2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Understanding Seated Stability in Spinal Cord Injury using an asymmetric two arms Biomechanical observer

Thierry-Marie Guerra, Hajer Srihi, Antoine Dequidt, Anh-Tu Nguyen, Philippe Pudlo

***Abstract* — Various pathologies and physical impairments diminish the capacity to maintain a seated balance, with spinal cord injury serving as a clinical example. Individuals with this condition often lose control of muscles below the injury level and commonly rely on a wheelchair for mobility. The impact of such injuries on seated postural control necessitates the development of new stabilization strategies in response to disturbances. These strategies differ significantly from those used by asymptomatic individuals. Particularly, they rely on upper limb movements as the primary means of control, given the absence of control from the trunk. Reconstructing the produced active joint torques at the shoulder and arm levels or the passive torque at the lumbo-sacral level is crucial to understand compensatory strategies and developing innovative monitoring techniques in rehabilitation exercises. The methodology starts from a nonlinear model with 5 Degrees of Freedom called "Trunk-2-Arms" (T2A) and proposes an observer based on quasi-LPV and LMI problem synthesis. The design of the observer uses a cascade of 3 models (trunk and the 2 arms) in order to reduce the design complexity. Therefore, local PI-observers are derived that allows to estimate the state and the human joint torques. The global estimation error convergence of the cascaded observer scheme is guaranteed using a separation principle and the Lyapunov theory. The methodology is validated through simulations and using real clinical data collected on 26 SCI people.**

I. INTRODUCTION

Individuals experiencing a spinal cord injury (SCI) often exhibit a substantial reduction or complete absence of muscular activity in the abdominal and trunk areas, sometimes coupled with partial or complete loss of sensation below the level of injury. Consequently, this leads to diminished postural control and significantly increases the risk of falling [1] [2]. The consequences of such injuries are substantial, impacting sensorimotor, cardiovascular, respiratory, and digestive functions. The impact of the injury on the vertebral stabilizing system generates new stabilization strategies, which are different from the asymptomatic strategies of healthy subjects. These strategies which compensate for the absence of voluntary control are essentially based on the movements of the upper limbs.

Understanding these strategies as well as quantifying the resulting joint forces are necessary in order to assess the impact of the injury and to propose news methods facilitating the control of seated stability. One of the challenges is to estimate the variables that generate these efforts, the torques that are not measured and *not measurable*. The paper follows an alternative to classic inverse dynamics [3-5]. This alternative starts from a biomechanical model and use nonlinear PI-observer design. The framework of this design uses the so-called quasi-LPV (or Takagi-Sugeno model) formalism via exact polytopic descriptions of nonlinear systems in a compact set of the state and LMI constraints problems [6]. In order to reduce the complexity and to get less conservative result a descriptor form is considered, that is also very well-suited to mechanical models [7].

In order to assess the methodology, an experimental campaign has been carried out in Montreal [8] on 26 SCI people delivering numerous workable data. The first trials to understand the SCI seated position were using reduced models: the "Head-2-Arms-Trunk" (H2AT) with 2 Degrees-of-Freedom (DoF) [3] and the "Seated-3-segments" (S3S) model involving 3 DoF [5] [8]. The S3S model was validated with real data, however, it considers (for simplification issues) symmetric arms behavior for the SCI. Indeed, a large part of the experimental data (when the arms behave asymmetrically) could not be exploited. This work answers to this issue by introducing an asymmetric 2 arms model called "Trunk-2-Arms" (T2A).

There are several challenges to go from S3S model to T2A. A first challenge is related to complexity; an observer for the S3S model [5] [8] produces an LMI constraint problem that is already close to the limitations of current solvers with a standard computer (see Remark 3). A second challenge concerns genericity, the methodology must remain valid even with additional degrees of freedom. The last one is to carry out the synthesis of the observer using the nonlinear model without simplification (linearization, neglecting terms, small angles assumption, etc.). It also has to solve the problem of unmeasured scheduling variable without using neither norm bounds [9] nor using Lipschitz-like conditions [10].

Part II presents the notations, part III the biomechanical model T2A and the problem formulation. To deal with the numerical complexity of the observer design, we follow the idea of cascaded nonlinear observer [11] [5]. To this end, part IV presents the T2A model rewritten in the form of a cascaded model with three subsystems: trunk, left and right arms. A PI-observer [12] is developed for each system and using Lyapunov stability theory, the convergence of the estimation

This work was supported in part by the RITMEA research program funded by the European Regional Development Fund, the State, and the Hauts-de-France Region. This work was also sponsored by the Ministry of Higher Education and Research and the French National Center for Scientific Research. It is also supported by the Research National Agency under the project HM-Science ANR-21-CE48-0021.The authors are with the laboratory LAMIH UMR CNRS 8201, Université Polytechnique Hauts-de-France, 59313 Valenciennes, France. H. Srihi, A. Dequidt, A.-T. Nguyen and P. Pudlo are also with the INSA Hauts-de-France, Valenciennes, France. Corresponding author: guerra@ uphf.fr.

error is obtained. Finally, part V gives illustrative results especially with clinical data and part VI concludes the work.

II. NOTATIONS AND MATERIALS

The following notations are considered in the paper. The symbol $(\cdot)$ in an expression might be used either for brevity or to signify that the dependence of a variable will be defined later. $I_n \in \mathbb{R}^{n\times n}$ stands for the identity matrix. Subscripts and exponents h and v stands for polytopic representation of a variable, i.e., $E_v = \sum_{i=1}^{r_v} v_i(\cdot) E_i$, $A_h^v = \sum_{i=1}^{r}\sum_{j=1}^{r_v} h_i(\cdot) v_j(\cdot) A_i^j$ and the functions share the convex sum property $h_i(\cdot) \ge 0$, $i \in \{1\ldots r\}$, $\sum_{i=1}^{r} h_i(\cdot) = 1$. The symbol $(*)$ in an expression stands for an expression induced by symmetry, e.g., $\begin{bmatrix} A+(*) & (*) \\ C & B \end{bmatrix} = \begin{bmatrix} A+A^T & C^T \\ C & B \end{bmatrix}$. We also recall the following relaxation lemma from [13].

Lemma 1. For $i,j \in \{1\ldots r\}$ and $k \in \{1\ldots r_e\}$. The condition $\Upsilon_{hh}^{v} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i(\cdot) h_j(\cdot) v_k(\cdot) \Upsilon_{ij}^{k} < 0$ is verified if

$$2\Upsilon_{ii}^{k} + (r-1)\left(\Upsilon_{ij}^{k} + \Upsilon_{ji}^{k}\right) < 0, \quad i,j \in \{1\ldots r\}, \; k \in \{1\ldots r_e\} \quad (1)$$

III. BIOMECHANICAL MODELLING

A. SCI modelling: From H2AT model to T2A model

Postural control modeling generally focuses on standing and often relies on variations of inverted pendulum models [14] [15]. There are very few works related to modeling in the sitting position [16], and [17] appears to be one of the rare fully nonlinear models dedicated to sitting control. However, in the latter case, the control is solely at the lumbar joint, and the model is not specifically intended for experimental validations. Therefore, we have developed our own fully nonlinear models for sitting positions. The initial versions are H2AT model [3] and S3S model [5], Fig. 1.

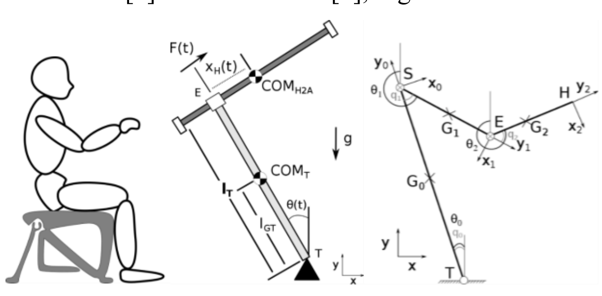

Fig. 1. Sitting posture (left) modelling, H2AT (middle), S3S (right)

The comprehensive description of these nonlinear models and their Euler-Lagrange-based derivations are beyond the scope of this paper due to space constraints. The new T2A model, depicted in Fig. 2, extends the S3S model by incorporating a second arm, expanding the model from 3 degrees-of-freedom (DoF) to 5 DoF.

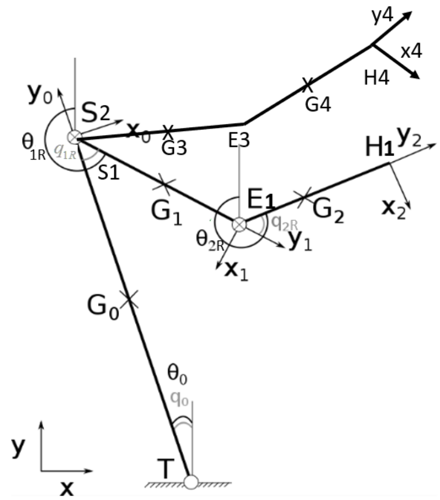

Fig. 2. The Trunk-2-arms (T2A) model

The T2A model is formulated in a descriptor form as

$$\begin{cases} E(q(t))\dot{x}(t) = A(x(t))x(t) + S(q(t)) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

The coordinates $q(t)$ are relative joint positions such that: $q(t) = \begin{bmatrix} q_0(t) & q_{1R}(t) & q_{2R}(t) & q_{1L}(t) & q_{2L}(t) \end{bmatrix}^T$.

$x = \begin{bmatrix} \theta_0 & \theta_{1R} & \theta_{2R} & \theta_{1L} & \theta_{2L} & \dot{\theta}_0 & \dot{\theta}_{1R} & \dot{\theta}_{2R} & \dot{\theta}_{1L} & \dot{\theta}_{2L} \end{bmatrix}^T$ is the T2A state composed of absolute position coordinates such that: $q_0 = \theta_0$, $q_{1S} = \theta_{1S} - \theta_0$, $q_{2S} = \theta_{2S} - \theta_{1S}$, $S = R/L$.

The five torque inputs $u(t) = \begin{bmatrix} \Gamma_0 & \Gamma_{1R} & \Gamma_{2R} & \Gamma_{1L} & \Gamma_{2L} \end{bmatrix}^T$ correspond to the joint torques, where Γ_0 is the torque at the trunk, Γ_{1R} (respectively Γ_{1L}) is the torque at the right shoulder (respectively left shoulder), and Γ_{2R} (respectively Γ_{2L}) is the torque at the right elbow (respectively left elbow). The state-space matrices of the T2A model (2) are given by

$$E(q) = \begin{bmatrix} I_5 & 0_5 \\ 0_5 & \tilde{E}(q) \end{bmatrix}, \; A(x) = \begin{bmatrix} 0_5 & I_5 \\ 0_5 & \tilde{A}(x) \end{bmatrix}, \; S(q) = \begin{bmatrix} 0_{5\times 1} \\ \tilde{S}(q) \end{bmatrix},$$

$B = \begin{bmatrix} 0_{5\times 4} \\ \tilde{B} \end{bmatrix}$, $C = \begin{bmatrix} I_5 & 0_5 \end{bmatrix}$. The entries of these matrices are described equations (3) to (5). Subscript R stands for right, L for left, $l_\bullet$ is a length, $m_\bullet$ a mass, $I_\bullet$ an inertia, $a_\bullet$ inertial constants related to the Center of Mass, $b_\bullet = l_\bullet - a_\bullet$; subscript 0 is used for the trunk, 1 for the shoulder and 2 for the arm. For example, l_0 is the length of the trunk, m_{2L} the mass of the left arm. At last, the region of possible movements for a human [18] are defined using the compact set Ω_x:

$$\Omega_x = \begin{Bmatrix} -5° \le q_0 \le 5° & \|\dot{q}_0\| \le 29\ °/s \\ -20° \le q_{1R/L} \le 60°, & \|\dot{q}_{1R/L}\| \le 57\ °/s \\ -10° \le q_{2R/L} \le 45° & \|\dot{q}_{2R/L}\| \le 57\ °/s \end{Bmatrix} \quad (3)$$

$$\tilde{A}(x)=\begin{bmatrix} 0 & -p_{10R}\sin(q_{1R})\dot{\theta}_{1R} & -p_{9R}\sin(q_{1R}+q_{2R})\dot{\theta}_{2R} & -p_{10L}\sin(q_{1L})\dot{\theta}_{1L} & -p_{9L}\sin(q_{1L}+q_{2L})\dot{\theta}_{2L} \\ p_{8R}\dot{\theta}_0\sin(q_{1R}) & 0 & p_{4R}\sin(q_{2R})\dot{\theta}_{2R} & 0 & 0 \\ p_{9R}\dot{\theta}_0\sin(q_{1R}+q_{2R}) & -p_{4R}\sin(q_{2R})\dot{\theta}_{1R} & 0 & 0 & 0 \\ p_{8L}\dot{\theta}_0\sin(q_{1L}) & 0 & 0 & 0 & p_{4L}\sin(q_{2L})\dot{\theta}_{2L} \\ p_{9L}\dot{\theta}_0\sin(q_{1L}+q_{2L}) & 0 & 0 & -p_{4L}\sin(q_{2L})\dot{\theta}_{1L} & 0 \end{bmatrix} \tag{4}$$

$$\tilde{E}(q)=\begin{bmatrix} p_1 & -p_{10R}\cos(q_{1R}) & -p_{9R}\cos(q_{1R}+q_{2R}) & -p_{10L}\cos(q_{1L}) & -p_{9L}\cos(q_{1L}+q_{2L}) \\ -p_{8R}\cos(q_{1R}) & p_{3R} & p_{4R}\cos(q_{2R}) & 0 & 0 \\ -p_{9R}\cos(q_{1R}+q_{2R}) & p_{4R}\cos(q_{2R}) & p_{5R} & 0 & 0 \\ -p_{8L}\cos(q_{1L}) & 0 & 0 & p_{3L} & p_{4L}\cos(q_{2L}) \\ -p_{9L}\cos(q_{1L}+q_{2L}) & 0 & 0 & p_{4L}\cos(q_{2L}) & p_{5L} \end{bmatrix} \tag{5}$$

$$\tilde{S}(q)=\begin{bmatrix} p_2\sin(\theta_0) \\ p_{6R}\sin(\theta_{1R}) \\ p_{7R}\sin(\theta_{2R}) \\ p_{6L}\sin(\theta_{1L}) \\ p_{7L}\sin(\theta_{2L}) \end{bmatrix},\ \tilde{B}=\begin{bmatrix} 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \begin{array}{l} p_1 = I_0 + m_0 a_0^2 + (m_{1R}+m_{1L}+m_{2R}+m_{2L})l_0^2, \\ p_2 = (m_0 a_0 + (m_{1R}+m_{1L}+m_{2R}+m_{2L})l_0)g, \\ p_{3R} = I_{1R} + m_{1R}a_{1R}^2 + m_{2R}l_{1R}^2,\ p_{4R} = m_{2R}a_{2R}l_{1R},\ p_{5R} = I_{2R} + m_{2R}a_{2R}^2, \\ p_{6R} = (-m_{1R}b_{1R} + (m_{1R}+m_{2R})l_{1R})g,\ p_{7R} = m_{2R}a_{2R}g, \\ p_{8R} = m_{1R}a_{1R}l_0 + m_{2R}l_0 l_{1R},\ p_{9R} = m_{2R}a_{2R}l_0,\ p_{10R} = m_{1R}a_{1R} + m_{2R}l_{1R} \end{array} \tag{6}$$

p_{3L} to p_{10L} are defined similarly as p_{3R} to p_{10R} in (6), replacing the subscript R with L.

Remark 1: notice that $E(q(t))$ in (2) is invertible for each $q(t)$. Nevertheless, for such mechanical models [7] the number of nonlinearities increases in a huge way if we use a standard description involving $E^{-1}(q(t))$. It results in an increase of vertices to describe the nonlinear model and generally ends with more conservative results.

Remark 2: The fifth input Γ_0 corresponds to the trunk torque which cannot be mobilized in the case of an SCI whereas it is the most important input for a valid person. Thus, it results in an unstable or weakly stabilizable system as the SCI can only use the upper body (the shoulders Γ_{1L}, Γ_{1R} and the arms Γ_{2L} and Γ_{2R}) to stabilize itself.

B. The problem formulation

The goal is to derive an observer for system described by (4) to (6) inside the compact state region Ω_x (3):

a) that guarantees the state error observation to converge;
b) that estimates slow varying input torques;
c) can be tested on data recorded on different SCI people.

The technique used is based on polytopic observers representing, in a compact set of the system, exactly the nonlinear system without approximation. Nevertheless, even if a direct formulation can be obtained via nonlinear sectors, it appears out of the possibility of the actual solvers.

Remark 3: To give an idea of the complexity, consider the S3S model [3] (Fig. 1 right) and a polytopic observer based on an exact representation of the biomechanical model. Writing of the problems a) and b) requires 128 LMI constraints of size 12×12 and 1518 decision variables. Following the same approach for the system T2A would end with 4096 LMI constraints of size 20×20 and more than 14000 variables, considering the poorest code, without any performance and no relaxation and with no guarantee to have a solution. Apart the fact that it approaches the numerical limits of solvers, adding performances, or other DoF (rotations for example), this direct way cannot be followed.

Therefore, similar to the approach in [5] the design of a cascade of observer [11] can come at hand. The steps to design the cascade of observer are:

1. rewrite model T2A under a cascaded form of 3 models;
2. extend these 3 "local" models to include the slow varying input torques, problem b);
3. show that the cascade obtained holds the properties of a separation principle for the "local" observers' design;
4. design for each model a PI-observer with guarantee of convergence of the "local" error. This step includes non-measured scheduling variables issues.

C. Step 1: Cascade representation of T2A model

A cascade of three sub-systems, Fig. 3, is built, Σ_T represents the trunk, Σ_{AR} represents the right arm, and Σ_{AL} the left arm. The trunk model Σ_T corresponds to:

$$p_1\ddot{\theta}_0 = p_2\sin(\theta_0) + \Gamma_c + \Gamma_0 \tag{7}$$

The right arm model Σ_{AR} corresponds to

$$\begin{cases} E_R(q)\dot{x}_R = A_R(x_R)x_R + S_R(q) + B_R u + D_R\begin{bmatrix}\dot{\theta}_0^2 \\ \ddot{\theta}_0\end{bmatrix} \\ y_R = \begin{bmatrix} I_2 & 0\end{bmatrix}x_{AR} = \theta_R = \begin{bmatrix}\theta_{1R} \\ \theta_{2R}\end{bmatrix} \end{cases} \tag{8}$$

with $x_R = \begin{bmatrix}\theta_{1R} & \theta_{2R} & \dot{\theta}_{1R} & \dot{\theta}_{2R}\end{bmatrix}^T$, and

$$A_R(x_{AR}) = \begin{bmatrix} 0 & I \\ 0 & \begin{bmatrix} 0 & p_{4R}\sin(q_{2R})\dot{\theta}_{2R} \\ -p_{4R}\sin(q_{2R})\dot{\theta}_{1R} & 0 \end{bmatrix} \end{bmatrix}$$

$$B_R=\begin{bmatrix} 0 & 0 \\ \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} & 0 \end{bmatrix},\ S_R(q)=\begin{bmatrix} 0_{2\times 1} \\ p_{6R}\sin(\theta_{1R}) \\ p_{7R}\sin(\theta_{2R}) \end{bmatrix},$$

$$D_R(q)=\begin{bmatrix} 0_2 \\ \begin{bmatrix} p_{8R}\sin(q_{1R}) & p_{8R}\cos(q_{1R}) \\ p_{9R}\sin(q_{1R}+q_{2R}) & p_{9R}\cos(q_{1R}+q_{2R}) \end{bmatrix} \end{bmatrix},$$

$$E_R(\theta_{2R})=\begin{bmatrix} I & 0 \\ 0 & \begin{bmatrix} p_{3R} & p_{4R}\cos(q_{2R}) \\ p_{4R}\cos(q_{2R}) & p_{5R} \end{bmatrix} \end{bmatrix}.$$

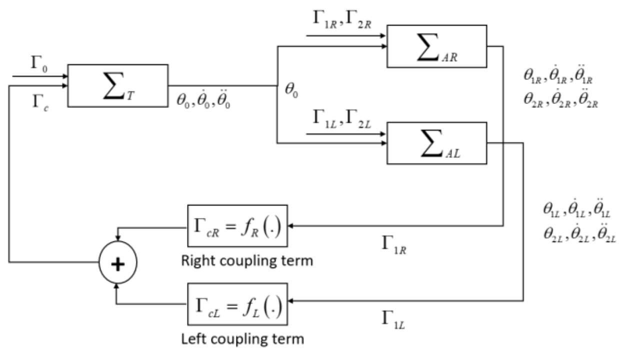

Fig. 3. Cascade representation of the T2A model with Σ_T the trunk, Σ_{AR}, Σ_{AL} the right and left arms and the 2 static functions.

The coupling equation between the right arm Σ_{AR} and the trunk Σ_T is given by:

$$\Gamma_{cR}=f_R(.)=\begin{bmatrix} -p_{10R}\sin(q_{1R})\dot{\theta}_{1R} \\ -p_{9R}\sin(q_{1R}+q_{2R})\dot{\theta}_{2R} \\ p_{10R}\cos(q_{1R}) \\ p_{9R}\cos(q_{1R}+q_{2R}) \end{bmatrix}^T \begin{bmatrix} \dot{\theta}_{1R} \\ \dot{\theta}_{2R} \\ \ddot{\theta}_{1R} \\ \ddot{\theta}_{2R} \end{bmatrix}-\Gamma_{1R}. \tag{9}$$

The definition of the left arm Σ_{LR} follows the same procedure as described in (8) for the model Σ_{AR}, and its interaction with the trunk Γ_{cL} in (9). The equations are not reiterated here; simply substitute the subscript "R" with "L" in (8) and (9).

D. *Step 2: Extended systems – slow varying inputs*

Designing a classical Unknown Input Observer (UIO) is not feasible due to unsatisfied rank conditions [19]. Hence, in line with our prior studies, in order to design a PI-observer, we consider $\ddot{\Gamma}_{(.)}\approx 0$ as an "accurate" representation of the input torques behavior.

Remark 4: This choice corresponds to the fact that the human torque variations are sufficiently slow, problem b), to be captured via a second order dynamic. It has been validated in simulation and on real data [8].

Therefore, we introduce the second order dynamic for the trunk Σ_T to get:

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & p_1 \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{\theta}_0 \\ \ddot{\theta}_0 \\ \dot{\Omega}_T \end{bmatrix}=\begin{bmatrix} \begin{bmatrix} 0 & 1 \\ p_2\mathrm{sinc}(\theta_0) & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \\ 0 & J \end{bmatrix}\begin{bmatrix} \theta_0 \\ \dot{\theta}_0 \\ \Omega_T \end{bmatrix} \tag{10}$$

with $J=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\Omega_T=\begin{bmatrix} \Gamma_U \\ \dot{\Gamma}_U \end{bmatrix}$, and $\Gamma_U=\Gamma_c+\Gamma_0$. Defining $x_T=\begin{bmatrix} \theta_0 \\ \dot{\theta}_0 \end{bmatrix}$, $A_T(\theta_0)=\begin{bmatrix} 1 & 0 \\ 0 & p_1^{-1} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ p_2\mathrm{sinc}(\theta_0) & 0 \end{bmatrix}$ and $B_T=\begin{bmatrix} 1 & 0 \\ 0 & p_1^{-1} \end{bmatrix}\begin{bmatrix} B & 0 \end{bmatrix}$, then expression (10) is equivalent to

$$\begin{bmatrix} \dot{x}_T \\ \dot{\Omega}_T \end{bmatrix}=\begin{bmatrix} A_T(\theta_0) & B_T \\ 0 & J \end{bmatrix}\begin{bmatrix} x_T \\ \Omega_T \end{bmatrix}. \tag{11}$$

Identically, for the right arm (8), we get:

$$\begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}_R \\ \dot{\Omega}_R \end{bmatrix}=\begin{bmatrix} A_R(x_R) & B_R \\ 0 & J_R \end{bmatrix}\begin{bmatrix} x_R \\ \Omega_R \end{bmatrix}+\begin{bmatrix} 0 \\ D_R(\theta_R) \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ C(\theta_R) \\ 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_0^2 \\ \ddot{\theta}_0 \end{bmatrix}$$

$$y_R=\begin{bmatrix} I_2 & 0 \end{bmatrix}\begin{bmatrix} x_R \\ \Omega_R \end{bmatrix}=\theta_R=\begin{bmatrix} \theta_{1R} \\ \theta_{2R} \end{bmatrix} \tag{12}$$

with $J_R=\begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix}$, $x_{AR}=\begin{bmatrix} \theta_{1R} & \theta_{2R} & \dot{\theta}_{1R} & \dot{\theta}_{2R} \end{bmatrix}^T$, and $\Omega_R=\begin{bmatrix} \Gamma_{1R} & \dot{\Gamma}_{1R} & \Gamma_{2R} & \dot{\Gamma}_{2R} \end{bmatrix}^T$. Notice that for the arms (12) following Remark 1, we keep a descriptor structure. From (11) and (12), the steps 3 and 4 can be initiated.

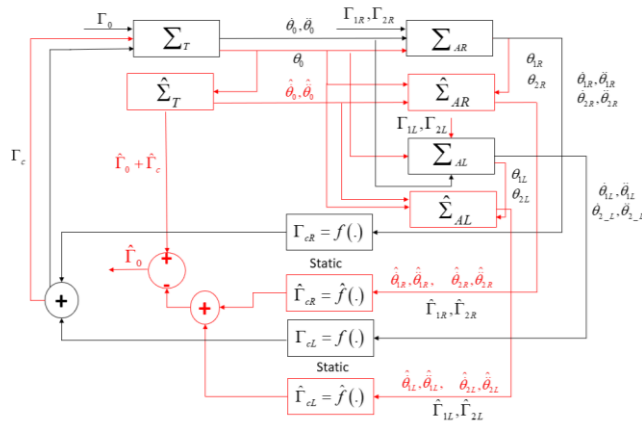

Fig. 4. T2A cascade observer with $\hat{\Sigma}_T$ the observer for the trunk, $\hat{\Sigma}_{AR}$ $\hat{\Sigma}_{AL}$ for the right and left arms and the 2 static functions.

IV. DESIGN OF THE CASCADE OF OBSERVERS

The global scheme of the cascade of observers, $\hat{\Sigma}_T$ and $\hat{\Sigma}_{AR}$, $\hat{\Sigma}_{AL}$ is depicted Fig. 4. The black arrows of Fig. 4. indicate measurable variables, the red arrows indicate estimated variables used by the local observers of the cascade. Step 3: Observation problem formulation

For the trunk, a direct form can be obtained from (11) as

$$\begin{bmatrix} \dot{\hat{x}}_T \\ \dot{\hat{\Omega}}_T \end{bmatrix}=\begin{bmatrix} A_T(\theta_0) & B_T \\ 0 & J \end{bmatrix}\begin{bmatrix} \hat{x}_T \\ \hat{\Omega}_T \end{bmatrix}+K_T(\theta_0)(y_T-\hat{y}_T). \tag{13}$$

Define the state observation error as $e_T=\begin{bmatrix} x_T \\ \Omega_T \end{bmatrix}-\begin{bmatrix} \hat{x}_T \\ \hat{\Omega}_T \end{bmatrix}$. It follows directly from (11) and (13) that

$$\dot{e}_T = \left(\begin{bmatrix} A_T(\theta_0) & B_T \\ 0 & J \end{bmatrix} - K_T(\theta_0) C_T \right) e_T = \psi_T(\theta_0) e_T. \quad (14)$$

The observer design problem resumes in finding $K_T(\theta_0)$ such that (14) is asymptotically stable.

For the arms (12), there is an issue as the matrix $A_R(x_{AR})$, see its definition after (8) (same for $A_L(x_{AL})$), depends on the non-measured scheduling variables $\dot{\theta}_{1R}, \dot{\theta}_{2R}$ ($\dot{\theta}_{1L}, \dot{\theta}_{2L}$). The problem of non-measured variables for the design of an observer, in a general framework, is still an open problem. We provide in our case a way to solve it without neither norm bounds [9] nor using Lipschitz-like conditions [10]. For the right arm, an observer can be written as:

$$\begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{\hat{x}}_R \\ \dot{\hat{\Omega}}_R \end{bmatrix} = \begin{bmatrix} A_R(\hat{x}_R) & B_A \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \hat{x}_R \\ \hat{\Omega}_R \end{bmatrix} + \begin{bmatrix} 0 \\ D_a(\theta_R) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ C(\theta_R) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{\theta}}_0^2 \\ \ddot{\hat{\theta}}_0 \end{bmatrix} + K_R(\cdot)(y_R - \hat{y}_R) \quad (15)$$

Notice that since the variables $\dot{\theta}_0$ and $\ddot{\theta}_0$ are non-measured, they are replaced with their respective estimates $\dot{\hat{\theta}}_0$ and $\ddot{\hat{\theta}}_0$ coming from the trunk observer $\hat{\Sigma}_T$. We define the state estimation error as $e_R = \begin{bmatrix} x_R \\ \Omega_R \end{bmatrix} - \begin{bmatrix} \hat{x}_R \\ \hat{\Omega}_R \end{bmatrix}$. Since $\begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix}$ is well-defined for every θ_{2R}, the convergence of e_R is ensured if system (16) converges

$$\begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix} \dot{e}_R = \begin{bmatrix} A_R(x_R) & B_R \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} x_R \\ \Omega_R \end{bmatrix} - \begin{bmatrix} A_R(\hat{x}_R) & B_R \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \hat{x}_R \\ \hat{\Omega}_R \end{bmatrix} - K_R(\cdot) C e_R + \begin{bmatrix} 0 \\ C(\theta_R) \\ 0 \end{bmatrix} \left(\begin{bmatrix} \dot{\theta}_0^2 \\ \ddot{\theta}_0 \end{bmatrix} - \begin{bmatrix} \dot{\hat{\theta}}_0^2 \\ \ddot{\hat{\theta}}_0 \end{bmatrix} \right) \quad (16)$$

In (16), the issues are to treat both quantities:

$$\Delta A_R(\cdot) = A_R(x_R) - A_R(\hat{x}_R) \text{ and } \Delta\theta_T = \begin{bmatrix} \dot{\theta}_0^2 \\ \ddot{\theta}_0 \end{bmatrix} - \begin{bmatrix} \dot{\hat{\theta}}_0^2 \\ \ddot{\hat{\theta}}_0 \end{bmatrix} \quad (17)$$

as functions of the state errors. The first term comes from $A_R(x_R)x_R - A_R(\hat{x}_R)\hat{x}_R = A_R(x_R)e_R + \Delta A_R(\cdot)\hat{x}_R$ and:

$$\Delta A_R(\cdot) = p_{4R} \sin(q_{2R}) \begin{bmatrix} 0 & 0 \\ 0 & \begin{bmatrix} 0 & \dot{\theta}_{2R} - \dot{\hat{\theta}}_{2R} \\ -\dot{\theta}_{1R} + \dot{\hat{\theta}}_{1R} & 0 \end{bmatrix} \end{bmatrix}.$$

Then, it follows after technical manipulations that:

$$A_R(x_R)e_R + \Delta A_R(\cdot)\hat{x}_R \triangleq H(x_R, \hat{x}_R)e_R = \begin{bmatrix} 0 & I \\ 0 & p_{4R}\sin(q_{2R}) \begin{bmatrix} 0 & \dot{\theta}_{2R} + \dot{\hat{\theta}}_{2R} \\ -\dot{\theta}_{1R} - \dot{\hat{\theta}}_{1R} & 0 \end{bmatrix} \end{bmatrix} e_R \quad (18)$$

The second term of (17) writes directly

$$\begin{bmatrix} \dot{\theta}_0^2 \\ \ddot{\theta}_0 \end{bmatrix} - \begin{bmatrix} \dot{\hat{\theta}}_0^2 \\ \ddot{\hat{\theta}}_0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_0 + \dot{\hat{\theta}}_0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 - \dot{\hat{\theta}}_0 \\ \ddot{\theta}_0 - \ddot{\hat{\theta}}_0 \end{bmatrix}. \quad (19)$$

Then, it follows from (19) that

$$\begin{bmatrix} 0 \\ C(\theta_R) \\ 0 \end{bmatrix} \left(\begin{bmatrix} \dot{\theta}_0^2 \\ \ddot{\theta}_0 \end{bmatrix} - \begin{bmatrix} \dot{\hat{\theta}}_0^2 \\ \ddot{\hat{\theta}}_0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ C(\theta_R) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 + \dot{\hat{\theta}}_0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 - \dot{\hat{\theta}}_0 \\ \ddot{\theta}_0 - \ddot{\hat{\theta}}_0 \\ \Omega_T - \hat{\Omega}_T \end{bmatrix} \triangleq -E_{12R}\left(\theta_R, \dot{\theta}_0, \dot{\hat{\theta}}_0\right)\dot{e}_T \quad (20)$$

For the right arm, it follows from (16) and (20) that

$$E_{12R}(\cdot)\dot{e}_T + \begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix} \dot{e}_R = \left(\begin{bmatrix} H(\cdot) & B_R \\ 0 & J_{2R} \end{bmatrix} - K_R(\cdot) C \right) e_R$$

or equivalently introducing the notation $\phi_R(\cdot)$ and $\psi_R(\cdot)$:

$$E_{12R}(\cdot)\dot{e}_T + \phi_R(\cdot)\dot{e}_R = \psi_R(\cdot)e_R. \quad (21)$$

The left arm can be easily defined by symmetry, i.e., with matrices $H(x_L, \hat{x}_L)$ and $E_{12L}\left(\theta_L, \dot{\theta}_0, \dot{\hat{\theta}}_0\right)$:

$$E_{12L}(\cdot)\dot{e}_T + \phi_L(\cdot)\dot{e}_L = \psi_L(\cdot)e_L. \quad (22)$$

Remark 5: Remember that problem a) requires that we guarantee asymptotic convergence of the state error in Ω_x. The way to describe (21) and (22) using the transformations (18) and (20) is indeed crucial as they introduce no approximation and are only state error dependent.

Now, the full observation problem can be written as

$$\begin{bmatrix} I & 0 & 0 \\ E_{12R}(\cdot) & \phi_R & 0 \\ E_{12L}(\cdot) & 0 & \phi_L \end{bmatrix} \begin{bmatrix} \dot{e}_T \\ \dot{e}_R \\ \dot{e}_L \end{bmatrix} = \begin{bmatrix} \psi_T(\theta_0) & 0 & 0 \\ 0 & \psi_R(x) & 0 \\ 0 & 0 & \psi_L(x) \end{bmatrix} \begin{bmatrix} e_T \\ e_R \\ e_L \end{bmatrix} \quad (23)$$

where $\psi_T(\theta_0)$ is defined in (14), $\phi_R(\cdot)$ and $\psi_R(\cdot)$ are defined in (21). We recall the following result for descriptor cascaded observers [5].

Theorem 1 [5]. Consider two descriptor systems, $E_i(\cdot)$ being always regular $(E_i(\cdot), A_i(\cdot), C_i(\cdot))$, $i \in \{1,2\}$, such that it exists matrices $K_i(\cdot)$, $i \in \{1,2\}$ ensuring that the two estimation error systems defined by

$$E_i(\cdot)\dot{\varepsilon}_i = (A_i(\cdot) - K_i(\cdot)C_i(\cdot))\varepsilon_i \quad (24)$$

are Globally Asymptotically Stable (GAS). Consider the augmented system:

$$\begin{bmatrix} E_1(\cdot) & 0 \\ E_{12}(\cdot) & E_2(\cdot) \end{bmatrix} \dot{e} = \begin{bmatrix} A_1(\cdot) - K_1(\cdot)C_1(\cdot) & 0 \\ A_{12}(\cdot) & A_2(\cdot) - K_2(\cdot)C_2(\cdot) \end{bmatrix} e \quad (25)$$

with $E_{12}(\cdot)$ and $A_{12}(\cdot)$ are norm-bounded matrices. Then, the error system (25) is also Globally Asymptotically Stable.

It is direct to see that problem (23) corresponds to Theorem 1 conditions, as $A_{12}(\cdot) = 0$, $E_{12R}(\cdot)$ and $E_{12L}(\cdot)$ are norm-bounded matrices. Therefore, sufficient conditions for (23) to be GAS are that the 3 following systems are GAS:

$$\dot{e}_T = \left(\begin{bmatrix} A_T(\theta_0) & B_T \\ 0 & J \end{bmatrix} - K_T(\theta_0) C_T \right) e_T. \tag{26}$$

$$\begin{bmatrix} E_R(\theta_{2R}) & 0 \\ 0 & I \end{bmatrix} \dot{\varepsilon}_R = \left(\begin{bmatrix} H(x_R, \hat{x}_R) & B_R \\ 0 & J_{2R} \end{bmatrix} - K_R(\cdot) C \right) \varepsilon_R \tag{27}$$

$$\begin{bmatrix} E_L(\theta_{2L}) & 0 \\ 0 & I \end{bmatrix} \dot{\varepsilon}_L = \left(\begin{bmatrix} H(x_L, \hat{x}_L) & B_L \\ 0 & J_{2L} \end{bmatrix} - K_L(\cdot) C \right) \varepsilon_L \tag{28}$$

where ε_R and ε_L are dummy variables, as they do not represent the full state errors e_R and e_L of the right (left) arm. Nevertheless, finding $K_T(\theta_0)$, $K_R(\cdot)$ and $K_L(\cdot)$ such that $e_T, \varepsilon_R, \varepsilon_L \xrightarrow[t\to\infty]{} 0$ ensures (23) to be GAS.

A. Step 4: LMI design

For system (26), we can use a "linearizing" expression of the gain $K_T(\theta_0) = \begin{bmatrix} 0 & \beta p_1^{-1} p_2 \sin(\theta_0)/\theta_0 & 0 & 0 \end{bmatrix}^T + K_{LT}$ to get a linear state observation error as

$$\dot{e}_T = \left(\begin{bmatrix} J & B_T \\ 0 & J \end{bmatrix} - K_{LT} C_T \right) e_T \tag{29}$$

Therefore, a single pole placement can be used to obtain

$$K_{LT} = 10^2 \begin{bmatrix} 10.2 & 9.14 & 4.1\times 10^2 & 5.36\times 10^2 \end{bmatrix}^T \tag{30}$$

For (27) and (28), we use a polytopic description of the descriptors using the compact state region Ω_x (3). For (27), with $\cos(q_2) \in [v, 1]$, $\sin(q_2)\left(\dot{\theta}_i + \dot{\hat{\theta}}_i\right) \in \left[\underline{\rho}_i, \bar{\rho}_i\right]$, $i \in \{1, 2\}$, it corresponds to

$$\begin{gathered} \sum_{i=1}^{2} v_i(q_2) \begin{bmatrix} E_{Ri} & 0 \\ 0 & I \end{bmatrix} \dot{\varepsilon}_R = \\ \sum_{i=1}^{2} \sum_{j=1}^{2} w_{1i}(x_R, \hat{x}_R) w_{2j}(x_R, \hat{x}_R) \left(\begin{bmatrix} H_{ij} & B_R \\ 0 & J_2 \end{bmatrix} - K_R(\cdot) C \right) \varepsilon_R \end{gathered} \tag{31}$$

with $v_1(q_2) = \dfrac{1-\cos(q_2)}{1-v} = 1 - v_2(q_2)$ $\quad 1 - w_{i2}(x_R, \hat{x}_R) = w_{i1}(x_R, \hat{x}_R) = \dfrac{\bar{\rho}_i - \sin(q_2)\left(\dot{\theta}_i + \dot{\hat{\theta}}_i\right)}{\bar{\rho}_i - \underline{\rho}_i}$. From (31), we use the result of [7] for observation of descriptor systems.

Theorem 2 [7]: Consider a descriptor model

$$E_v \dot{x} = A_h x + B_h u, \; y = C_h x \tag{32}$$

together with the observer:

$$\begin{cases} E_v \dot{\hat{x}} = A_h \hat{x} + B_h u + \begin{bmatrix} E_v & I \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_{3h} & P_{4h} \end{bmatrix}^{-T} \begin{bmatrix} K_{1hv} \\ K_{2hv} \end{bmatrix} (y - \hat{y}) \\ \hat{y} = C_h \hat{x} \end{cases} \tag{33}$$

If there exist $P_1 = P_1^T > 0$, P_{3j}, P_{4j}, K_{1jk}, K_{2jk}, $j \in \{1 \dots r\}$ and $k \in \{1 \dots r_e\}$, such that conditions in (33) holds with

$$\Upsilon_{ij}^k = \begin{bmatrix} P_{3j}^T A_i - K_{1jk} C_i + (*) & (*) \\ P_{4j}^T A_i - K_{2jk} C_i + P_1 - E_k^T P_{3j} & -P_{4j}^T E_k + (*) \end{bmatrix}. \tag{34}$$

Then, the estimation error is asymptotically stable.

The LMI problem (34) applied to (31) gives an observer solution in the whole state compact set Ω_x, defined in (3):

$$\begin{aligned} K_{11}^T &= \begin{bmatrix} 158 & -45 & 51 & -11.8 & 9e^{-3} & 7^{-3} & 8e^{-3} & e^{-3} \\ -13.2 & 117.3 & 9.91 & 12.2 & 5e^{-3} & -5e^{-3} & 9e^{-3} & -9e^{-3} \end{bmatrix} \\ K_{12}^T &= \begin{bmatrix} 91.2 & -14.9 & 19.2 & -5.3 & 7e^{-3} & 9e^{-3} & 2e^{-3} & e^{-4} \\ -81 & 122.4 & -6.2 & 0.7 & 9e^{-3} & -4.1 & 9.2e^{-3} & -7e^{-4} \end{bmatrix} \\ K_{21}^T &= \begin{bmatrix} 47.3 & -8.1 & -0.41 & -0.09 & 4e^{-4} & 5e^{-3} & 4e^{-4} & e^{-3} \\ -2.7 & 21.3 & 0.09 & -0.16 & 5.7e^{-2} & 8e^{-3} & 4e^{-3} & -3e^{-4} \end{bmatrix} \\ K_{22}^T &= \begin{bmatrix} 13.4 & 5.8 & -0.07 & -5.2e^{-3} & 6e^{-4} & 9.4e^{-3} & e^{-4} & e^{-3} \\ -4.7 & 11.5 & 1.16 & -0.09 & 8e^{-3} & 2e^{-3} & 7.8e^{-2} & -0.12 \end{bmatrix} \end{aligned} \tag{35}$$

Therefore, gains (30) for $\hat{\Sigma}_T$, (35) for $\hat{\Sigma}_{AR}$ and its equivalent for the left arm $\hat{\Sigma}_{LR}$ ensure that $e_T, \varepsilon_R, \varepsilon_L \xrightarrow[t\to\infty]{} 0$.

V. Illustrative Results

The first validations are done in simulation as the unmeasurable inputs (torques) are perfectly known and can therefore, validate the approach. To leverage the T2A model, all the results presented consider asymmetric upper limbs movements.

A. Simulation results

Additionally, to be close to the real experiments (described in the next section) we simulate a disturbance on the back. It corresponds to a sinusoidal signal with a frequency of 10Hz added to the angular speed of the trunk between times 2.8s and 5.8s. The methodology has been intensively tested in simulation with various parameters representing different SCI people and behaviors. In order to illustrate the results, we just present Fig. 5 one of the trials. The torque signals of the upper limbs are given, simulated in black, estimated in red.

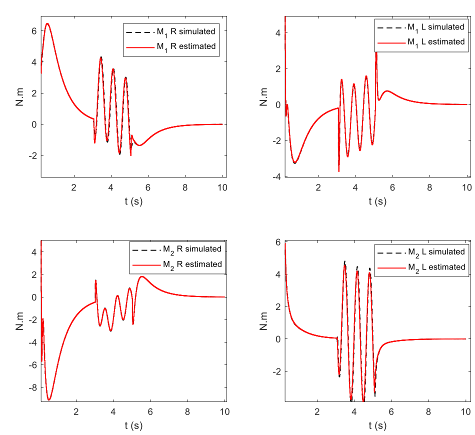

Fig. 5. Estimated torques at left and right shoulder and elbow levels by T2A observers.

B. Experimental data results

A set of experimental tests on 26 subjects with SCI were carried out in Montreal. Ethical approval has been obtained from the Research Ethics Committee of the Centre for Interdisciplinary Research in Rehabilitation of Greater Montreal (CRIR-1083-0515R). The participants read and signed the informed consent form prior to initiating the measurements. The experiments consist in a person with SCI starting in a sitting position with the upper limbs flexed to 90°. Then a destabilizing force is applied at the level of the 3[rd] thoracic vertebra between the scapulae [8], it can be seen after 2s on Fig 6. The participant has to maintain his/her stability, and to recover the flexed position at 90°.

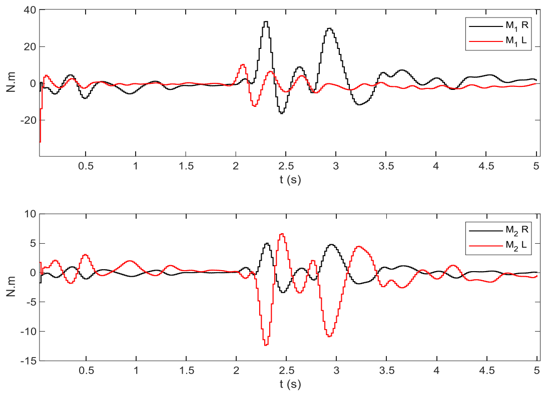

Fig. 6. Estimated upper limbs torques.

Fig. 6 shows the estimated right and left active torques at the shoulder (top) and the elbow (bottom). It shows clearly the dissymmetrical behavior on the upper limbs. Fig. 7 also gives the estimated passive torque at the trunk level according to the relation $\hat{\Gamma}_0 = \hat{\Gamma}_U - \hat{\Gamma}_c$.

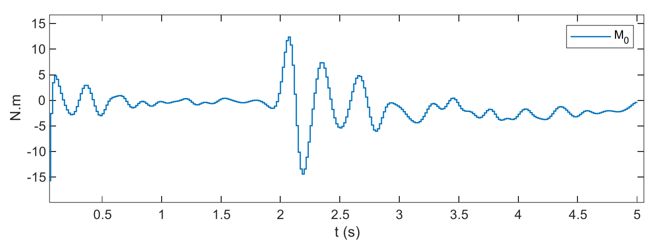

Fig. 7. Estimated trunk torque.

The active torques of the upper limbs have minimal variation from the start of the acquisition to the disturbance application, When the destabilizing force is applied, the active torques rapidly adjust to counter it. After, as the subject regains seated balance, the active torques gradually decrease. The effectiveness of postural control in a seated position becomes evident through the subject's corrective stabilization movements. In particular, the asymmetry in the active torques of the shoulders and elbows justifies the utilization of an observer based on the T2A model.

VI. CONCLUSIONS

A nonlinear observer method is presented to reconstruct the internal variables of SCI patients, especially the joint torques. The observer design is based on the improved biomechanical T2A model, taking into account the flexion and extension movements of the right and left arms in an asymmetrical manner. Due to the involved numerical complexities of the observer design, the T2A model is rewritten in the form of a cascade of three subsystems: trunk, left arm and right arm. Then, an PI observer is developed for each subsystem. Via Lyapunov stability theory, the estimation convergence of the cascaded observer is guaranteed and the observer design is expressed in terms of LMI constraints. Both numerical simulations and data-based validations are performed to show the effectiveness of the proposed cascade observer design method. Future works focus on nonlinear observer design based on biomechanical models, considering the 3D movements of SCI patients.

REFERENCES

[1] Boswell-Ruys, C., Harvey, L., Delbaere, K., Lord, S. (2010) “A falls concern scale for people with spinal cord injury”. Spinal Cord. 48 (9), 704–709.

[2] Nelson, A., Groer, S., Palacios, P., Mitchell, D., Sabharwal, S., Kirby, R., Gavin-Dreschnack, D., Powell-Cope, G. (2010) “Wheelchair-Related Falls in Veterans with Spinal Cord Injury Residing in the Community: A Prospective Cohort Study”. Arch. Phys. Med. Rehabil. 91 (8), 1166–1173.

[3] Blandeau, M., Estrada-Manzo, V., Guerra, T.-M., Pudlo, P., Gabrielli, F., (2018) “Fuzzy unknown input observer for understanding sitting control of persons living with spinal cord injury”. Engineering Applications of Artificial Intelligence 67, 381–389.

[4] Guerra T.-M., Blandeau M., Nguyen A. T., Srihi H., Dequidt A. (2020) “Stabilizing unstable biomechanical model to understand sitting stability for persons with spinal cord injury”. IFAC-PapersOnLine 53 (2), 8001-8006

[5] Srihi H., Guerra T. M., Nguyen A. T., Pudlo P., Dequidt A. (2021) “Cascade descriptor observers: application to understanding sitting control of persons living with spinal cord injury,” Frontiers in Control Engineering 2, 710271.

[6] Boyd S., Ghaoui L. E., Feron E., Balakrishnan V., Linear Matrix Inequalities in System and Control Theory. Society for Industrial and Applied Mathematics, 1994

[7] Guerra T.-M., Estrada-Manzo V., Lendek Z. (2015). “Observer design for Takagi–Sugeno descriptor models: An LMI approach”. Automatica. 52154–159.

[8] Blandeau, Mathias. Modélisation et caractérisation de la stabilité en position assise chez les personnes vivant avec une lésion de la moelle épinière. Biomécanique [physics.med-ph]. U. Valenciennes et du Hainaut-Cambresis, 2018. NNT: 2018VALE0021. tel-01861272.

[9] Zemouche A., Boutayeb M., Bara G.I. (2008). Observers for a class of Lipschitz systems with extension to H1 performance analysis. Systems & Control Letters, 57(1):18–27.

[10] Zemouche A., Boutayeb M. (2013). On LMI conditions to design observers for Lipschitz nonlinear systems. Automatica, 49, 585-591

[11] Lendek Zs., Babuska R., De Schutter B. (2008). Stability of cascaded fuzzy systems and observers. 515 IEEE Fuzzy Systems 17 (3), 641-653

[12] Koenig, D., Mammar, S. (2002) Design of proportional-integral observer for unknown input descriptor systems. IEEE Trans. Autom. Control, vol. 47, no 12, p. 2057–2062.

[13] Tuan, H,. Apkarian, P., Narikiyo, T., & Yamamoto, Y. (2001). “Parameterized linear matrix inequalities techniques in fuzzy control system design”. IEEE Trans. Fuzzy Syst., 9(2), 324-332

[14] Johansson R., Magnusson M., Akesson M. (1988). “Identification of human postural dynamics”, IEEE Trans. Biomed. Eng., 35 (10) 858-869.

[15] Winter D., Patla A., Prince F., Ishac M., Gielo-Perczak K. (1998). “Stiffness control of balance in quiet standing”, J. Neurophysiol., vol. 80, no 3, p. 1211-1221.

[16] Reeves N., Cholewicki J., Narendra K. (2009). “Effects of reflex delays on postural control during unstable seated balance”, J. Biomech., vol. 42, 2, p. 164-170.

[17] Tanaka M., Ross S. D., Nussbaum M. (2010). “Mathematical modeling and simulation of seated stability”, J. Biomech., vol. 43, no 5, p. 906-912, 2010.

[18] Kapandji, A. I. (2005a) Anatomie fonctionnelle I : Membres supérieurs. Physiologie de l’appareil locomoteur. 6th edition. Maloine.

[19] Ichalal D., Mammar S. (2015). On Unknown Input Observers for LPV Systems. IEEE Trans. Industrial 508 Electronics, 62 (9), 5870-5880