approach [17], Euler-Bernoulli beam theory [18], Hamilton's principle [19], Lagrangian dynamics [20], Newton-Euler-FEM method [21], [22], Finite Element Method (FEM) [10], [11], assumed-modes method [12]–[15].

Due to the flexibility of the link, the tip position of a flexible link robot manipulator depends on both the joint angle and the link deformation variable. Even a small link deformation has a very significant impact on the tip position. Therefore, a control input must be designed to drive the tip to the desired trajectory to perform the specific operation using the flexible link manipulator. However, due to the deformation in the link, the existing control algorithms are insufficient to efficiently control the flexible link manipulator [6], [23].

The most desirable characteristics of a control system are a simple design, fast response, and robustness to uncertainties and disturbances. The dynamic model of a flexible link manipulator has inherent, unmodelled uncertainties.

Therefore, a robust control design is preferable for such a system. Sliding mode control (SMC) is one of the most used control schemes for uncertain nonlinear systems to provide robustness and faster system response [24], [25]. The SMC scheme is a model-based feedback control technique. This paper uses the state feedback sliding mode control design because of its simplistic design. Therefore, it is required that the system states needed in the control input be available for feedback control design. Nevertheless, in a flexible link manipulator, all the states can never be available for the feedback design. Hence, traditional SMC can not fit such a system well. Therefore, an observer is designed to estimate the unmeasurable system state by utilising the knowledge of input and output. However, typically, a linear feedback control law needs to estimate some linear function of states of the form $Kx(t)$. The estimation of the linear function of the state vector can be done using a minimal-order observer. Therefore, a functional observer is designed in this paper to estimate the linear function of the state vector required in the SMC design.

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A functional observer uses system outputs to estimate the function of the linear combination of states required in the control input. Previous works have demonstrated several techniques to design functional observers for linear time-invariant (LTI) systems [26]–[29].

A functional observer estimates linear functions of states, which are then used in the sliding mode controller. This composite control strategy is the Functional Observer-based Sliding Mode Control (FO-SMC) scheme. It has been demonstrated that FO-SMC works well for controlling the position of flexible link manipulators.

In this paper, we proposed FO-SMC to control the position of a single-link flexible manipulator. Numerical simulations are utilized to verify the proposed control scheme. The results illustrate that the proposed FO-SMC technique can precisely and successfully control the position of the single-link flexible manipulator.

The contributions of this paper are:

- The FO-SMC based composite control scheme is designed considering the reduced-order dynamics.
- Numerical simulation is used to validate the proposed control scheme for a larger-order model.
- The demonstration of the efficacy of the proposed FO-SMC scheme in controlling the position of a SLFM.

II. Dynamic Model of Single Link Flexible Manipulator

The flexible manipulator having a single link is shown in figure 1. The link under investigation has mass uniformly distributed across its length $l(m)$ with $\rho(kg/m)$ as linear mass density. In figure 1 CoM represents the position of centre of mass. The SLFM is modeled with the following assumptions taken into account.

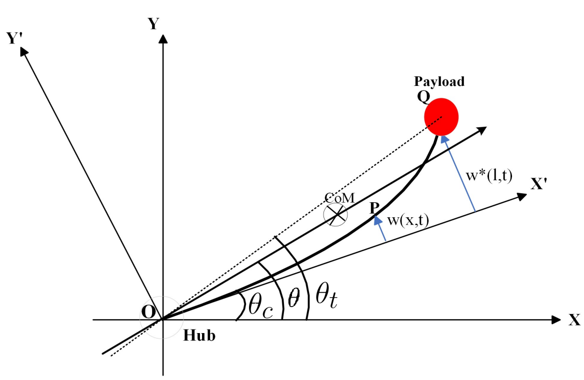

Fig. 1. Single-Link Flexible Manipulator

Assumption 1: The mass is evenly distributed along the entire length of the link.

Assumption 2: The link exhibits small deformation due to pure bending. (Without any torsion or compression)

The link under investigation is modelled as an Euler-Bernoulli beam with a cross-sectional moment of inertia I and Young's modulus E. The payload carried by the manipulator has both mass $m_p(kg)$ and inertia $J_p(kg-m^2)$. Torque (τ N-m) is supplied to the manipulator by an electrical motor connected at its base with inertia $J_0(kg-m^2)$.

The dynamic equations of SLFM are derived using Hamilton's principle and assumed mode method [30]. The Euler-Lagrange equations for the generalized $(n+1)$ coordinates $q=(\theta,\zeta_i)=(\theta,\zeta_1,\cdots,\zeta_n)$ are given as:

$$M\ddot{q}(t)+\bar{D}\dot{q}(t)+Kq(t)=\bar{B}\tau(t) \tag{1}$$

Where, ζ_i represents the i^{th} vibrational mode.

$$M=\begin{bmatrix}J & 0\\ 0 & I\end{bmatrix}, \bar{D}=\begin{bmatrix}0 & 0\\ 0 & 2\xi\Omega\end{bmatrix},$$
$$K=\begin{bmatrix}0 & 0\\ 0 & \Omega^2\end{bmatrix}, \bar{B}=\begin{bmatrix}1\\ \phi'(0)\end{bmatrix}$$

$\Omega = \text{diag}\{\omega_1,\cdots,\omega_n\}$, $\phi'(0) = (\phi'_1(0),\cdots,\phi'_n(0))^T$. Where, $J=J_0+\rho\frac{l^3}{3}+J_p+m_pl^2$, $\phi'_i(0)=\frac{\partial\phi_i(x)}{\partial x}$ at $l=0$ and $i=1,2,\cdots,n$ denotes the assumed modes, $\phi(x)$ is the shape function and ξ denotes the damping coefficient. The state space representation of the dynamic model in (1) can be written as:

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{2}$$
$$y(t)=Cx(t) \tag{3}$$

where, $x(t)\in\mathbb{R}^{(2n+2)}$, $A\in\mathbb{R}^{(2n+2)\times(2n+2)}$, $B\in\mathbb{R}^{(2n+2)\times1}$, $C\in\mathbb{R}^{2\times(2n+2)}$, $y(t)\in\mathbb{R}^2$ denotes the output of the system, and $u(t)\in\mathbb{R}$ represents the input to the system.

$$x(t)=\begin{bmatrix}\vartheta(t) & \dot{\vartheta}(t)\end{bmatrix}^T, \qquad y(t)=\begin{bmatrix}\theta_c(t) & \theta_t(t)\end{bmatrix}^T$$

Where, $\vartheta(t)=[\theta(t),\zeta_1(t),\zeta_2(t),\cdots,\zeta_n(t)]^T$ and $\theta_c(t)$ and $\theta_t(t)$ are the clamped joint angle and tip position angle respectively.

$$A=\begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}\bar{D}\end{bmatrix}, \qquad B=\begin{bmatrix}0\\ M^{-1}\bar{B}\end{bmatrix} \tag{4}$$
$$C=\begin{bmatrix}C_1 & C_2\end{bmatrix} \tag{5}$$
$$u(t)=\tau(t)$$

where, $C_1=\begin{bmatrix}1 & \phi'_1(0) & \phi'_2(0) & \cdots & \phi'_n(0)\\ 1 & \frac{\phi_1(l)}{l} & \frac{\phi_2(l)}{l} & \cdots & \frac{\phi_n(l)}{l}\end{bmatrix}$, and $C_2=0$; $C_1\in\mathbb{R}^{2\times(n+1)}$ and $C_2\in\mathbb{R}^{2\times(n+1)}$

III. Composite Control Design

A. Sliding Mode Control Design

In this section, the SMC law is designed to control the position of the SLFM system. The sliding function is chosen as follows:

$$\sigma(t)=\Gamma\left[x(t)-x_d(t)\right] \tag{6}$$

where, $x_d(t)$ is the desired position of states and $\Gamma\in\mathbb{R}^{1\times(2n+2)}$ is a constant, which is to be designed such that the system is stable when confined to $\sigma(t)=0$.

Differentiating $\sigma(t)$ in (6) with respect to time:

$$\begin{aligned}\dot{\sigma}(t)&=\Gamma\left[\dot{x}(t)-\dot{x_d}(t)\right]\\ &=\Gamma\left[Ax(t)+Bu(t)-\dot{x_d}(t)\right]\end{aligned} \tag{7}$$

The proposed control law $u(t)$ has two components: nominal control u_{nom} and discontinuous control u_{disc}. The expression for $u(t)$ is given as:

$$u(t) = \underbrace{(\Gamma B)^{-1}\left[-\Gamma A x(t) + \Gamma \dot{x}_d(t)\right]}_{u_{\text{nom}}} - \underbrace{(\Gamma B)^{-1}\left[k_1\sigma(t) + k_2 \text{sgn}(\sigma(t))\right]}_{u_{\text{disc}}} \quad (8)$$

Where, $k_1, k_2 > 0 (\in \mathbb{R})$ are constants to be designed and $(\Gamma B)^{-1}$ is invertible.

Lemma 1 (Finite-time convergence lemma [31]): Consider a continuous time system $\dot{\Psi} = f(\Psi)$, $\Psi \in \mathbb{R}^n$ with zero as the only equilibrium point. Consider a positive definite Lyapunov candidate function $\mathcal{V}(\Psi) : \mathbb{R}^n \to \mathbb{R}$, with $\alpha_1 > 0$, $\alpha_2 > 0$, $\chi \in (0, 1)$, and an open vicinity of origin $\Delta_0 \subseteq \mathbb{R}^n$, such that the inequality in (9) is satisfied.

$$\dot{\mathcal{V}}(\Psi) \leq -\alpha_1 \mathcal{V}(\Psi) - \alpha_2 \mathcal{V}^{\chi}(\Psi); \quad \Psi \in \Delta_0 \ \{0\}, \quad (9)$$

then the equilibrium point is finite-time stable. Further, if $\Delta_0 = \mathbb{R}^n$, then the global finite-time stability of the equilibrium point is guaranteed.

Theorem 2: Consider the state space model in (2) and the sliding function in (6). With the application of the proposed controller, (8), the sliding phase will be attained in finite time (i.e., $\sigma(t) = 0,\ \forall\, t > T,\ T < \infty$), and the system states will converge asymptotically to the desired position.

Proof: Define a Lyapunov function V_1 as:

$$V_1(t) = \frac{1}{2}\sigma^2(t). \quad (10)$$

The time derivative of $V_1(t)$ gives

$$\dot{V}_1(t) = \sigma(t)\dot{\sigma}(t). \quad (11)$$

From equation (7) put $\dot{\sigma}(t)$ in (11) and use (2):

$$\dot{V}_1(t) = \sigma(t)\Gamma\left(Ax(t) + Bu(t) - \dot{x}_d(t)\right). \quad (12)$$

Putting $u(t)$ from (8) into (12):

$$\begin{aligned}
\dot{V}_1(t) &= \sigma(t)\left(-k_1\sigma(t) - k_2 \text{sgn}(\sigma(t))\right) \\
&= -k_1\sigma^2(t) - k_2|\sigma(t)| \\
&= -2k_1\frac{\sigma^2(t)}{2} - \sqrt{2}k_2\left(\frac{|\sigma^2(t)|}{2}\right)^{\frac{1}{2}} \\
\dot{V}_1(t) &= -\alpha_1 V_1(t) - \alpha_2 V_1^{\frac{1}{2}}(t) \quad (13)
\end{aligned}$$

where $\alpha_1 = 2k_1$, $\alpha_2 = \sqrt{2}k_2$ and $\chi = 1/2$. From equation (13) it is clearly visible that it satisfies the inequality condition in lemma 1's finite time inequality equation. Thus, it can be inferred that the sliding variable in equation (6) converges to zero in finite time, thereby guaranteeing the convergence of system state $x(t)$ to the desired position $x_d(t)$ as sliding function $\sigma(t)$ has been designed in a manner that the system dynamics is asymptotically stable when confined to $\sigma(t) = 0$. ■

The control input in equation (8) can be equivalently written as:

$$\begin{aligned}
u(t) = &-(\Gamma B)^{-1}\left[\Gamma A + k_1\Gamma\right]x(t) \\
&- (\Gamma B)^{-1}\left[k_2 \text{sgn}(\Gamma x(t) - \Gamma x_d(t))\right] \\
&+ (\Gamma B)^{-1}\left[\Gamma \dot{x}_d(t) + k_1\Gamma x_d(t)\right] \quad (14)
\end{aligned}$$

The SMC law in (14) needs the system states for closed-loop design. But the system under consideration does not have all the required states available for the measurement. Therefore, an observer is to be designed to implement the control law. As the control input in (14) needs estimation of some linear function of states, a linear state function observer is proposed such that the output of the functional observer can be directly used in the controller.

B. Functional Observer

This section introduces a functional observer that estimates the linear combination of states required by the control input function.

In order to implement the control in (14), we need two linear functionals of the state, which are $g_1(t) = F_1 x(t)$ and $g_2(t) = F_2 x(t)$. Where F_1 and F_2 are given as:

$$F_1 = -(\Gamma B)^{-1}\left[\Gamma A + k_1\Gamma\right]; \quad F_2 = \Gamma$$

Then control law can be written as:

$$\begin{aligned}
u(t) = &\begin{bmatrix} I & 0 \end{bmatrix} g(t) - (\Gamma B)^{-1}[k_2 \text{sgn}(\begin{bmatrix} 0 & I \end{bmatrix} g(t) - \\
&\Gamma x_d(t))] \\
&+ (\Gamma B)^{-1}\left[\Gamma \dot{x}_d(t) + k_1\Gamma x_d(t)\right] \quad (15)
\end{aligned}$$

where, I represents the identity matrix.

Now, the requirement boils down to designing a functional observer that estimates

$$g(t) = \begin{bmatrix} g_1(t) & g_2(t) \end{bmatrix}^T = \begin{bmatrix} F_1 & F_2 \end{bmatrix}^T x(t) = Fx(t) \quad (16)$$

where, $F \in \mathbb{R}^{2\times(2n+2)}$ and $g(t) \in \mathbb{R}^2$.

In order to achieve this linear state function estimation, an observer of the form (17) needs to be designed.

$$\dot{\hat{\eta}}(t) = N\hat{\eta}(t) + Ly(t) + Hu(t) \quad (17a)$$

$$\hat{g}(t) = Gy(t) + D\hat{\eta}(t) \quad (17b)$$

where, $\hat{\eta}(t) \in \mathbb{R}^v$ is a state vector. $\hat{g}(t) \in \mathbb{R}^2$ is the desired estimate of functional. $N \in \mathbb{R}^{v\times v}$, $L \in \mathbb{R}^{v\times 2}$, $H \in \mathbb{R}^v$, $G \in \mathbb{R}^{2\times 2}$, and $D \in \mathbb{R}^{2\times v}$ are unknown matrices.
The output $\hat{g}(t)$ of (17b) is said to estimate $Fx(t)$ in an asymptotic manner if

$$\lim_{t\to\infty}\left[\hat{g}(t) - Fx(t)\right] = 0 \quad (18)$$

Now let us suppose that if $\hat{\eta}(t)$ estimates the linear function of $x(t)$ as $\eta(t) = Tx(t)$ (where $T \in \mathbb{R}^{v\times(2n+2)}$) then, $\hat{g}(t)$ estimates the $Fx(t)$ for which we have the theorem 3.

Theorem 3: The completely observable v^{th} order observer will estimate $g(t) = Fx(t)$ if and only if the following conditions are satisfied:

1) N is a Hurwitz matrix.

2) $TA - NT - LC = 0$
3) $H = TB$
4) $F = GC + DT$
5) $v \geq rank(F - GC)$

where $F \in \mathbb{R}^{2\times(2n+2)}$ is the linear state function gain matrix and $T \in \mathbb{R}^{v\times(2n+2)}$ is the unknown matrix which is to be determined.

Proof: The proof of the theorem is given in [28]. ■

C. Proposed Functional Observer-based Sliding Mode Control

This section proposes a composite control law using the sliding mode design and functional observer output. The error between the linear function estimates is expressed as e(t) as:

$$e(t) = \eta(t) - \hat{\eta}(t) = Tx(t) - \hat{\eta}(t) \tag{19}$$

Using equations (2), (17a) in the derivative of $e(t)$ in (19), we get:

$$\dot{e}(t) = T\dot{x}(t) - \dot{\hat{\eta}}(t) \tag{20}$$

On simplifying equation (20) and using Theorem 3 we get:

$$\dot{e}(t) = Ne(t) \tag{21}$$

Using the results in theorem 3 control input $u(t)$ can be rewritten as:

$$\begin{aligned} u(t) = [I \quad 0]\, Fx(t) - [I \quad 0]\, De(t) \\ - \underbrace{(\Gamma B)^{-1}\left[k_2 \text{sgn}(\Gamma x(t) - \Gamma x_d(t))\right]}_{u_{\text{switching control}}} + \\ \underbrace{(\Gamma B)^{-1}\left[\Gamma \dot{x}_d(t) + k_1 \Gamma x_d(t)\right]}_{u_{\text{reference}}} \end{aligned} \tag{22}$$

Now equation (2) is rewritten using (22).

$$\begin{aligned} \dot{x}(t) = Ax(t) + B\,[I \quad 0]\, Fx(t) - B\,[I \quad 0]\, De(t) \\ + B\underbrace{\left[u_{\text{switching control}} - u_{\text{reference}}\right]}_{u_{\text{bounded}}} \end{aligned} \tag{23}$$

A composite system is formed using (21) and (23).

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix} = \underbrace{\begin{bmatrix} A + B\,[I \quad 0]\,F & -B\,[I \quad 0]\,D \\ 0 & N\end{bmatrix}}_{A_C}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} + \underbrace{\begin{bmatrix}B\\ 0\end{bmatrix}}_{B_C} u_{bounded} \tag{24}$$

where, $A_C \in \mathbb{R}^{(2n+2+v)\times(2n+2+v)}$, and $B_C \in \mathbb{R}^{(2n+2+v)}$.

If observer matrix N and system matrix A have distinct eigenvalues, then $TA - NT - LC = 0$ will have a solution for T. Also, if the composite system matrix A_C has all the eigenvalues in the plane's left half, the system will be uniformly ultimate bounded. Hence, the observer matrix N is chosen such that the composite system matrix has stable eigenvalues.

By using the theorem 3 and the condition of stable eigenvalues for the composite system matrix A_C in (24), the observer matrices can be obtained. Hence, the control input $u(t)$ can be further rewritten using the observer output obtained in (17b).

$$\begin{aligned} u(t) = \begin{bmatrix}1 & 0\end{bmatrix}\hat{g}(t) - (\Gamma B)^{-1}[k_2 \text{sgn}(\begin{bmatrix}0 & 1\end{bmatrix}\hat{g}(t) - \\ \Gamma x_d(t))] + (\Gamma B)^{-1}\left[\Gamma \dot{x}_d(t) + k_1 \Gamma x_d(t)\right] \end{aligned} \tag{25}$$

The state space model in (2) is of $(2n+2)$ order, designing the control for a large value of n results in a complex and difficult-to-implement control law. Therefore, in this paper, the proposed control in (25) is designed for (i) *two* $(n = 2)$ vibratory modes and (ii) *three*$(n = 3)$ vibratory modes.

The proposed control law in (25) designed for the systems having *two* and *three* assumed modes are separately tested on the larger-order system $(n > 3)$, i.e. considering the dynamic model with more number of modes.

IV. Simulation and Results

This section includes the numerical simulations and results that demonstrate the effectiveness of the presented control approach for the SLFM. This paper simulates the developed control law on the system having first five vibrational modes. The physical parameter specifications of the SLFM are given in Table I.

TABLE I
PHYSICAL PARAMETERS OF SINGLE-LINK FLEXIBLE MANIPULATOR

Parameters	Value	Parameters	Value	Parameters	Value
ρ	0.5	ω_2	55.88	$\phi_4'(0)$	3.8529
1	1	ω_3	101.36	$\phi_5'(0)$	2.4422
m_p	0	ω_4	177.66	$\phi_1(l)$	0.3214
J_p	0	ω_5	286.84	$\phi_2(l)$	-1.6407
J_0	0.002	$\phi_1'(0)$	32.8184	$\phi_3(l)$	2.4586
EI	1	$\phi_2'(0)$	10.4096	$\phi_4(l)$	-2.3010
ω_1	20.53	$\phi_3'(0)$	6.1588	$\phi_5(l)$	2.1568
ζ	0.05				

The observer designed for the state space model in (2) and (3) by considering $n = 2$ and $n = 3$ are of order *two*.

The observer matrices are chosen such that the composite matrix A_C has all its eigenvalues in the left-half plane, which guarantees the stability of the composite system.

The control input (25) designed for $n = 2$ and $n = 3$ is applied to the model having five vibration modes. The simulation is being performed for both regulation and tracking problems.

A. Regulation Problem

The reference values for the angle are chosen as:

$$\theta_d = \frac{\pi}{4} \quad \text{rad.}$$

Figure 2 shows the convergence of tip position $\theta_t(t)$ to the desired position θ_d with vibrations suppressed for both the designed controllers. The figure also shows the plots for clamped joint angle $\theta_c(t)$. The output response for the controller designed considering $n = 3$ is more smooth and transient free as compared to the controller designed considering $n = 2$. But this comes at the cost of increased computational complexity. Furthermore, the convergence time is quite similar for both the controllers.

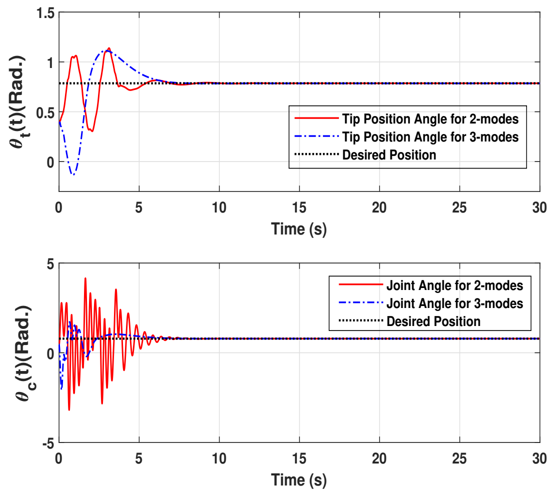

Fig. 2. Output Plot for Regulation Problem

The plot for the sliding variable versus time for both two and three oscillatory modes based controllers is shown in figure 3. The figure indicates that the sliding variable for the controllers converges to zero in finite time.

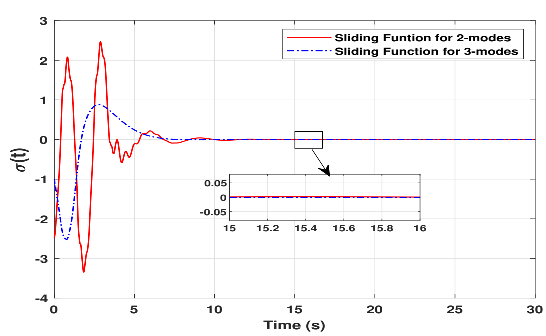

Fig. 3. Sliding Function Plot for Regulation Problem

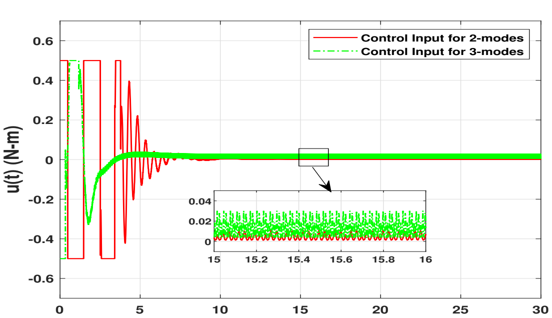

Fig. 4. Control Input for Regulation Problem

Figure 4 shows the actuator torque applied to the manipulator. The actuator torque applied is well within the bound of $\pm 0.5N - m$, i.e., the applied control input is bounded. The response of the controller designed using three vibratory modes have significant amount of chattering compared to the controller designed with two oscillatory modes.

B. Tracking Problem

The desired trajectory for the position of a manipulator is chosen as:

$$\theta_d(t) = e^{-0.5t}\sin(t) + (1 - e^{-0.5t}) \text{ rad.}$$

Figure 5 shows the convergence of tip position $\theta_t(t)$ to the desired trajectory $\theta_d(t)$ smoothly for both the designed controllers. The figure also shows the trajectory for clamped joint angle $\theta_c(t)$. The tip position angle converges to the desired trajectory faster with the controller designed with two modes of vibration. But the output transient response is better for the controller designed using three modes of vibration.

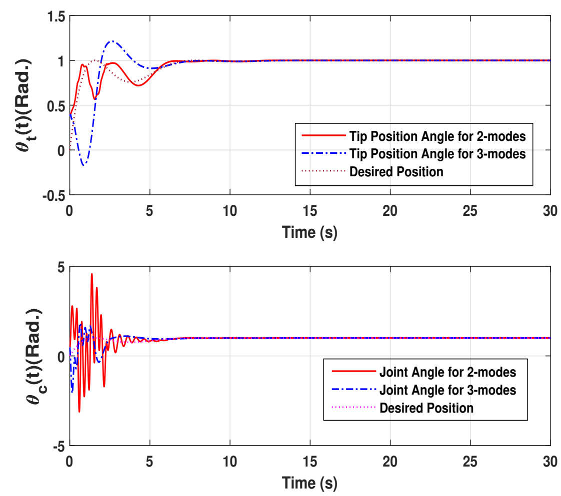

Fig. 5. Output Plot for Tracking Problem

The plot of the sliding variable vs time is shown in figure 6 for both the controllers. It is evident from the figure that the sliding variable converges to zero in a finite amount of time.

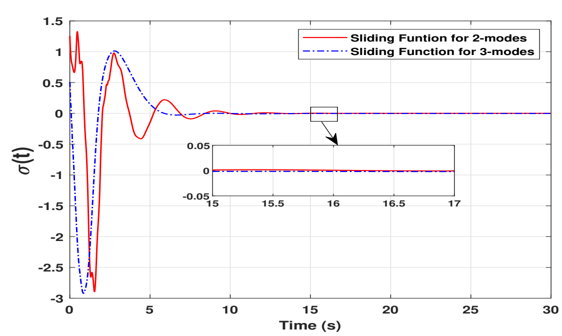

Fig. 6. Sliding Function Plot for Tracking Problem

The plot of actuator torque applied to the manipulator with respect to time is shown in figure 7. The figure shows that the actuator torque has a range of $\pm 0.5\ N-m$. The control input response for the controller designed considering $n = 3$ have quite high chattering component compared to the controller designed considering $n = 2$.

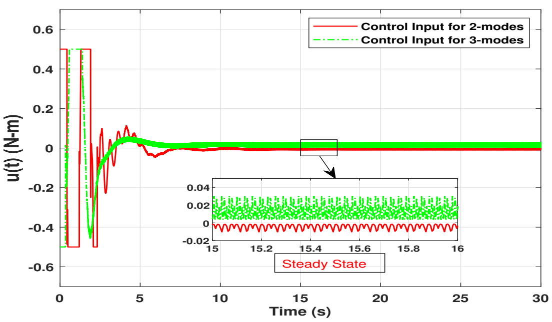

Fig. 7. Control Input for Tracking Problem

V. Conclusion

The paper proposes a sliding mode approach based on a functional observer for controlling the position of the SLFM system. In this paper, the controller is designed by considering two ($n = 2$) and three ($n = 3$) vibratory modes in the system model and the designed control is validated using numerical simulation for the dynamic model with the first five vibration modes considered in the system modelling. The simulation results for both the controllers are presented. The response of the system for the controller designed using three oscillatory modes is more smooth and transient free compared to the controller designed using two oscillatory modes, but that comes with increased computational cost.

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