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A State-Constrained Control Scheme for Hysteresis Control Systems with Unknown Hysteresis Parameters

Jenq-Lang Wu and Arumugam Arunkumar

***Abstract*— This research examines the issue of state-constrained stabilizing controllers for Bouc-Wen hysteresis control systems with all hysteresis parameters being unknown. We develop a novel hysteresis estimator for estimating the virtual hysteresis state. Employing the L_2-gain control approach effectively mitigates the impact of estimation errors on the system. With barrier functions, even when the hysteresis parameters are unknown, we can formulate state-constrained stabilizing controllers using solutions to linear matrix inequalities. Ultimately, we showcase the efficacy and practicality of this control strategy with the help of a numerical example.**

***Index Terms*— Hysteresis systems, barrier function, Lyapunov function, state constraints.**

I. INTRODUCTION

Hysteresis manifests as a phenomenon observed across a broad spectrum of systems, encompassing electromagnetic actuators, smart materials, and electromechanical devices, among others [1]. Its presence can significantly impact control performance due to its inherent nonlinearity, and in some severe cases, it can even lead to system instability. Furthermore, hysteresis introduces complexity because its output is influenced not only by the input but also by changes in the input [2]. Consequently, compensating for hysteresis poses additional challenges. Nevertheless, through the collective endeavors of numerous researchers, significant progress has been made in hysteresis compensation over the past year.

The classification of hysteresis models typically spans across two main categories: physical and mathematical models. The most significant physical model is the Jile-Atherthon model [3], with prevalent mathematical models encompassing the Preisach model [1], the Prandtl-Ishlinskii (PI) model [2], the Krasnosel'skii-Pokrovkii hysteron [4], and the Bouc-Wen model [5-6]. Among the numerous approaches to characterizing hysteresis, the Bouc-Wen model stands out for its ability to succinctly describe hysteresis using a first-order nonlinear differential expression. To date, a considerable body of literature has been dedicated to identifying and controlling the Bouc-Wen model for managing hysteresis systems [7-8].

However, the significance of state constraints in control systems is substantial, impacting practical applications as well as fundamental control theories. The safety of human operators and the system itself is jeopardized when specific unsafe states are encountered, especially in safety-critical systems such as robotic systems, autonomous vehicles, and chemical plants [9]. Therefore, it is imperative for the designed controller to guarantee compliance with state constraints while effectively managing the system. Several classical methods have been developed to tackle this challenge, which encompasses set invariance control [10], model predictive control [11], and reference governors [12]. In recent developments, innovative design strategies have surfaced, incorporating control Lyapunov functions and barrier functions to guarantee the non-violation of these constraints [13]. In [14], the application of the barrier Lyapunov function (BLF) technique was explored for single-input single-output systems with an adaptive control scheme and full state constraints. To ensure safe control, the authors in [15] investigated a CBF approach for designing controllers for nonlinear systems. In [9], a novel approach was introduced for nonlinear control-affine systems, which integrated a control Lyapunov function (CLF) and a control barrier function (CBF) into a unified concept known as a control Lyapunov barrier function (CLBF). Subsequently, continuous controllers were devised using Sontag's formula to ensure both safety and stability. Furthermore, a novel CLBF approach was discussed in [16] for a class of nonlinear control-affine systems subject to event-triggered control schemes and state constraints.

Additionally, a multitude of studies have substantiated that the adoption of Zeroing CBF (ZCBF) can concurrently enhance robustness and stability while ensuring safety. A quadratic programming (QP)-based control synthesis

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approach was developed to ensure compliance with state constraints by combining CLFs and ZCBFs, as demonstrated in [17]. Recently, Wu et al. [18] tackled the L_2-gain control issue for nonlinear control-affine systems with state constraints, employing an innovative approach based on the control storage function (CSF) methodology. However, all the above-mentioned results focus on nonlinear control systems without considering hysteresis behaviors. Very recently, Wu and Arunkumar [19] investigated the state-constrained control problem for Bouc-Wen hysteresis control systems with known hysteresis parameters. As of now, to the extent of our understanding, investigations into the state constraints problem for the Bouc-Wen model with unknown hysteresis parameters have yet to yield any results.

Inspired by the aforementioned factors, the current study delves into the design of state-constrained state feedback stabilizing controllers for Bouc-Wen hysteresis control systems with unknown hysteresis parameters. Designing state-constrained controllers becomes significantly challenging in practical systems where accurately obtaining the values of hysteresis parameters is difficult. In this paper, we introduce a novel hysteresis state estimator that enables the estimation of the virtual Bouc-Wen hysteresis state without prior knowledge of the hysteresis parameters. The estimation error is then treated as a disturbance, and an L_2-gain control method is employed to reduce the impact of the estimation error on the hysteresis system. The proposed approach simplifies the process of deriving state-constrained controllers through the solution of LMIs by combining barrier functions. Finally, we propose sufficient conditions for the existence of state-constrained controllers in Bouc-Wen hysteresis control systems, even when the hysteresis parameters are unknown. To validate the efficacy of this control approach, we provide a numerical illustration from a real-world application, specifically the piezo-positioning mechanical system.

II. Problem Formulation And Preliminaries

In this paper, the hysteresis linear control systems can be described as follows:

$$\begin{aligned} \dot{x} &= Ax + BH(u) \\ z &= Cx \end{aligned} \tag{1}$$

where $x \in \mathcal{R}^n$ represents the state of the system, while $u \in \mathcal{R}$ stands for the ideal control signal produced by the controller. Additionally, $v = H(u) \in \mathcal{R}$ represents the actual acting control force exerted due to the hysteresis behavior of the actuator, and $z \in \mathcal{R}^m$ signifies the controlled output. Furthermore, we have known real constant matrices $A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^n$ and $C \in \mathcal{R}^{m\times n}$. The outline of the Bouc-Wen hysteresis model can be summarized as follows:

$$v = H(u) = \mu_1 u + \mu_2 \varsigma \tag{2}$$

where $\mu_1 > 0$ and $\mu_2 > 0$ represent hysteresis parameters, and $\varsigma \in \mathcal{R}$ serves as a virtual auxiliary variable known as the hysteresis state. The determination of the hysteresis state relies on the subsequent first-order differential equation [8]:

$$\dot{\varsigma} = \dot{u} - \beta|\dot{u}||\varsigma|^{r-1}\varsigma - \chi\dot{u}\,|\varsigma|^r, \varsigma(t_0) = 0 \tag{3}$$

where β, χ, and r, are parameters to characterize the shape and magnitude of the hysteresis phenomenon. Assume $\beta > |\chi|$ and $r \geq 1$. Moreover, in this paper, we assume μ_2 is unknown and $\mu_1 = \hat{\mu}_1 + \Delta\mu_1$ for a known $\hat{\mu}_1 > 0$ and unknown $\Delta\mu_1$ satisfying $|\Delta\mu_1| < \bar{\mu}$ with known $\bar{\mu} < \hat{\mu}_1$. In [8], it was established that the solution $\varsigma(t)$ of equation (3) remains bounded and fulfills

$$|\varsigma(t)| \leq \sqrt[r]{\frac{1}{\beta + \mathcal{X}}}. \tag{4}$$

From the above discussions, the dynamics of the system can be articulated in the following manner:

$$\begin{aligned} \dot{x} &= Ax + B(\mu_1 u + \mu_2 \varsigma) \\ \dot{\varsigma} &= \dot{u} - \beta|\dot{u}||\varsigma|^{r-1}\varsigma - \chi\dot{u}\,|\varsigma|^r. \end{aligned} \tag{5}$$

The state-constrained region is defined as [19]

$$\mathcal{D} \equiv \{x \in R^n | s_i(x) \equiv S_i x + c_i > 0, i = 1,2,\dots,q\} \tag{6}$$

where S_i, for $i = 1,2,\dots,q$, represents constant row vectors then c_i, for $i = 1,2,\dots,q$, denotes positive scalars. We presume that $\mathcal{D}$ serves as a connected region and the origin is located within its interior. Similar to [19], define $\mathcal{D}_i \equiv \{x \in R^n | S_i x + c_i > 0\}$, $\partial\mathcal{D}_i \equiv \{x \in R^n | S_i x + c_i = 0\}, i = 1,2,\dots,q$. Then,

$$\begin{aligned} \partial\mathcal{D} &\equiv \{x \in \bar{\mathcal{D}} | S_i x + c_i = 0, \text{for some } i \in \{1,2,\dots,q\}\} \\ &= \bar{\mathcal{D}} \cap (\partial\mathcal{D}_1 \cup \dots \cup \partial\mathcal{D}_q). \end{aligned}$$

The primary aim of this paper is to design a controller that guarantees convergence of the state trajectory of the hysteresis control system (5) towards the origin, while satisfying the state constraint $x(t) \in \mathcal{D}$ for all $t \geq 0$ with $x(0) \in \mathcal{D}$. In such instances, system (5) is said to be stable with respect to $\mathcal{D}$.

III. Main Results

In the subsequent section, we develop a method for designing state-constrained controllers for hysteresis control systems when the hysteresis parameters are unknown. Note that μ_1 and μ_2 are unknown, and μ_1 satisfies the conditions presented in Section II. Assume that β, χ, r are also unknown but satisfy $r \leq \bar{r}$ and $\beta + \mathcal{X} \geq \delta$ for known $\bar{r} > 0$ and $\delta > 0$. Then, it can be shown that $|\varsigma(t)| \leq \bar{\delta}(\bar{r}, \delta)$, where

$$\bar{\delta}(\bar{r}, \delta) = \begin{cases} \frac{1}{\sqrt[\bar{r}]{\delta}}, \text{if } \delta < 1 \\ \frac{1}{\sqrt{\delta}}, \text{if } \delta \geq 1. \end{cases} \tag{7}$$

A. Hysteresis Observer

Suppose that $v = Kx$ is a stabilizing controller for the hysteresis-free system

$$\dot{x} = Ax + Bv. \tag{8}$$

That is, $A + BK$ is Hurwitz. However, utilizing $u = Kx$ to stabilize the hysteresis system (5) is impractical due to the influence of the unknown hysteresis state ς on the system. Our objective is to counteract the impact of ς, a virtual state beyond the reach of feedback mechanisms. In [19], an innovative method is developed to estimate the virtual hysteresis state in which the parameters are known. Here, we extend the approach in [19] to design a hysteresis state estimator in the case that the hysteresis parameters are unknown. We estimate the hysteresis state and subsequently incorporate it into our controller design. While there are several estimation techniques in the literature on hysteresis, they focus on the case of $r = 1$ and of known hysteresis parameters . Unfortunately, limited results are available for the case of $r > 1$ or unknown hysteresis parameters. In this subsection, we introduce a novel approach for estimating $\mu_2\varsigma$ in scenarios where r can be larger than 1 and the hysteresis parameters remain unknown.

We can conclude from (5) that

$$(\hat{\mu}_1 + \Delta\mu_1)Bu + \mu_2 B\varsigma = \dot{x} - Ax \tag{9}$$

Multiplying both sides of the equation (9) by B^T, we easily derive that

$$(\hat{\mu}_1 + \Delta\mu_1)\|B\|^2 u + \mu_2\|B\|^2\varsigma = B^T\dot{x} - B^T Ax. \tag{10}$$

Therefore, we can estimate $\mu_2\varsigma$ as follows:

$$\tilde{\varsigma}(t) = \frac{B^T\dot{x}(t) - B^T Ax(t) - \hat{\mu}_1\|B\|^2 u(t^-)}{\|B\|^2}. \tag{11}$$

Nonetheless, practical implementation is challenged by the sensitivity of this estimation to measurement disturbances arising from the utilization of a differentiator, potentially amplifying noise and compromising control performance. To address this challenge, we can enhance the estimation as outlined below (assuming $\alpha > 0$ and noting $|\varsigma(t)| \leq \bar{\delta}(\bar{r}, \delta)$):

$$\begin{aligned}&\dot{\hat{x}}(t) = -\alpha\hat{x}(t) + \alpha B^T x(t),\\ &\hat{\varsigma}(t) = sat\left(\frac{\alpha(B^T x(t) - \hat{x}(t)) - B^T Ax(t) - \hat{\mu}_1\|B\|^2 u(t^-)}{\|B\|^2}, \bar{\delta}(\bar{r}, \delta)\right).\end{aligned} \tag{12}$$

where

$$sat(h, \alpha) = \begin{cases} \alpha, & \text{if } h \geq \alpha \\ h, & \text{if } -\alpha < h < \alpha \\ -\alpha, & \text{if } h \leq -\alpha. \end{cases}$$

Here, $\hat{x}$ is the estimation of $B^T x$ and then $\alpha(B^T x - \hat{x}) = \dot{\hat{x}}$ is the estimation of $B\dot{x}$. Note that here $\hat{\varsigma}$ is the estimation of $\mu_2\varsigma$ but not only the hysteresis state ς.

B. Design of Stabilizing Controllers

In the following subsection, we will focus on designing controllers that stabilize the hysteresis system described by equation (5) without considering the state constraints. We define the estimation error as $d = \mu_2\varsigma - \hat{\varsigma}$. As a result,

$$\dot{x} = Ax + B\left((\hat{\mu}_1 + \Delta\mu_1)u + \hat{\varsigma}\right) + Bd. \tag{13}$$

Let

$$u = \frac{1}{\hat{\mu}_1}(v - \hat{\varsigma}) \tag{14}$$

We have

$$\dot{x} = Ax + B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v + B\hat{d} \tag{15}$$

where $\hat{d} = d - \frac{\Delta\mu_1}{\hat{\mu}_1}\hat{\varsigma}$. Then, $\hat{d} \in L_2[0, t_f]$ for any $t_f > 0$ because that d and $\hat{\varsigma}$ are bounded. We need to devise a controller to ensure that the $\hat{d}$ minimally impacts the controlled output $z = Cx$. That is, we want to achieve the following L_2-gain requirement:

$$\int_0^{t_f} z^T(t)z(t)dt < \gamma^2\int_0^{t_f}\hat{d}^T(t)\hat{d}(t)dt + \hat{L}(\|x(0)\|), \quad \text{for all } t_f > 0, \tag{16}$$

for a specified value of $\gamma > 0$ and a certain class $\mathcal{K}$ function $\hat{L}$.

Lemma 1: There exists a control law $v = Fx$ for system (15) such that the closed-loop system is asymptotically stable under $\hat{d} = 0$ and satisfies the L_2-gain requirement (16), provided that a positive definite matrix X, a matrix M with appropriate dimensions, and a positive scalar κ satisfy the following LMI:

$$\begin{bmatrix} XA^T + AX + BM + M^TB^T & XC^T & B & \kappa\frac{\bar{\mu}}{\hat{\mu}_1}B & M^T \\ * & -I & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & -\kappa & 0 \\ * & * & * & * & -\kappa \end{bmatrix} < 0; \tag{17}$$

and, $v = Fx = MX^{-1}x$ is such that $A + BF$ is Hurwitz and requirement (16) holds.

Proof: Suppose that (17) holds. We have

$$\begin{aligned}&XA^T + AX + BM + M^TB^T + XC^TCX\\ &\quad + \frac{1}{\gamma^2}BB^T + \rho\frac{\bar{\mu}^2}{\hat{\mu}_1^2}BB^T + \frac{1}{\rho}M^TM < 0.\end{aligned}$$

Define $P = X^{-1}$ and $\varepsilon = \frac{1}{\kappa}$. It can be demonstrated that (17) indicates

$$\begin{aligned}&A^TP + PA + PM^TB^TP + PBMP + C^TC\\ &\quad + \left(\frac{1}{\gamma^2} + \frac{1}{\varepsilon}\frac{\bar{\mu}^2}{\hat{\mu}_1^2}\right)PBB^TP + \varepsilon PM^TMP < 0.\end{aligned} \tag{18}$$

Let $V_s(x) = x^TPx$. With $v = Fx = MPx$, we have

$$\begin{aligned}&\dot{V}_S(x) + z^Tz - \gamma^2\hat{d}^2\\ &= 2x^TP\dot{x} + z^Tz - \gamma^2\hat{d}^2\\ &= 2x^TP\left(Ax + B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v + B\hat{d}\right) + z^Tz - \gamma^2\hat{d}^2\\ &= 2x^TPAx + x^TC^TCx + 2x^TPB\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v\\ &\quad + \frac{1}{\gamma^2}x^TPBB^TPx - \gamma^2\left(\hat{d} - \hat{d}_*\right)^2\\ &\leq 2x^TPAx + x^TC^TCx + \frac{1}{\gamma^2}x^TPBB^TPx\\ &\quad + 2x^TPBMPx + 2\frac{\Delta\mu_1}{\hat{\mu}_1}x^TPBMPx\\ &\leq x^T(A^TP + PA + PBMP + PM^TB^TP\end{aligned}$$

$$+C^TC + \left(\frac{1}{\gamma^2} + \frac{1}{\varepsilon}\frac{\bar{\mu}^2}{\hat{\mu}_1^2}\right)PBB^TP + \varepsilon PM^TMP\Big)x$$
$$< 0, \forall x \in \mathcal{D}\backslash\{0\},$$

where

$$\hat{d}_* = \frac{1}{\gamma^2}B^TPx.$$

Consequently, we deduce the validity of (16). Therefore, in accordance with (18), we acquire the following.

$$(A+BF)^TP + P(A+BF) + C^TC + \left(\frac{1}{\gamma^2} + \frac{1}{\varepsilon}\frac{\bar{\mu}^2}{\hat{\mu}_1^2}\right)PBB^TP + \varepsilon PM^TMP < 0.$$

That is, $A+BF$ is Hurwitz because that (C, A) is detectable. This completes the proof. □

Upon acquiring the feedback gain $F = MX^{-1}$, the actual controller to hysteresis system (15) becomes

$$u = \frac{1}{\hat{\mu}_1}(v - \hat{\varsigma}) = \frac{1}{\hat{\mu}_1}Fx - \frac{1}{\hat{\mu}_1}\hat{\varsigma} \qquad (19)$$

where $\hat{\varsigma}$ is obtained by (12).

C. Design of State-Constrained Controllers

In this subsection, we investigate the development of state-constrained controllers tailored for managing the hysteresis system (5), in which the parameters governing hysteresis remain unknown.

Define

$$V(x) = x^TPx\left(1 + \sum_{i=1}^{q}\frac{\varepsilon_i}{S_ix + c_i}\right), \qquad (20)$$

where $P = X^{-1}$. The term $\sum_{i=1}^{q}\frac{\varepsilon_i}{S_ix+c_i}$ is added to ensure that $V(x) \to \infty$ as $x \to \partial\mathcal{D}$. Ensuring that the time derivative of $V(x)$is negative will prevent the state trajectory from crossing the boundary and entering the unsafe area. Let $\Delta V(x) = \frac{\partial V(x)}{\partial x}$. We have

$$\Delta V(x) = 2\left(1 + \sum_{i=1}^{q}\frac{\varepsilon_i}{S_ix+c_i}\right)x^TP - x^TPx\sum_{i=1}^{q}\frac{\varepsilon_i}{(S_ix+c_i)^2}S_i.$$

Theorem 1: Consider the system (1). Suppose that LMI (17) is feasible. A feedback law $v = p(x)$ exists such that the closed-loop hysteresis system is stable with respect to $\mathcal{D}$ when $\hat{d} = 0$ and meets the L_2-gain requirement (16), if there is an $\varepsilon > 0$ such that

$$\Delta V(x)Ax + x^TC^TCx + \left(\frac{1}{4\gamma^2} + \frac{1}{4\varepsilon}\left(\frac{\bar{\mu}^2}{\hat{\mu}_1^2} - 1\right)\right)(\Delta V(x)B)^2 < 0 \quad \forall x \in \mathcal{D}\backslash\{0\}. \qquad (21)$$

Proof: Under (21), we have

$$\begin{aligned}
&\dot{V}(x) + z^Tz - \gamma^2\hat{d}^2 \\
&= \Delta V(x)\dot{x} + z^Tz - \gamma^2\hat{d}^2 \\
&= \Delta V(x)\left(Ax + B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v + B\hat{d}\right) + z^Tz - \gamma^2\hat{d}^2 \\
&= \Delta V(x)Ax + x^TC^TCx + \Delta V(x)B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v \\
&\quad + \frac{1}{4\gamma^2}(\Delta V(x)B)^2 - \gamma^2(\hat{d} - \hat{d}_*)^2 \\
&\le \Delta V(x)Ax + x^TC^TCx + \frac{1}{4\gamma^2}(\Delta V(x)B)^2 \\
&\quad + \Delta V(x)B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)v \\
&\le \Delta V(x)Ax + x^TC^TCx + \frac{1}{4\gamma^2}(\Delta V(x)B)^2 \\
&\quad + \Delta V(x)Bv + \frac{1}{4\varepsilon}\frac{\bar{\mu}^2}{\hat{\mu}_1^2}(\Delta V(x)B)^2 + \varepsilon v^Tv \\
&= \Delta V(x)Ax + x^TC^TCx + \left(\frac{1}{4\gamma^2} + \frac{1}{4\varepsilon}\frac{\bar{\mu}^2}{\hat{\mu}_1^2}\right)(\Delta V(x)B)^2 \\
&\quad - \frac{1}{4\varepsilon}(\Delta V(x)B)^2 + \varepsilon(v - v_*)^T(v - v_*) \\
&\le \Delta V(x)Ax + x^TC^TCx + \left(\frac{1}{4\gamma^2} + \frac{1}{4\varepsilon}\left(\frac{\bar{\mu}^2}{\hat{\mu}_1^2} - 1\right)\right)(\Delta V(x)B)^2 \\
&< 0, \forall x \in \mathcal{D}\backslash\{0\},
\end{aligned}$$

where the worst-case disturbance is

$$\hat{d}_* = \frac{1}{2\gamma^2}\Delta V(x)B \qquad (22)$$

and the optimal controller is

$$v = v_* = p(x) = -\frac{1}{2\varepsilon}\Delta V(x)B. \qquad (23)$$

Then, according to the feedback law $v = p(x)$, we obtain

$$\dot{V}(x) < -z^Tz + \gamma^2\hat{d}^2, \forall x \in \mathcal{D}\backslash\{0\}. \qquad (24)$$

Based on (24), it is evident that (16) holds. Similar to the proof in [19], since $\hat{d}$ is bounded, equation (24) indicates that, throughout the state trajectory of $\dot{x} = Ax + B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)p(x) + B\hat{d}$ with the initial condition $x(0) \in \mathcal{D}\backslash\{0\}$, the function $V(x)$ remains bounded. Consequently, the state trajectory $x(t)$ is restricted from crossing the boundary of $\mathcal{D}$, due to $V(x) \to \infty$ as $x \to \partial\mathcal{D}$. When $\hat{d} = 0$ and $x \in \mathcal{D}\backslash\{0\}$, $\dot{V}(x) < -z^Tz$, indicating that the system $\dot{x} = Ax + B\left(1 + \frac{\Delta\mu_1}{\hat{\mu}_1}\right)p(x)$ is stable with respect to the $\mathcal{D}$. This concludes the proof. □

It is important to emphasize that the actual control signal to the hysteresis system (4) is

$$u = \frac{1}{\hat{\mu}_1}(v - \hat{\varsigma}) = \frac{1}{\hat{\mu}_1}p(x) - \frac{1}{\hat{\mu}_1}\hat{\varsigma}. \qquad (25)$$

where $\hat{\varsigma}$ is obtained by (12) and $p(x)$ is defined in (23).

IV. Numerical Example

To provide additional validation of the method's effectiveness in practical scenarios, we simulate the approach within a piezo-positioning mechanical system affected by hysteresis nonlinearity, as described in (1) and (2) and considered in [8]. The accompanying model is described as follows:

$$\boldsymbol{M}\ddot{y}(t) + \boldsymbol{D}\dot{y}(t) + \boldsymbol{F}y(t) = H(u), \; v = H(u), \qquad (26)$$

where $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$ correspond to the system's position, velocity, and acceleration, respectively, while u

represents the voltage signal applied to the piezo-positioning mechanism [19]. Additionally, $\boldsymbol{M}$, $\boldsymbol{D}$ and $\boldsymbol{F}$ denote the mass, damping coefficient, and stiffness coefficient, respectively. As in [19], we define $x_1(t) = \psi(t)$ and $x_2(t) = \dot{\psi}(t)$, then by selecting the state variables $x(t) = [x_1(t) \quad x_2(t)]^T$, we get the following state space model [8],

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\boldsymbol{F}}{\boldsymbol{M}} & -\frac{\boldsymbol{D}}{\boldsymbol{M}} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\boldsymbol{M}} \end{bmatrix} H(u). \qquad (27)$$

Here the actual parameters are chosen as $\boldsymbol{M} = 1\,kg$, $\boldsymbol{D} = 0.15\,Ns/m$ and $\boldsymbol{F} = 1\,N/m$ [8]. Moreover, for the simulation purpose, the unknown hysteresis parameters are provided as $\beta = 1$, $\chi = 0.5, r = 2$, $\bar{\mu} = 0.3$ and $\hat{\mu}_1 = 0.55$ and the remaining system parameter as $C = [1\ 0]$. To showcase the efficacy of the achieved outcomes, we will develop state-constrained controllers for the hysteresis system (27) with unknown hysteresis parameters. The state constraint is (the same as set in [19]):

$$x(t) \in \mathcal{D} \equiv \mathcal{D}_1 \cap \mathcal{D}_2 \cap \mathcal{D}_3, \qquad (28)$$

where $\mathcal{D} \equiv \{x \in R^3 | S_i x + c_i > 0, i = 1,2,3\}$ with $S_1 = [1 \quad 3], S_2 = [1 \quad -1], S_3 = [-3 \quad -1]$, $c_1 = 2, c_2 = 2, c_3 = 10$. In order to achieve the desired results, the following parameters have been chosen: $\alpha = 1000, \varepsilon_i = 0.001$, $\gamma = 1$ and $\kappa = \frac{1}{\varepsilon} = 1.6$. Solving LMI (17) results in the following solution:

$$P = \begin{bmatrix} 2.7954 & 0.7450 \\ 0.7450 & 2.7789 \end{bmatrix}.$$

In light of this,

$$F = [-1.1857 \ -4.4832].$$

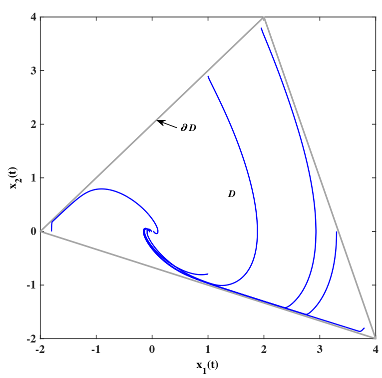
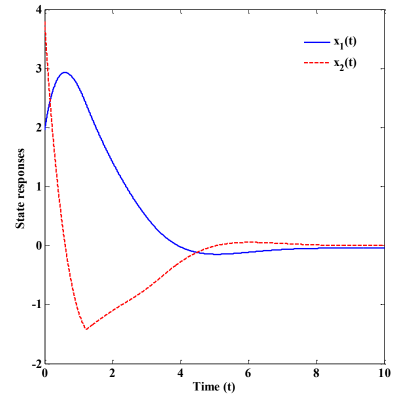
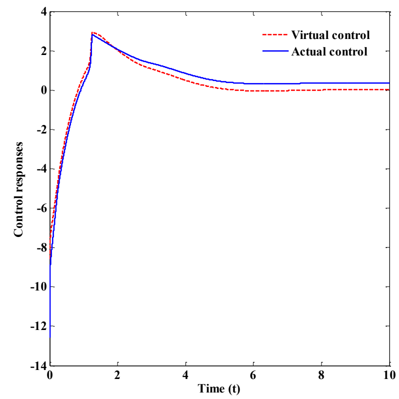


Fig. 1. State behavior of the state-constrained hysteresis system (27)-(25) under different initial conditions

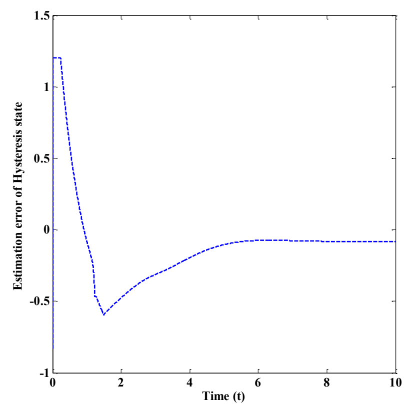
It can be verified that the condition in Theorem 1 holds. Fig. 1 depicts the state trajectories of the hysteresis system (27) under the control of (25), starting with several initial states in $\mathcal{D}$ that are close to $\partial\mathcal{D}$. Clearly, every state trajectory converges towards the origin while satisfying the state constraint (28). For the initial state $x(0) = [1.95 \quad 3.8]^T$, Fig. 2 depicts the state and input response of the closed-loop hysteresis system and the estimation error of the hysteresis state respectively.



(a) State response



(b) Control response



(c) Estimation error

Fig. 2. Responses of the closed-loop hysteresis system

For comparisons, with different values of κ, the state trajectories of the hysteresis system (27) with controller (25), controller (19), and without control are depicted in Fig. 3. The blue solid curves in Fig. 3 illustrate the state trajectories of the system (27) controlled by (25) that reach the origin and satisfy the state constraint (28). In this situation, the trajectories might approach $\partial\mathcal{D}$ closely but never breach into the unsafe region. The red dashed curves in Fig. 3 illustrate the state trajectories of the system (27) controlled by (19) that reach the origin but do not satisfy the state constraint (28). However, the green dashed curve in Fig. 3 makes it evident that the open-loop system's state trajectories cross through unsafe zones and violate the condition in (28). From Fig. 3, it is obvious that all trajectories are kept in the safe region when the state-constrained controller (25) is used and that no violations of the state-constrained law (28); however, in both the open- and the closed-loop system without considering the state constraints, trajectories breach into the unsafe region, which demonstrates the applicability of the proposed state-constrained controller design.

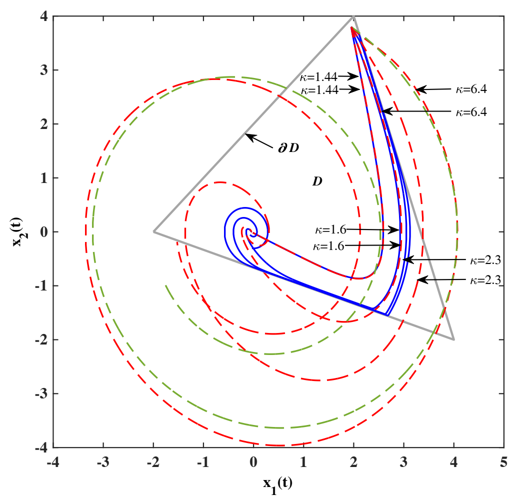


Fig. 3. State trajectories of the hysteresis system under closed- and open-loop control

V. Conclusions

Newly proposed state-constrained stabilizing control strategies have been introduced for a specific category of Bouc-Wen hysteresis control systems with unknown hysteresis parameters. New hysteresis estimators have been developed to estimate the unknown hysteresis state. State-constrained stabilizing controllers can be derived through the solution of LMIs and the incorporation of barrier functions. Ultimately, the efficacy of the suggested controller has been showcased via a numerical illustration. In the future, the suggested method can be expanded to accommodate time-varying hysteresis parameters. Should the parameters shift gradually, the proposed method can be applied with certain adjustments. However, in cases of rapid parameter changes, accurately estimating the hysteresis state will pose a significant challenge.

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