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Data-driven Event-Triggered Control for Discrete-time LTI Systems with Exogenous Inputs

Geethanjali Ravi*[1], Vijayanand Digge*[2] , Durgesh Singh[3] and Ramkrishna Pasumarthy[4]

***Abstract*— Many industrial processes, computing devices, and networks, such as traffic systems and power grids, exhibit complex dynamics. The model capturing the true dynamics may not be readily available for these systems. Due to advanced sensing technologies, it is possible to collect large amounts of real-time data. Leveraging this data offers an opportunity to design data-driven control without constructing an explicit model for the system. In this paper, we derive data-dependent matrices from an ensemble of input-state trajectories to parameterize the closed-loop Linear Time-Invariant (LTI) system, accounting for exogenous inputs. The stabilizing control law is designed in such a way that it gets updated based on events, where an event refers to the violation of certain performance conditions. The results proposed are implemented on a computing system, in particular demonstrating auto-scaling of web servers hosted on a private cloud.**

Keywords: Data-driven control, Event-triggered control, LTI System, Virtual machine scheduling, Web-servers

I. INTRODUCTION

Many complex systems such as power grids [1], computing systems [2], [3], asynchronous machine systems [4], have exogenous inputs that cannot be manipulated, and affect the system's performance. In literature, researchers have employed various strategies to design control laws for systems with exogenous inputs. For instance, the authors in [5] have designed appropriate feedback gains to nullify the effect of the exogenous inputs, which involves designing different gains, one for controlling the system and the other for canceling the effect of exogenous input. In [1], the cost of exogenous input is added to minimize the overall quadratic cost function under optimal control formulation.

These approaches depend on having precise system models, typically derived from first principles or system identification methods. In many cases, like in biological or computing systems, a model derived from first principles may not be readily available. In other cases where models are available, the system size and complexities may hinder using the first principle-based models. For systems that generate a rich amount of data, data-driven approaches for controller design and implementation have gained significant traction. This has found wide acceptance in the community, owing to the simplified procedure of designing high-performance control systems without identifying a model [6].

An extension of this approach, for the case of linear parameter varying (LPV) systems, is discussed in [7]. The control of complex networks without the knowledge of the network dynamics is studied in the data-driven framework in [8] and [9]. Many data-driven formulations and control design methods result in semi-definite programs using data-dependent LMI conditions as in [10]. However, this article does not consider systems with exogenous inputs under a data-driven framework. In a recent work [11], a data-driven LQR controller is designed, considering explicitly the effect of exogenous inputs. This approach attenuates the effect of exogenous inputs on the states and minimizes the net costs associated with the control efforts and state deviations.

The popular approach for the digital implementation of controllers is to update the control in a time-triggered or periodic manner [12]. This approach is often conservative and does not fully account for resource limitations. The main objective is to determine how frequently the control law needs to be executed to achieve the desired system performance with judicious utilization of resources. Instead of a periodic implementation, event-based control and communication are the techniques employed to optimize resource utilization, a topic that has been extensively explored within the control systems community. The key idea behind event-based strategies is to employ resources or implement changes only when certain stability or performance conditions are violated. [13].

Event-based control involves the continuous or periodic evaluation of a condition known as the triggering condition. The thresholding conditions are designed to capture unforeseen alterations in measurements, parameters crossing some specified thresholds, or faults, leading the system towards instability [14]. When these conditions are breached, feedback signals are prompted to be transmitted to the controller, triggering the update of the control system. The authors in [15] show that resource utilization is better in event-based implementation.

This motivates us to generalize data-driven event-triggered control techniques to be applicable to systems with exogenous inputs. Many real-world processes incorporate exogenous inputs as a fundamental part of the system. It is crucial that the data acquisition phase and the closed-loop operations remain unaffected by the presence of these exogenous inputs. In this context, we present a data-driven approach to design state-feedback event-triggered controllers for stabilizable discrete-time linear systems, with exogenous

Department of Electrical Engineering, IIT Madras, Tamil Nadu, India
[1]Geethanjali Ravi - ee21s085@smail.iitm.ac.in
[2]Vijayanand Digge- vijayanand4085@gmail.com
[3]Durgesh Singh- 257durgesh@gmail.com
[4]Ramkrishna Pasumarthy- ramkrishna@ee.iitm.ac.in
[4] Also with the Robert Bosch Center for Data Science and AI at IIT Madras
* Equal contribution by both authors

inputs whose explicit model is unknown. The exogenous input data is conscientiously considered both during data acquisition and in the implementation phase. The main contributions of this paper are as follows:

1) Relative thresholding-based event-triggered control is designed for discrete-time linear systems with exogenous inputs operating within a data-driven framework.
2) A data-driven controller is designed to accommodate exogenous inputs and performance specifications. These constitute extensions of the results in [10] and are crucial in designing a relative thresholding-based event-triggered implementation.
3) We validate the results through an experimental setup: a web-server system without a readily available model.

II. Notation and Preliminaries

A. Notation

The notation $\mathbb{R}$ denotes the set of real numbers. The Euclidean vector norm or the induced matrix norm is represented by $\|\cdot\|$, depending on its argument. I refers to the identity matrix with an appropriate dimension. $\mathbb{S}^n$ denotes the set of all $n \times n$ real symmetric matrices.

B. Preliminaries

Definition 2.1 ([10]): Given a signal, $z : \mathbb{Z} \in \mathbb{R}^p$, a vectorized form of z in the restricted interval $[k, k+T] \cap \mathbb{Z}$ is denoted by $z_{[k,k+T]}$ where $k \in \mathbb{Z}, T \in \mathbb{N}$. Furthermore, the signal $z_{[0,T-1]} \in \mathbb{R}^p$ is considered persistently exciting of order L if the Hankel matrix associated with $z_{[0,T-1]}$,

$$\mathrm{Z}_{0,L,T-L+1} = \begin{bmatrix} z(0) & z(1) & \dots & z(T-L) \\ z(1) & z(2) & \dots & z(T-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ z(L-1) & z(L) & \dots & z(T-1) \end{bmatrix}$$

has a full rank pL.

For a signal z to be persistently exciting of order L, it must be sufficiently long with the number of samples $T \geq (p+1)L-1$. The next section delves into concepts from the literature used in deriving the results presented in this article.

C. Persistency of Excitation

Consider a discrete-time linear system of the form

$$x(k+1) = Ax(k) + Bu(k) + Ew(k) \tag{1}$$

where, $x(k) \in \mathbb{R}^n$ denotes the state, $u(k) \in \mathbb{R}^{m_1}$ denotes the control inputs and $w(k) \in \mathbb{R}^{m_2}$ denotes the exogenous inputs to the system. $w(k)$ is assumed to be bounded, that is,

$$\|w(k)\| \leq \gamma \|x(k)\|, \ \forall\ k \geq 0, \tag{2}$$

where $\gamma > 0$. This manuscript assumes that the system matrices are unknown and that only the state and the input measurements from open-loop experiments are available. It is also assumed that the system is controllable and that the values of $x(\cdot)$ and $w(\cdot)$ are known or can be predicted at each time instant. Let $\{x(k)\}_{k=0}^{k=T}$ denote the sequence of states collected during an open-loop experiment from application of input sequences $\{u(k)\}_{k=0}^{k=T-1}$ and $\{w(k)\}_{k=0}^{k=T-1}$, where T denotes the final time of the experiment.

Employing the structure of the Hankel matrix for input and state data collected, we construct the following data matrices

$$U = \begin{bmatrix} u(0) & \dots & u(T-1) \end{bmatrix}, W = \begin{bmatrix} w(0) & \dots & w(T-1) \end{bmatrix}$$
$$X_- = \begin{bmatrix} x(0) & \dots & x(T-1) \end{bmatrix}, X_+ = \begin{bmatrix} x(1) & \dots & x(T) \end{bmatrix}.$$

The notations mentioned above are used in the rest of this paper. The results in [10] guarantees the rank condition

$$\operatorname{rank} \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top = n + m_1 + m_2 \tag{3}$$

under the assumption that $\{u(k)\}_{k=0}^{k=T-1}$ and $\{w(k)\}_{k=0}^{k=T-1}$ are persistently exciting input sequences. In the open-loop experiment, $T > (m_1+m_2+1)n + (m_1+m_2)$ is necessary for the persistency of excitation to hold. The rank condition (3) implies that a finite set of system trajectories can represent a linear system's whole set of trajectories.

D. Event-triggered Control

Event-triggered control is a control strategy that offers the potential to reduce the communication and computation load in networked control systems, thereby enhancing their efficiency and responsiveness. It is a valuable approach in scenarios where resource efficiency and reduced data transmission are essential. At the same time, careful engineering and analysis are required to ensure system stability and desired performance. In event-triggered control, the triggering condition is periodically checked, and upon violation, the feedback signals are sent to the controller for update. Thus, in event-triggered control, the control input is held constant until the next event occurrence,

$$u(k) = Kx(k_i), \quad k \in [k_i,\ k_{i+1}) \tag{4}$$

where k_i and k_{i+1} are the event instants. The nature of control update (4) induces a state measurement error

$$e(k) = x(k_i) - x(k) \tag{5}$$

where $x(k_i)$ is the last state instant of control update and $x(k)$ is current value of state. In this work, the relative thresholding event condition is considered of the form

$$\|e(k)\| \leq \sigma \|x(k)\| \tag{6}$$

where $\sigma > 0$ is the threshold parameter.

III. Problem Formulation

Consider the system (1) with state-feedback control law,

$$u(k) = Kx(k) \tag{7}$$

where K is the feedback gain matrix. Now, the closed loop system is given by,

$$x(k+1) = (A+BK)x(k) + Ew(k). \tag{8}$$

In an event-triggered framework, the control signal is updated whenever condition (6) is violated. Now, $\forall\, k \in [k_i, k_{i+1})$, the closed-loop system (8) under event-triggered controller (4) with state measurement error (5) is described by

$$x(k+1) = (A+BK)x(k) + BKe(k) + Ew(k). \quad (9)$$

We formulate the data-driven event-triggered control problem for discrete-time LTI systems with exogenous inputs. As the matrices (A, B, E) are unknown, we derive a data-based representation of the closed-loop system dynamics (9). This representation is instrumental in formulating an event-triggered control law. The goal is to ensure the stability and optimal performance of the system under exogenous inputs.

IV. Data-driven Representation of Closed-loop System

In this section, the equivalent data-driven representation of the system (9) without state measurement errors is derived, enabling the design of a data-driven stabilizing controller.

Lemma 1: The equivalent data-driven representation of the system (8) is

$$x(k+1) = X_+ G_1 x(k) + X_+ H w(k) \quad (10)$$

where G_1 and H are matrices satisfying

$$\begin{bmatrix} I & K^\top & 0 \\ 0 & 0 & I \end{bmatrix}^\top = \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top \begin{bmatrix} G_1 & H \end{bmatrix}. \quad (11)$$

Proof: The rank condition (3) implies that the matrix $\begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top$ has full row rank and therefore there always exist matrices G_1 and H satisfying (11). System (8) can be written as

$$\begin{aligned} x(k+1) = &\begin{bmatrix} A & B & E \end{bmatrix} \begin{bmatrix} I & K^\top & 0 \end{bmatrix}^\top x(k) \\ &+ \begin{bmatrix} A & B & E \end{bmatrix} \begin{bmatrix} 0 & 0 & I \end{bmatrix}^\top w(k). \end{aligned}$$

Using (11), the closed-loop system has the equivalent data-driven representation:

$$\begin{aligned} x(k+1) = &\begin{bmatrix} A & B & E \end{bmatrix} \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top G_1 x(k) \\ &+ \begin{bmatrix} A & B & E \end{bmatrix} \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top H w(k). \end{aligned}$$

Note that $X_+ = \begin{bmatrix} A & B & E \end{bmatrix} \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top$. Therefore, $x(k+1) = X_+ G_1 x(k) + X_+ H w(k)$. ■

On similar lines as Lemma 1, the data-driven equivalent representation of the closed-loop system with state measurement errors (9) is arrived at as

$$x(k+1) = X_+ G_1 x(k) + X_+ G_2 e(k) + X_+ H w(k) \quad (12)$$

where G_2 satisfies

$$\begin{bmatrix} 0 & K^\top & 0 \end{bmatrix}^\top = \begin{bmatrix} X_-^\top & U^\top & W^\top \end{bmatrix}^\top G_2. \quad (13)$$

Next, we derive conditions under which the closed-loop system (12) is exponentially stable. The problem of interest here is to design an appropriate state-feedback control law (4) to maintain system stability within an event-triggered framework. After introducing the concepts of event-triggered control and the equivalent data-driven representation of a closed-loop system with state measurement errors, the next section presents an optimization problem to determine the suitable range for the thresholding parameter σ.

Optimizing thresholding parameters within event-triggered feedback control is essential to balance control effectiveness and resource efficiency. Notably, the triggering parameters in the data-driven event-triggered scheme are predefined, eliminating the need for repetitive experiments to optimize this parameter, σ. This predefined setup ensures the stability of the system (12) under the state-feedback controller (4) meeting a given performance parameter λ and with a bounding parameter γ of the exogenous input.

V. Data-driven Controller Design and Event-triggered Implementation

In the state-feedback design paradigm, it is assumed all the inputs to the system are control inputs, which means that the controller can manipulate them. However, in practical scenarios, this may not always be the case, and some inputs to the system may be from external sources that can neither be disabled nor manipulated.

In this section, state-feedback gain K as in (7) is designed for (10) in the presence of exogenous inputs and then relative threshold parameter σ of (6) for event-triggered implementation of the controller is calculated.

A. Computation of State-feedback Gain

Theorem 1: Let rank condition (3) hold. For a given γ, and performance parameter λ, let Q_1 be any matrix satisfying

$$\begin{bmatrix} (1-\lambda)X_- Q_1 & 0 & Q_1^\top X_+^\top & X_- Q_1 \\ 0 & qI_n & Q_2^\top X_+^\top & 0 \\ X_+ Q_1 & X_+ Q_2 & X_- Q_1 & 0 \\ X_- Q_1 & 0 & 0 & \frac{q}{\gamma} I_n \end{bmatrix} \succeq 0 \quad (14)$$

$$q > 0,\; WQ_1 = 0,\; \begin{bmatrix} X^\top & U^\top & W^\top \end{bmatrix}^\top Q_2 = \begin{bmatrix} 0 & 0 & qI \end{bmatrix}^\top.$$

Then, the stabilizing state-feedback gain for (10) is given by

$$K = UQ_1 (X_- Q_1)^{-1}. \quad (15)$$

Proof: Consider a candidate Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$ defined by $V(x(k)) = x^\top(k) P x(k)$, $P = P^\top \succ 0$. To prove stability of the system (10), we need $P \succ 0$ and the evolution of the Lyapunov function satisfies $V(k+1) - V(k) \leq -\lambda V(k)$ for all x with $\lambda > 0$. Plugging the value of $V(k)$ and $V(k+1)$ along the trajectories of the closed-loop system (10), we obtain

$$\begin{aligned} &V(k+1) - (1-\lambda)V(k) \\ &\quad = x(k+1)^\top P x(k+1) - (1-\lambda) x^\top(k) P x(k) \\ &\quad = \begin{bmatrix} x \\ w \end{bmatrix}^\top F_2 \begin{bmatrix} x \\ w \end{bmatrix} \end{aligned}$$

where $F_2 = \begin{bmatrix} G_1^\top X_+^\top P X_+ G_1 - (1-\lambda)P & G_1^\top X_+^\top P X_+ H \\ H^\top X_+^\top P X_+ G_1 & H^\top X_+^\top P X_+ H \end{bmatrix}$, and the exogenous input w is bounded as given by (2). To guarantee stability, it is required to get a $P \succ 0$ and

$$\begin{bmatrix} x \\ w \end{bmatrix}^\top F_2 \begin{bmatrix} x \\ w \end{bmatrix} \leq 0, \text{ when } \begin{bmatrix} x \\ w \end{bmatrix}^\top \begin{bmatrix} -\gamma I_n & 0 \\ 0 & I_n \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \leq 0. \quad (16)$$

The S-procedure [16] ensures that (16) holds. Thus, the sufficient condition for the existence of a quadratic Lyapunov function is given by

$$\varepsilon\begin{bmatrix}-\gamma I_n & 0\\ 0 & I_n\end{bmatrix}-F_2\succeq 0 \tag{17}$$

for some $\varepsilon\geq 0$. Applying Schur's complement, we obtain

$$\begin{bmatrix}-\varepsilon\gamma I_n+(1-\lambda)P & 0 & G_1^\top X_+^\top\\ 0 & \varepsilon I_n & H^\top X_+^\top\\ X_+G_1 & X_+H & P^{-1}\end{bmatrix}\succeq 0.$$

Pre and post-multiplying by the positive definite matrix diag$(P^{-1},\ I_n,\ I_n)$, and applying Schur's complement again,

$$\begin{bmatrix}(1-\lambda)P^{-1} & 0 & P^{-1}G_1^\top X_+^\top & P^{-1}\\ 0 & \varepsilon I_n & H^\top X_+^\top & 0\\ X_+G_1P^{-1} & X_+H & P^{-1} & 0\\ P^{-1} & 0 & 0 & \frac{\varepsilon^{-1}}{\gamma}I_n\end{bmatrix}\succeq 0.$$

Pre and post-multiplying by the positive definite matrix diag$(I_n,\ \varepsilon^{-1}I_n,\ I_n,\ I_n)$, we obtain

$$\begin{cases}\begin{bmatrix}(1-\lambda)P^{-1} & 0 & P^{-1}G_1^\top X_+^\top & P^{-1}\\ 0 & \varepsilon^{-1}I_n & \varepsilon^{-1}H^\top X_+^\top & 0\\ X_+G_1P^{-1} & X_+H\varepsilon^{-1} & P^{-1} & 0\\ P^{-1} & 0 & 0 & \frac{\varepsilon^{-1}}{\gamma}I_n\end{bmatrix}\succeq 0\\ X_-G_1P^{-1}=P^{-1},\ UG_1P^{-1}=KP^{-1}\end{cases}$$

where the two equality constraints are obtained from (11) after multiplying both sides with P^{-1}. We define the change of variables, $Q_1=G_1P^{-1}$, $Q_2=\varepsilon^{-1}H$, $q=\varepsilon^{-1}$. Then, we have $P^{-1}=X_-Q_1$ from the equality constraint. Thus, we arrive at an LMI (14) by incorporating all the variables, while the remaining equality constraints $WQ_1=0$ and (11) are satisfied a posteriori with the choice $K=UQ_1(X_-Q_1)^{-1}$. ■

Theorem 1 provides a sufficient condition guaranteeing the stability of system (10). The next section implements the controller (15) under an event-triggered framework (6).

B. Design of Relative Thresholding Parameter

This section solves an optimization problem, obtaining a range for the thresholding parameter (σ), ensuring the stability of system (12) under event-triggered implementation. We maximize the relative thresholding parameter, leading to larger inter-event times and better utilization of resources.

Theorem 2: Let the rank condition (3) hold. Then, the relative thresholding parameter of event-triggered implementation (6) with controller (15), obtained by solving

$$\max_{q_2,\ Q_2,\ Q_3}\quad\sigma \tag{18}$$

subject to

$$\begin{bmatrix}(1-\lambda)X_-Q_1 & 0 & 0 & Q_1^\top X_+^\top & X_-Q_1 & \sigma X_-Q_1\\ 0 & q_2I_n & 0 & Q_3^\top X_+^\top & 0 & 0\\ 0 & 0 & q_1I_n & Q_2^\top X_+^\top & 0 & 0\\ X_+Q_1 & X_+Q_3 & X_+Q_2 & X_-Q_1 & 0 & 0\\ X_-Q_1 & 0 & 0 & 0 & \frac{q_1}{\gamma}I_n & 0\\ \sigma X_-Q_1 & 0 & 0 & 0 & 0 & q_2I_n\end{bmatrix}\succeq 0$$

$$\begin{bmatrix}X^\top & U^\top & W^\top\end{bmatrix}^\top Q_3=\begin{bmatrix}0 & q_2K^\top & 0\end{bmatrix}^\top$$
$$\begin{bmatrix}X^\top & U^\top & W^\top\end{bmatrix}^\top Q_2=\begin{bmatrix}0 & 0 & q_1I^\top\end{bmatrix}^\top,$$

stabilizes the system (12).

Proof: Using the equivalent data-driven representation of a closed-loop system under an event-triggered framework (6), the difference of Lyapunov function $V(k+1)-(1-\lambda)V(k)$ along the trajectories of the system (12) is

$$V(k+1)-(1-\lambda)V(k)=y^\top F_1y \tag{19}$$

where $F_1=\begin{bmatrix}G_1^\top X_+^\top PX_+G_1-(1-\lambda)P & G_1^\top X_+^\top PX_+G_2 & G_1^\top X_+^\top PX_+H\\ G_2^\top X_+^\top PX_+G_1 & G_2^\top X_+^\top PX_+G_2 & G_2^\top X_+^\top PX_+H\\ H^\top X_+^\top PX_+G_1 & H^\top X_+^\top PX_+G_2 & H^\top X_+^\top PX_+H\end{bmatrix}$.

and $y:=\begin{bmatrix}x^\top & e^\top & w^\top\end{bmatrix}^\top$. The state measurement error $e(k)$ shown in (5) and exogenous input $w(k)$ satisfy (6) and (2) respectively. To guarantee stability of the system (12), we need $P\succ 0$ and $y^\top F_1y\leq 0$, when

$$y^\top\begin{bmatrix}-\gamma I_n & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & I_n\end{bmatrix}y\leq 0,\ y^\top\begin{bmatrix}-\sigma^2I_n & 0 & 0\\ 0 & I_n & 0\\ 0 & 0 & 0\end{bmatrix}y\leq 0 \tag{20}$$

where both inequalities in (20) are obtained by recasting (2) and (6) in quadratic form, respectively, using the S-procedure. Thus, the sufficient condition for the existence of a quadratic Lyapunov function is given by (21) for some $\tau_1\geq 0,\ \tau_2\geq 0$.

$$\begin{bmatrix}G_1^\top X_+^\top PX_+G_1-(1-\lambda)P & G_1^\top X_+^\top PX_+G_2 & G_1^\top X_+^\top PX_+H\\ G_2^\top X_+^\top PX_+G_1 & G_2^\top X_+^\top PX_+G_2 & G_2^\top X_+^\top PX_+H\\ H^\top X_+^\top PX_+G_1 & H^\top X_+^\top PX_+G_2 & H^\top X_+^\top PX_+H\end{bmatrix}$$
$$+\tau_1\begin{bmatrix}\gamma I_n & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -I_n\end{bmatrix}+\tau_2\begin{bmatrix}\sigma^2I & 0 & 0\\ 0 & -I & 0\\ 0 & 0 & 0\end{bmatrix}\preceq 0. \tag{21}$$

On simplification of (21) and with the application of Schur's complement to the result, we get

$$\begin{bmatrix}(1-\lambda)P-\tau_2\sigma^2I_n-\tau_1\gamma I_n & 0 & 0 & G_1^\top X_+^\top\\ 0 & \tau_2I_n & 0 & G_2^\top X_+^\top\\ 0 & 0 & \tau_1I_n & H^\top X_+^\top\\ X_+G_1 & X_+G_2 & X_+H & P^{-1}\end{bmatrix}\succeq 0.$$

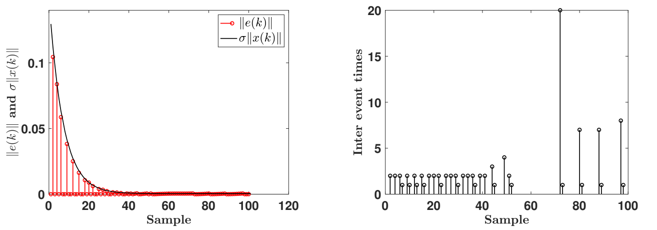
Pre and post-multiplying by the positive definite matrix diag $(P^{-1},\ I_n,\ I_n,\ I_n)$ and applying Schur's complement again,

$$\begin{bmatrix}(1-\lambda)P^{-1} & 0 & 0 & P^{-1}G_1^\top X_+^\top & P^{-1} & \sigma P^{-1}\\ 0 & \tau_2I_n & 0 & G_2^\top X_+^\top & 0 & 0\\ 0 & 0 & \tau_1I_n & H^\top X_+^\top & 0 & 0\\ X_+G_1P^{-1} & X_+G_2 & X_+H & P^{-1} & 0 & 0\\ P^{-1} & 0 & 0 & 0 & \frac{\tau_1^{-1}}{\gamma}I_n & 0\\ \sigma P^{-1} & 0 & 0 & 0 & 0 & \tau_2^{-1}I_n\end{bmatrix}\succeq 0.$$

Pre- and post-multiplying by $(I_n,\tau_2^{-1}I_n,\tau_1^{-1}I_n,I_n,I_n,I_n)$ and letting $Q_3=\tau_2^{-1}G_2,\ Q_2=\tau_1^{-1}H,\ q_1=\tau_1^{-1},\ q_2=\tau_2^{-1}$ we obtain the inequality in (18).

Maximizing σ over the free variables q_2, Q_2 and Q_3 leads to the optimization problem (18), where the condition on G_2 from (13) is satisfied a posteriori as equality constraints. The parameter Q_1 is obtained from solving (14). ■

Therefore, any $\sigma\in(0,\sigma^*]$ stabilizes the system (12) under event-triggered framework, where σ^* is the maximum threshold parameter.



(a) Evolution of error norm and threshold

(b) Inter event times between each controller updates

Fig. 1: Simulation results of the event-triggered implementation

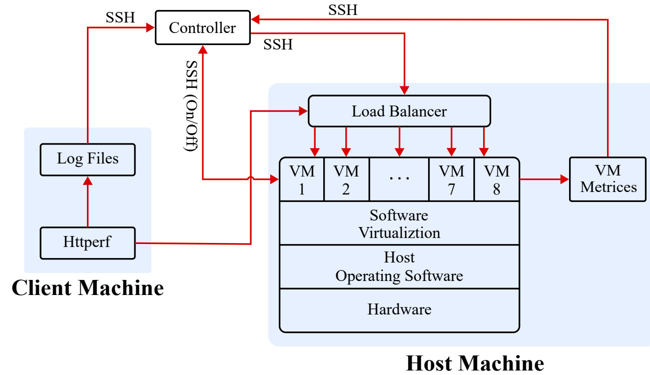


Fig. 2: Experimental testbed of web-server system

C. Numerical Example

The above results are validated using an illustrative example of a discrete-time linear system of the form (1) with

$$A = \begin{bmatrix} 1.0018 & 0.001 \\ 0.36 & 1.0018 \end{bmatrix}, \; B = E = \begin{bmatrix} -0.10 \\ -0.184 \end{bmatrix} \quad (22)$$

The system to be controlled is open-loop unstable with poles at 1.0208 and 0.9828. The open-loop experiment is carried out by applying random input and exogenous input sequence of length $T = 30$, while the system states data is collected using MATLAB. CVX [17] is used to solve (14) with design parameter $\lambda = 0.001, \gamma = 1$, and obtain $K = [4.3012 \;\; 2.4988]$, which places the location of the closed-loop poles at 0.8849 and 0.2288. Considering that λ is associated with the exponential stability of the system and γ represents the bound on exogenous input, it is imperative to select small values for these parameters to yield compact LMI (Linear Matrix Inequality) gains. Further, the optimization problem (18) is solved by applying the state-feedback controller K in an event-triggered setup to obtain $\sigma^* = 0.1498$. The simulation results are presented. In Fig. 1a-1b, simulation results with artificial injection of the exogenous input $w(k) = \sin(\frac{2\pi}{1000}k)$ are presented. In Fig 1a, the red stems represent the evolution of the norm of the error $e(k)$ defined in (5), which stays below the threshold $\sigma^*\|x(k)\|$ represented by the black line. Fig 1b shows that non-trivial triggering is obtained, with maximum inter-event times of 20 samples and an average inter-event-time at 3 samples.

VI. Application: Auto-scaling in Cloud

In recent years, cloud computing has experienced remarkable growth, with approximately 83% of companies delegating their computing needs to cloud-based solutions. The appeal of cloud computing lies in its ability to offer subscribers a versatile array of on-demand resources, including computing capabilities, storage, and network services. Moreover, cloud platforms provide a critical feature known as elasticity, permitting subscribers to acquire and release instances dynamically.

A. States and Inputs of the System

The state variables describing the dynamics of a web-server system encompass the mean physical core utilization, denoted as $CPU(k)$, expressed as a percentage across all physical cores. The mean response time, represented as $RT(k)$, is measured in milliseconds (ms) across all Virtual Machines (VMs) in the system. Additionally, we introduce the exogenous input variable for our model, the request rate of the workload denoted as $WOR(k)$, quantified in requests per second (reqs/s). This input is considered an exogenous signal in the design of our controller. Conversely, the control input signifies the number of active virtual machines within the system, labeled as $VM(k)$. These can be written as:

$$x(k) = [CPU(k) \;\; RT(k)]^\top, \; u(k) = VM(k), \; w(k) = WOR(k).$$

A feedback control law can be implemented on the web-server system to ensure the allocation of an appropriate number of VMs and the adjustment to varying workloads while adhering to predefined performance criteria. Specifically, the goal is to guarantee that the response time remains below a defined threshold denoted as RT_f. Providing adequate VMs is necessary to address the over-provisioning or under-provisioning of VMs and maintain the desired response time. Consequently, we rely on the design of a feedback control law to ensure the performance guarantee of the web-server system. Given the unavailability of a model, we prefer a data-driven approach over a model-based one to design the feedback controller gains in this article.

B. Web-Server Testbed

Our experimental setup (Fig. 2) simulates real-world network conditions. The client machine, with 16 GB RAM and four 3.2GHz CPU cores, uses httperf to generate HTTP requests. The web-server host, running Ubuntu OS with 32 GB RAM, hosts 8 VMs allocated with one core and 2 GB RAM each. An HAProxy load balancer evenly distributes requests among VMs. A separate machine acts as a controller device and runs the MATLAB script to execute the control algorithm. All machines are communicating with each other via a dedicated 1 Gbps Ethernet connection.

C. Open-Loop Data-Acquisition

We begin by conducting an open-loop experiment to gather data from the web-server system. We systematically vary the workload and number of active VMs to create a comprehensive dataset covering a wide spectrum of operating conditions for the web-server system.

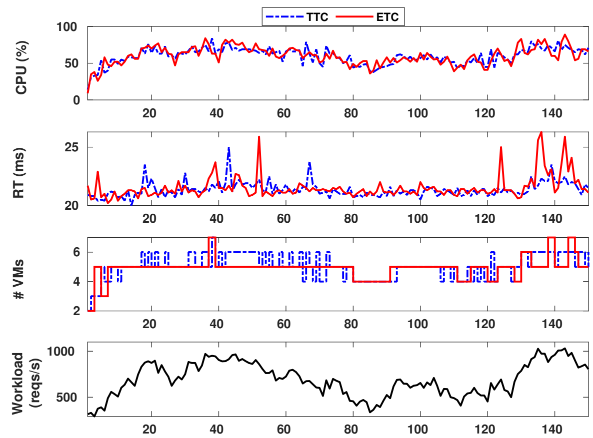


Fig. 3: Performance of data-driven controller

We make the following assumptions for the data acquisition process: a) we ensure that no CPU or memory-intensive background processes are running within the system to maintain a controlled environment; b) We establish a crucial threshold for the incoming request rate to prevent system overload, saturation, and potential failure.

D. Results

The data collected from the system is found to be persistently exciting, and the rank condition (3) holds. The assumptions of Theorem 1 are satisfied, and matrix Q_1 is obtained using CVX [17]. The state-feedback gain K that asymptotically stabilizes the system is found to be $K = \begin{bmatrix}0.0652 & 0.0303\end{bmatrix}$. To obtain the relative thresholding parameter σ^*, the optimization problem (18) is solved using CVX, and σ^* is found to be 0.24. Thus, the range of σ guaranteeing asymptotic stability is $(0\ ,\ 0.24]$.

We employ two distinct implementation frameworks to evaluate the controller's performance: time-triggered and event-triggered approaches. The time-triggered implementation updates the control law at each specified sample time. The performance assessment of our proposed method is illustrated in Fig. 3. Our objective is to achieve a response time (RT) under $30ms$. The desired RT is achieved when implemented, which proves the effectiveness of the proposed event-triggered approach. The Fig. 3 highlights the close alignment between the event-triggered and time-triggered implementations. It is evident that the event-based controller ensures infrequent alterations to the control law. Specifically, we observed only 19 updates for the given workload, whereas the time-triggered approach updated the control law 150 times. This results in significant savings in communication channel bandwidth and unnecessary updates of the control law. This result affirms the effectiveness of the data-driven event-triggered implementation, especially in complex systems such as cloud computing.

VII. Conclusion

In this work, an event-triggered implementation of controllers for discrete-time linear systems with exogenous input in a data-driven framework is introduced. The methods presented do not necessitate explicit model identification and rely on finite system data collected from open-loop experiments. We first established a data-driven representation of the closed-loop system, accounting for measurement errors. Subsequently, this representation was employed to design a relative threshold using S-procedure for event-triggered control implementation, with the relative threshold parameter determined through the resolution of an optimization problem involving data-dependent LMI constraints. The efficacy of the proposed approach was validated through numerical simulations and real-world experimentation on a cloud-hosted web-server system.

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