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Time-optimal real-time energy management strategies for hybrid battery packs in electric racing cars

Stefano Radrizzani, Giorgio Riva, Giulio Panzani, Matteo Corno, Sergio M. Savaresi

Abstract— **The increasing interest in hybridization and electrification of racing cars is pushing towards the design of dedicated energy storage systems. Among them, Hybrid Battery Packs (HBPs) represent an interesting solution, especially in racing, due to the joint presence of high-power and high-energy requirements, needed to guarantee the desired race mileage while maximizing performance. To realize a HBP, a real-time control law, named Energy Management Strategy (EMS), is pivotal to properly split the power while satisfying the driver's request. In this paper, we investigate whether the control laws that emerged for traditional vehicles can be employed in the racing scenario. Considering a Formula E case study, the well-known Equivalent Consumption Minimization Strategy (ECMS) and a classical filter-based approach are compared to the race time-optimal implicit power distribution. Analyses firstly evaluate the capability of each EMS in matching the implicit solution, showing the superior performance of ECMS. Then, explicitly including each real-time EMS in the time-optimal problem, the race times are re-optimized to evaluate the actual loss of performance. Finally, we highlight how the combination of each EMS with a dedicated battery sizing strategy can influence the overall performance, closing the gap among the different power split solutions.**

I. Introduction

The current hybridization and electrification trend in racing vehicles calls for an optimal battery design to push the vehicle performance to its limit of handling [1]. In this scenario, Hybrid Energy Storage Systems (HESSs) could play an important role, starting from Hybrid Battery Packs (HBPs), which can combine the advantages of high-power and high-energy cells technologies [2]. Indeed, both aspects are required in order to guarantee the mileage of a race and to maximize performance at the same time.

The maximization of the performance in racing vehicles is widely addressed in the literature as a global time-optimal problem, knowing the entire lap or race trajectory. This approach has been employed both for Formula 1 vehicles [3] and for electric racing cars [4], with either a focus on Formula E [1], [2] or endurance races [5]. The outcomes of these approaches are multiple: from the optimal size of the powertrain and energy storage system components to the speed profile. In addition, for both hybrid powertrains and energy storage systems, implicit Energy Management Strategies (EMSs) are obtained, which optimally distribute power among the different sources. Despite optimality, their implicit formulation entails the need to develop explicit and implementable logic, able to control the power flow in real-time on the vehicle, to satisfy the driver's request. In this scenario, different EMSs born for standard Hybrid Electric Vehicles (HEVs) and HESSs can be exploited.

In the literature, EMSs – for either HEVs or HESSs – are classified according to their level of optimality, expressed as energy saving performance, and they range from heuristic up to global optimal solutions. Starting from global optimal approaches, dynamic programming or Pontryagin's minimum principle are employed to compute the implicit solution that minimizes the energy consumption over a known driving-cycle. Having the complete driving-cycle as input, these solutions cannot be implemented in real-time applications; hence, they are generally used to analyze the best energy saving performance or to optimally design the powertrain size [6]. To the other extreme, there are heuristic approaches, which offer an easy real-time implementation on the vehicle control unit, thanks to their simplified nature, often based on explicit rules. In between, there is a broad range of policies, which still solve an optimization problem: either over a finite prediction horizon, e.g., in Model Predictive Control (MPC) [7]; or in the current operating point, as for the widespread Equivalent Consumption Minimization Strategy (ECMS) [8].

Concerning the racing scenario of electric cars with a HBP, the global time-optimal problem has been already deeply discussed in [2], computing the optimal speed profile, the implicit EMS, and designing the optimal battery to minimize the race time. The primary aim in racing is the pure performance, i.e., the minimization of the race time, and, differently from standard vehicles, the energy consumption is not directly taken into consideration in the cost function. However, it is intuitive that the two objectives are strictly related. Indeed, given that performance maximization asks for light and small battery packs capable of delivering enough power and energy, efficient management of both batteries is crucial, entailing that the obtained implicit EMS must be optimal also in terms of energy saving.

In this work, we take a step forward with respect to the Minimum Race Time (MRT) and battery sizing problem discussed in [2]. Indeed, the optimization in [2] returns an implicit EMS control logic to optimally handle the power split between the two batteries, calling for the development of explicit implementable logic. Given this motivation, we aim at evaluating the performance loss induced by an explicit EMS with respect to this implicit global time-optimal policy. As the main contribution, we develop a novel formulation of the global MRT problem in [2], including an explicit EMS through suitable additional constraints. Moreover, we address two standard approaches for traditional vehicles with a HESS: the ECMS and a filter-based heuristic solution

All the authors are with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Via Ponzio 34/5, 20133 Milan, Italy stefano.radrizzani@polimi.it

employing a Complementary Filter (CF) [9], which are representative respectively of optimization-based and heuristic families.

Considering the Formula E case study, we investigate whether the selected EMSs can be suitable also for the racing scenario, following a three-step analysis. Firstly, we compare their capabilities to mimic the power split of the implicit global optimal solution that minimizes the race time. Then, exploiting the proposed explicit-EMS formulation of the MRT problem, we evaluate the impact of each EMS in terms of race time. Finally, we address the sizing problem to understand if a proper size for each EMS can improve the performance to close the gap with the implicit MRT solution.

The paper continues with the review of ECMS and CF approaches in Section II, followed, in Section III, by the derivation of a modified MRT to take them into account. Finally, the analyses are carried out on the Gen3 Formula E case study in Section IV. The paper ends with some concluding remarks.

II. Energy Management Strategies for HESSs

In this section, we recall the ECMS and the CF before their application in Section III.

Equivalent consumption minimization strategy. ECMS is originally formulated for standard HEVs [8], where it minimizes at any time instant the equivalent fuel consumption, expressed as the sum of the fuel power P_{f} and battery power P_{b} consumption, weighted by the equivalence factor λ:

$$\min P_{\mathrm{f}}(t) + \lambda(t) P_{\mathrm{b}}(t) \quad \forall t, \tag{1}$$

subject to the powertrain operating limits. The equivalence factor plays multiple roles in the optimization problem: 1) *optimality*: it affects the energy saving performance of the ECMS solution, which can be very close to the global optimal one for a proper tuning [10]; 2) *physical interpretation*: in HEVs, it represents the efficiency encountered in recharging the battery through the engine or regenerative braking [11]; 3) *practical interpretation*: it can be used as a tuning parameter to mimic the global optimal solution or for additional purposes, like the battery charge level control [12]. In the case of HBPs, the equivalent consumption in the ECMS is the weighted sum of the power consumption $P_{\mathrm{bc},i}$ of each battery $i = 1, 2$:

$$\min P_{\mathrm{bc},1}(t) + \lambda(t) P_{\mathrm{bc},2}(t) \quad \forall t, \tag{2}$$

while satisfying the driver's power request P_{b} and subject to the battery operating limits. In the context of HESSs the equivalence factor is often used as a tuning parameter, e.g., [13], [14]. It is interesting to highlight that, a constant value of the equivalence factor can be optimal under some assumptions, and even when these conditions are not satisfied, a constant value has been shown to return performance anyhow close to the optimal solution in the HEVs scenario [10].

Complementary filter. The EMS based on a complementary filter [9] is expressed mathematically in Laplace domain, using the Laplace operator p:

$$P_{\mathrm{b},1} = H(p) P_{\mathrm{b}}, \quad P_{\mathrm{b},2} = P_{\mathrm{b}} - P_{\mathrm{b},1}, \tag{3}$$

where $P_{\mathrm{b},i}$ is the power at the terminals of the i-th battery, $H(p)$ is the filter transfer function used to split the driver's power request P_{b}, which is equal to the sum of $P_{\mathrm{b},1}$ and $P_{\mathrm{b},2}$ by construction. Differently to the ECMS, this heuristic solution is employed just for HESSs, because it has been designed to mimic the global optimal solution when high power energy storages are combined with high energy ones [15].

III. The MRT problem with explicit EMS

In this section, we discuss the structure of the MRT problem. Firstly, we summarize the baseline implicit-EMS formulation in [2], where no explicit EMS is enforced. Then, we show how it can be built up to derive an explicit-EMS formulation, including a specific EMS in the optimization problem. Finally, we detail how the CF and the ECMS can be actually implemented.

A. The implicit-EMS MRT problem: recap

The baseline implicit-EMS MRT problem in [2] represents the extension towards HESSs of the work in [1], where a space-reformulation of vehicle and battery dynamics is employed to optimize the overall race time along a circuit. Reminding that a rigorous and complete formulation is available in [2], we report here a lighter version to enhance readability for the upcoming discussion:

$$\begin{aligned} \min \quad & \text{race time} \\ \text{s.t.} \quad & \text{vehicle dynamics \& friction limits} \\ & \text{batteries dynamics \& limits} \\ & \text{battery pack-vehicle coupling.} \end{aligned} \tag{4}$$

In more detail, batteries and vehicle dynamics are modeled respectively through a static Equivalent Circuit Model (ECM) and a point-mass model moving on a fixed raceline, where the friction limit allows for linking and constraining longitudinal and lateral dynamics. The coupling between the battery pack and vehicle is highly simplified through a constant average efficiency, which includes the presence of electric motors and power-electronics devices. Finally, the batteries limits prevent State-of-Charge (SoC), current, and voltage from overcoming safety boundaries, and constrain the total power at the terminals to comply with possible limitations enforced by racing regulations. We stick with the power electronics configuration employed in [2], characterized by the presence of a single ideal dc/dc converter coupled with one of the two batteries. Albeit the dc/dc non-idealities may slightly influence the results, this choice is instrumental to solely evaluate the effect of the presence of an explicit EMS with respect to the implicit one.

B. The explicit-EMS MRT problem: general formulation

From the perspective of the optimization problem, an EMS can be simply seen as an additional constraint acting on the power delivered by the two batteries as a function of the total power request and possibly other variables, like the SoC. We recall that all the employed symbols are compliant with the ECM battery model described in [2].

To introduce the EMS in the optimization problem, only one additional constraint is required, e.g., on battery 1, because, as in [2], we force the sum of the two battery powers to be the total one, i.e., $P_{\mathrm{b}} = P_{\mathrm{b},1} + P_{\mathrm{b},2}$. As a consequence, the optimization problem in (4) is built up to include an explicit-EMS by adding the generic constraint

$$P_{\mathrm{b},1} = f_{\mathrm{EMS}}\left(P_{\mathrm{b}}, \dots\right), \tag{5}$$

where $f_{\mathrm{EMS}}\left(P_{\mathrm{b}}, \dots\right)$ is the explicit function describing the EMS policy under study. We remark that, in the case of a dynamic EMS, further additions to problem (4) may be required, as exemplified by the CF example.

Complementary Filter implementation: As detailed in Section II, the CF can be expressed via the equation

$$P_{\mathrm{b},1} = H(p) P_{\mathrm{b}}. \tag{6}$$

In our formulation, we consider a first-order low-pass filter $H(p, f_{\mathrm{lp}}) = \frac{1}{1+\frac{p}{2\pi f_{\mathrm{lp}}}}$, parameterized by a fixed filter frequency f_{lp}. Given the dynamic nature of the low-pass filter, an additional state variable must be added to the dynamical model employed in the optimization. We define $P_{\mathrm{b,lp}}$ as the low-pass filtered version of the total battery power P_{b}, whose dynamic equation in the space domain is obtained from a state-space representation of (6):

$$\frac{\mathrm{d}P_{\mathrm{b,lp}}}{\mathrm{d}s} = \frac{1}{v}\left(-2\pi f_{\mathrm{lp}} P_{\mathrm{b,lp}} + 2\pi f_{\mathrm{lp}} P_{\mathrm{b}}\right) \tag{7}$$

Finally, the generic EMS constraint in (5) can be simply expressed as:

$$P_{\mathrm{b},1} = P_{\mathrm{b,lp}}, \tag{8}$$

recalling that the battery operating limits are already included in (4).

ECMS implementation: starting from problem (2) in Section II, the complete ECMS, including the static Equivalent Circuit Model (ECM), is formulated as:

$$\begin{aligned}
\min_{P_{\mathrm{b},1}, P_{\mathrm{b},2}} \; & P_{\mathrm{oc},1} + \lambda P_{\mathrm{oc},2} \\
\text{s.t. } & P_{\mathrm{oc},i} = V_{\mathrm{oc},i}(\mathrm{SoC}_i) I_{\mathrm{b},i}, && i = 1,2 \\
& V_{\mathrm{b},i} = V_{\mathrm{oc},i}(\mathrm{SoC}_i) - R_{\mathrm{b},i} I_{\mathrm{b},i}, && i = 1,2 \\
& P_{\mathrm{b},i} = V_{\mathrm{b},i} I_{\mathrm{b},i}, && i = 1,2 \\
& V_{\mathrm{min},i} \le V_{\mathrm{b},i} \le V_{\mathrm{max},i}, && i = 1,2 \\
& I_{\mathrm{min},i} \le I_{\mathrm{b},i} \le I_{\mathrm{max},i}, && i = 1,2 \\
& P_{\mathrm{b}} = P_{\mathrm{b},1} + P_{\mathrm{b},2},
\end{aligned} \tag{9}$$

where the battery consumption $P_{\mathrm{bc},i}$ in (2) is modelled through the open-circuit power $P_{\mathrm{oc},i}$. Problem (9) cannot be directly expressed through an explicit control policy, while, the explicit-EMS MRT problem, implemented in CasADi [16] via an interior-point algorithm (IPOPT), requires the knowledge of analytic functions to compute gradients. As a consequence, neither the optimization problem in (9) nor its approximation through a look-up table can be employed, entailing the need for an analytic approximation. To this purpose, we employ Feed-Forward Neural Networks (FFNN) [17] due to their well-known approximation capabilities. Moreover, the risk of over-fitting is mitigated by the possibility of generating as many samples, i.e., solutions, as needed from (9). To summarize, given each battery configuration, a network described by function $f_{\mathrm{ffnn}}\left(P_{\mathrm{b}}, \mathrm{SoC}_1, \mathrm{SoC}_2, \lambda\right)$ is trained using (9) as data generator, and the generic EMS constraint in (5) becomes:

$$P_{\mathrm{b},1} = f_{\mathrm{ffnn}}\left(P_{\mathrm{b}}, \mathrm{SoC}_1, \mathrm{SoC}_2, \lambda\right). \tag{10}$$

An example of the selection of the FFNN structure, together with its approximation performance, is provided in the case study of Section IV.

IV. Case study: Gen3 Formula E

The case study in this section considers the Gen3 Formula E and the 23 laps Rome 2021 ePrix. This is the same scenario studied in [2], which is our starting point for the computation of the implicit-EMS solution, used as benchmark. To simplify, we consider only one technology for the High-Power Battery (HPB) and one for the High-Energy Battery (HEB). Specifically, we select the two technologies corresponding to the optimal solution computed in [2], considering cells illustrative of the current availability on the market. Indeed, the HEB is representative of the high-energy cells by Kokam, while the HPB represents the ultra high-power technology developed by Saft. Their main parameters are summarized in TABLE I, while all the vehicle parameters can be found in [2]. In this paper, we addressed the problem considering the power limit of 350 kW in traction, given by the current rules of the Gen3 Formula E, and discrete values for the number of cells in parallel $N_{\mathrm{p},1}$ and $N_{\mathrm{p},2}$, given that a real battery pack is composed of an integer number of cells. We recall that, as in [2], the size-dependent mass of the battery pack is added to the vehicle one.

The optimal battery size concerning the implicit-EMS MRT problem (4), i.e., the global optimal solution, is $N_{\mathrm{p},1} = 17$ and $N_{\mathrm{p},2} = 1$. The corresponding outcome is highlighted in Fig. 1 in terms of SoCs and powers behaviors. It is visible how the HEB battery has a full and constant, on average, depletion of the SoC, while the SoC of HPB cycles around a constant value, except for the very beginning and the end of the race. Concerning the power of the two batteries, it is

TABLE I
HEB and HPB cells parameters

	HEB	**HPB**	units
energy density	257	60	Wh/kg
weight	42	180	g
capacity	3000	3000	mAh
resistance	4.4	0.8	mΩ
max c-rate in traction	10	228	C
max c-rate in regeneration	1.5	228	C
voltage range	(2.5–4.2)		V

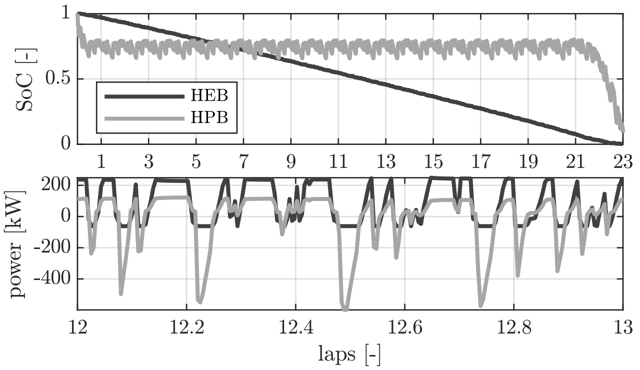

Fig. 1. The implicit-EMS Minimum Race Time: SoCs and power profiles for the global optimal battery size ($N_{\mathrm{p},1} = 17$, $N_{\mathrm{p},2} = 1$).

evident how both play a significant role in traction, while the HPB covers the high peaks during braking phases, thanks to its high recharge limit (see also Tab. I). Starting from this result, the first analysis consists of the tuning of both the ECMS and the CF to emulate the behavior of the optimal power distribution.

A. *EMS tuning on the implicit-EMS optimal profile*

To mimic how the battery power has been split by the implicit-EMS, as in Fig. 1, the two EMSs have been tuned in order to minimize their mismatch with respect to such a power distribution. Given that the total power is constrained to be equal to the one obtained by the MRT problem, a mismatch on $P_{\mathrm{b},1}$ equally reflects on $P_{\mathrm{b},2}$, so that the resulting optimization problem can be expressed only in terms of the HEB mismatch:

$$\begin{aligned} &\min_{\theta(s)} \sum_{s=0}^{s_{\mathrm{race}}} \left(P_{\mathrm{b},1}(s,\theta) - P_{\mathrm{b},1}^{\mathrm{free}}(s)\right)^2 \\ &\text{s.t.} \quad P_{\mathrm{b},1}(s,\theta) + P_{\mathrm{b},2}(s,\theta) = P_{\mathrm{b}}^{\mathrm{free}}(s) \ \forall s, \end{aligned} \tag{11}$$

where s is the discretization index along the race, and θ represents the tuning parameters, i.e., the frequency f_{lp} of the CF along with its initial condition $P_{\mathrm{b,lp}}(0)$, and, for the ECMS, a varying equivalence factor $\lambda(s)$ in (2).

Starting from the ECMS, Fig. 2 shows that the distribution of the equivalence factor $\lambda(s)$ has mean value $\bar{\lambda} = 0.979$ with a very limited variance, suggesting that a constant value could be able to mimic the optimal solution. We highlight that this result is in agreement with those obtained for HEVs, e.g., see [10]. Indeed, forcing $\lambda(s) = \lambda^* \ \forall s$ in (11), the optimization problem returns a value very close to the mean one: $\lambda^* = 0.981 \approx \bar{\lambda}$. It is also interesting to notice that this value is very close to 1, meaning that both batteries are equally weighted in the ECMS. Looking at Fig. 1, the absence of power exchange between the two batteries motivates this last result, since the equivalence factor represents, in classical HEVs, the efficiency chain in recharging one energy source (battery) through the other one (fuel).

The same procedure has been carried out also for the CF, even if there is no a-priori expectation of high mimic

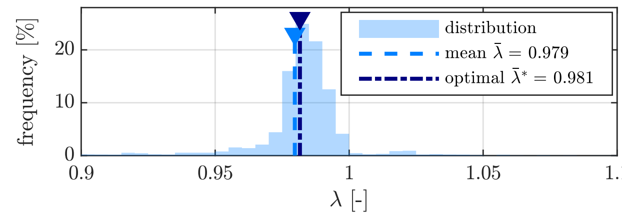

Fig. 2. Tuning on implicit-EMS: distribution of ECMS $\lambda(t)$.

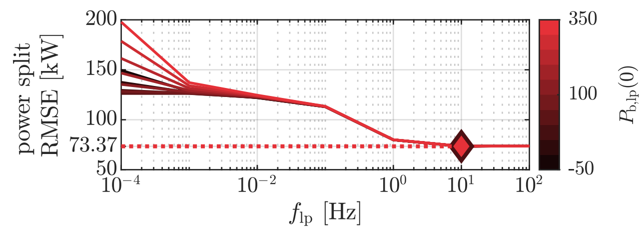

Fig. 3. Tuning on implicit-EMS: optimization of complementary frequency f_{lp} and initial condition $P_{\mathrm{b,lp}}(0)$.

capability. Indeed, even just by looking at the power profiles of Fig. 1, we can grasp that no frequency separation between the two batteries is present. Looking at Fig. 3, the optimal CF frequency results to be $f_{\mathrm{lp}}^* = 10$ Hz, independently of the initial condition, because the Root Mean Square Error (RMSE) of the power split mismatch is sensitive to it just for extremely low-pass filtering action.

Comparing the two approaches in Fig. 4, we can see how the distribution of the error is significantly closer to 0 for the ECMS, as also numerically evaluated by the RMSE of 16.5 kW for the ECMS and 73.37 kW for the CF. The good matching of the ECMS with respect to the CF can be visualized also in space domain in Fig. 5. Given the poor performance of the CF, the next discussion aims at evaluating the performance in terms of race time, when both the optimal speed profile and the total battery power request explicitly consider the EMS control policy.

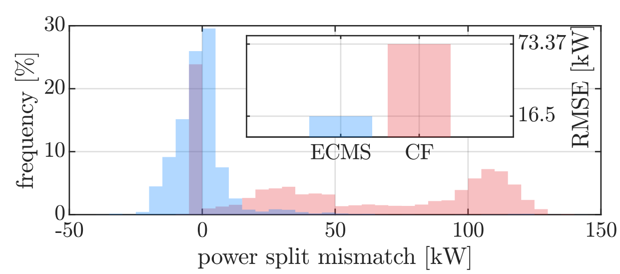

Fig. 4. Tuning on implicit-EMS: distribution of the power split mismatch, and ranking between ECMS and CF.

B. *The explicit-EMS MRT: fixed battery size*

Considering the optimal size of the battery pack computed with the implicit-EMS, the race time is recomputed forcing the two selected EMSs via the explicit-EMS MRT problem described in Section III-B. Starting from the ECMS, in

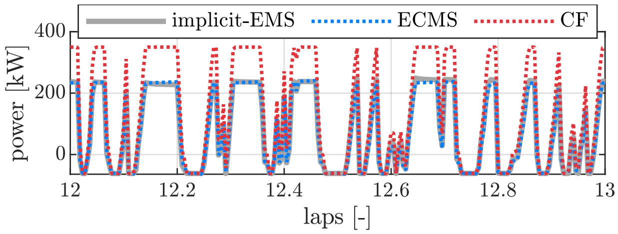

Fig. 5. Tuning on implicit-EMS: HEB power output comparison.

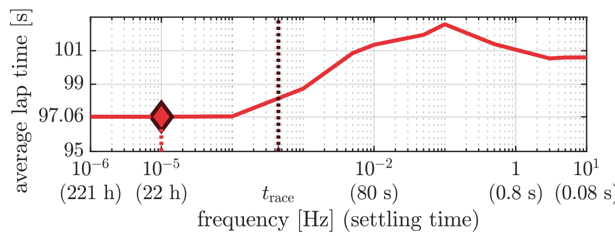

Fig. 6. The explicit-EMS MRT: CF frequency sensitivity for the global optimal battery size ($N_{\mathrm{p},1} = 17$ and $N_{\mathrm{p},2} = 1$).

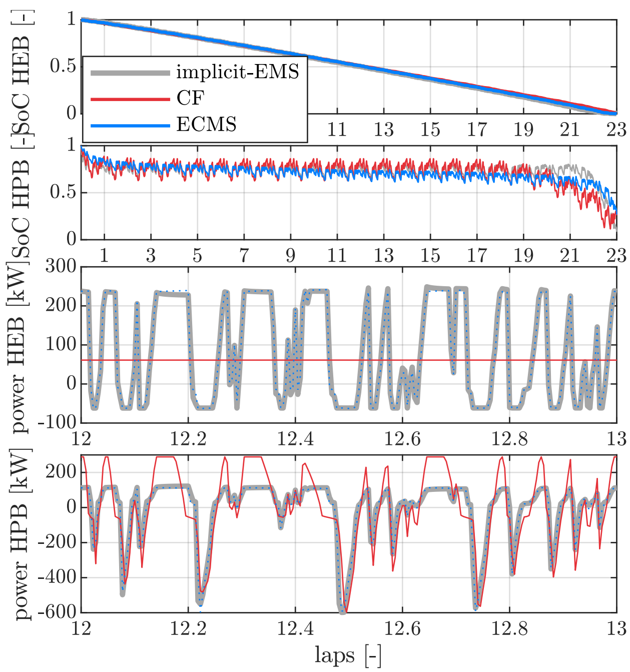

Fig. 7. The explicit-EMS MRT ($N_{\mathrm{p},1} = 17$, $N_{\mathrm{p},2} = 1$): SoCs and powers comparison of ECMS and CF respect to the implicit-EMS solution.

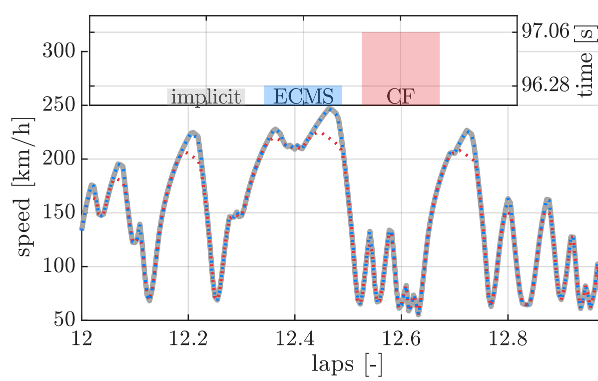

Fig. 8. The explicit-EMS MRT ($N_{\mathrm{p},1} = 17$, $N_{\mathrm{p},2} = 1$): speed profiles comparison and lap time ranking between implicit-EMS, ECMS, and CF.

this case study, we exploit a two-layer FFNN with 20 neurons each, implementing respectively sigmoid and relu activation functions, the latter useful to model saturations. The approximation performance of the employed network has been tested a-posteriori, compared with the true ECMS (9), on the achieved optimal profile, resulting in an average error of about -2.81 kW with a standard deviation of 7.26 kW. Moreover, driven by the previous results, we stuck with the assumption of a constant value of λ to be optimized. As expected from the capability of the ECMS in emulating the optimal solution, the constant value of the equivalence factor that minimizes the race time does not show significant changes, indeed, $\lambda^{\text{opt}} = 0.978 \approx \bar{\lambda} \approx \lambda^*$. On the contrary, the optimal tuning of the CF to minimize the race time reveals a completely different scenario. Indeed, Fig. 6 shows that the best CF tuning coincides with extremely low-pass filtering actions. Moreover, there is a significant change in the performance when the settling time of the low-pass filter is longer than the race itself. This means that practically the CF makes the HEB apply a constant power value. Looking at Fig. 7, we can see that all the solutions have a similar trend in terms of SoCs: the HEB is fully depleted during the race and the HPB acts as a power buffer. Considering the HEB and HPB powers, ECMS is very close to the implicit-EMS solution, while the CF has a higher power amplitude in the HPB, due to the constant power applied by the HPB. To summarize, the performance of each policy is shown in Fig. 8: ECMS has a speed profile and a race time much closer to the optimal one, while CF exhibits a slower race time, motivated by the lower top speed and the increased coasting. However, if on the one hand the CF could be considered a useless solution to mimic the implicit optimal one, on the other hand, this second analysis shows that it can achieve competitive performance if properly tuned. Given the very different behavior of the CF, it is natural to investigate if the race time can be further reduced with proper battery sizing.

C. The explicit-EMS MRT: battery size optimization

This last analysis extends the previous ones approaching the general sizing problem with an explicit EMS. We highlight that, for the ECMS, we employed the same FFNN architecture of the previous analysis, re-optimized for each dedicated battery size. The results are shown in Fig. 9, where the implicit-EMS surface relates to the results discussed in the introduction of Section IV: the ECMS again reveals to be very close in terms of race time to the optimal sizing for any couple of number of cells in parallel in the HEB and HPB. Given the small mismatch between the ECMS and the implicit-EMS experienced in previous analyses, the optimal sizing configuration does not change either. On the other side, considering the CF, we have shown in the previous section that, when optimized, it outputs a very different power distribution with respect to the other two approaches. As a consequence, the optimal size changes when the CF is employed, as highlighted in Fig. 9. Looking at the whole surface, we appreciate that all the solutions become closer, in terms of race time, when the battery pack size increases. Intuitively, this pattern is motivated by the reduced impact of the different levels of efficiency of each EMS on the vehicle performance, when applied to higher capacity battery packs. Finally, results revealed that the optimal equivalence factor and the optimal frequency of the CF are not that sensitive to the size of the battery, at least for the considered sizes. The final comparison in Fig. 10 shows that when the sizing strategy considers the EMSs, the optimal size changes, almost closing the gap between the different solutions. Indeed, with respect to Fig. 8, it is possible to see that the speed profiles and the race time are closer when a dedicated sizing for each EMS is considered.

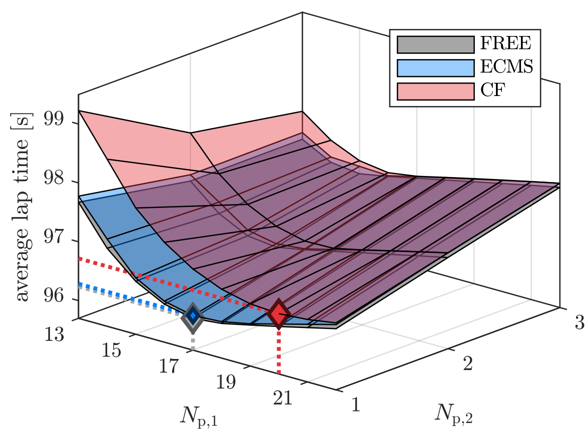

Fig. 9. Sizing surfaces comparison among the implicit-EMS solution and explicit-EMS MRT for ECMS and CF.

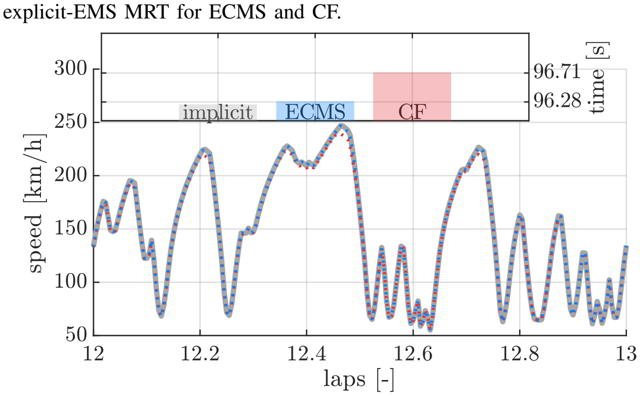

Fig. 10. The explicit-EMS MRT with optimal sizing: speed profiles comparison and lap time ranking between implicit-EMS, ECMS, and CF.

V. Conclusions

In this work, the impact on the race time of real-time EMSs for electric racing cars with a HBP has been analyzed. Towards this aim, the original implicit-EMS MRT problem has been properly modified to include explicit EMSs taken from the available literature on standard cars.

Results shown on the Gen3 Formula E case study are two-fold: i) the ECMS turns out to be an accurate approximation of the global optimal power distribution, confirming its optimality properties; ii) the CF strategy represents a viable heuristic solution despite its inadequacy in approximating the global optimum. Moreover, we showed that its performance gap with respect to ECMS can be further reduced if a dedicated sizing is selected.

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