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A distributed optimization approach for the adaptation of underwater acoustic communication protocols

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Abstract— **We propose a novel multi-agent approach for auto-adjusting OFDM parameters for underwater acoustic communication that utilizes distributed optimization to perform a collaborative choice. The algorithm enhances overall communication performance among all agents, and makes its decision based on environmental information that is first actively collected from each agent at the beginning of their mission, and then communicated via opportune statistics of such sampled information. The proposed method does not rely on link feedback from receivers; while based on distributed optimization (and thus requiring data transmission among the agents), the approach does not introduce any overhead during data transmission and can be used as a separate process at any preferred moment prior to the data transmission. We present numerical comparisons based on simulation results to demonstrate the dependence of the effectiveness of the proposed approach with respect to different marine conditions that may be encountered in field missions, and the dependence of its efficiency on which optimization algorithm is chosen. The overall results indicate that for a various set of conditions the approach may lead to a more effective usage of the underwater acoustic channel.**

I. Introduction

Acoustic communication is the primary means of communication underwater; however, due to its specific characteristics, such as limited communication bandwidth and time-varying multipath propagation, underwater acoustic communication can be quite challenging [1]. Considering the mentioned limitations and challenges, it is important to adjust the parameters of the used communication protocol to maximize its performance for the desired application and mission. One may then either adjust such parameters offline, i.e., based on the sea condition forecasts, or adapt them online, i.e., on the field and after having observed the sea conditions directly. This dichotomy raises a natural question: the online approach may take additional time, especially if the decision is performed distributedly and autonomously by the underwater agents themselves, and this additional time could be critical for missions where timely decision-making is crucial (e.g., a maritime mine countermeasures focused deployment). The question is thus whether it is worth spending time to sample the sea conditions, distributedly calculate a new set of communication parameters, and implement such a decision.

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Among different studies that focus on underwater communication enhancements, some target calculating and adjusting the communication setting before the mission. This adjustment can be primarily based on the information about the mission environment and conditions, which can be provided either by prediction based on historical data or by performing several trials or simulating the experiments to achieve the most efficient possible configuration. Similar work has been presented in [2]. Although the mentioned method can be helpful, it requires prior knowledge of the mission and its environment, which might not be available in all scenarios. Therefore different adaptation methods developed to improve communication by readjusting the communication parameters considering different factors. One common approach is to apply the parameter adjustment based on feedback from the receiver. The effectiveness of these approaches has been analyzed in different studies, both with simulated data such as [3], [4], and experimental results [5]. Since the mentioned methods mainly rely on feedback from the receiver, they require overhead for transferring the feedback data with the data transmission, which might not be the best choice, considering the limited bandwidth underwater. Also, this type of channel adaptation might not consider the overall network performance, but rather the performance of a specific link. On the other hand, many other studies utilize other techniques to improve underwater communication performance. For instance, [6] adjusts the power levels of subcarriers using game theory in a multi-agent fashion. Some other studies apply reinforcement learning [7] and machine learning [8] for communication performance improvement. A recent study uses the propagated geometrical data among an underwater swarm of AUVs to optimize the number of subcarriers and other communication parameters [9]. Or the work in [10] assesses an adaptive physical layer technique with spread-spectrum modulation. The packet used, which is called GUWAL, has a field in the header that is reserved for enabling adaptive modulation and coding. The information in this field lets the receiving node change the level of redundancy in the spread-spectrum modulation, which affects the data rate.

Statement of contributions: we here propose a distributed optimization approach that enables a set of underwater agents to autonomously select the most suitable Orthogonal Frequency-Division Multiplexing (OFDM) modulation parameters upon their deployment or prior to start of their mission. In the proposed approach, agents at the beginning of their deployments, sample some environmental information about the sea conditions where they find themselves. Utilizing the collected samples, in a distributed optimization mechanism, agents reach an agreement regarding the best communication channel parameters before they start their mission or on noticing environmental changes comparing to the last agreed conditions. We also analysed the condition that the communication benefits of utilizing the proposed method exceeding its computational costs. More precisely, we: 1) analyze how these agents may distributedly decide such OFDM parameters by means of a tunable-communication-complexity distributed optimization protocol that supports broadcast lossy asynchronous communications, and 2) investigate in which sea conditions/mission length / network configuration performing this type of tuning is beneficial (i.e., deciding the OFDM parameters online enables the agents to exchange more bits during the whole mission, than what they would if they were choosing such parameters offline). Note that this will mean evaluating also which complexity of the distributed optimization scheme can be advantageous for improving the overall mission-long communication throughput.

Structure of the manuscript: The paper is organized as follows: Section II provides a brief overview of the basics of underwater communication and distributed optimization. Section III presents the objective function definition and the rationale behind it. Section IV outlines the proposed parameter adaptation mechanism using distributed optimization. In section V, we compare the performance of different optimization algorithms for tuning the OFDM protocol. In section VI, we analyze the impact of communication protocols on selecting the most efficient optimization method. Section VII highlights the communication improvement achieved during the simulation. Section VIII explains the insignificant impact of some predictable environmental conditions on the communication performance, underscoring the importance of onsite parameter adaptation. Finally, in section IX, we conclude this paper and propose directions for future work.

II. BACKGROUND

A. Background on underwater acoustic communication

The purpose of this section is to provide a) some background on the concepts that will be laddered in Section III to create the local cost functions to be optimized, and b) references for the interested reader.

Acoustic communication uses sound waves to transmit information through a medium, such as water or air. The information can be encoded in the sound wave using various modulation techniques, such as pulse amplitude modulation (PAM) or frequency shift keying (FSK), and the receiving device decodes the information to recover the original message. The quality and clarity of the information transmitted depend on factors such as the strength and frequency of the sound wave, the distance between the sender and receiver, the properties of the medium through which the sound travels, and the modulation scheme used to encode / decode information packets. In acoustic communication, different modulation techniques can be utilized. Some of the most popular ones include PAM [11], FSK [12], Phase Shift Keying (PSK) [13], Quadrature Amplitude Modulation (QAM) [13], and OFDM [13]. OFDM splits the data into multiple subcarriers, each with its specific frequency. Although subcarriers are transmitted simultaneously, they typically do not interfere since they are orthogonal to each other. This allows for more efficient use of the available bandwidth and reduces the effects of multipath interference. Therefore, this modulation is widely used in cellular or underwater communication. Some key parameters for OFDM modulation include the number of subcarriers N, transmission power, symbols per packet m, modulation order M, and many other parameters. In OFDM, the data are modulated by QAM or PSK and assigned to one of many orthogonal subcarriers. Subcarriers separate neighbors by a given frequency, and some are null to reduce inter-carrier interference (ICI). The symbols are transformed into samples by passing them through Inverse Fast Fourier Transform (IFFT). The OFDM performance (in the sense of number of bits successfully transmitted per second) depends on both these parameters and the actual environmental conditions where the communicands are in. The two main purposes of this manuscript are thus both proposing a distributedly optimizable cost function for choosing suitable values for these quantities and investigating when performing such a choice online is meaningful from the standpoint of improving the overall throughput during a multi-agent mission whose time duration is fixed.

B. Background on distributed optimization

Distributed optimization means generally taking a collaborative approach to optimizing a distributable objective function (typically the sum of local costs). Various techniques have been developed in the years, the most famous ones likely being Distributed Stochastic Gradient Descent (DSGD) [14] and Alternating Direction Method of Multipliers (ADMM) [15]. We consider Robust Asynchronous Newton-Raphson Consensus (ra-NRC) [16], whose features enable us to test different communication topologies using virtually the same optimization structure. The algorithm on which ra-NRC is based, Distributed Newton-Raphson Consensus (DNRC) [17], is a second-order optimization method where agents exchange opportunely constructed Hessian matrices. In its original form, each local step towards the global optimum comes with a communication cost that scales with $O(n^2)$, where n is the number of variables involved in the local costs. This matrix may though be approximated in different ways, so that each local step may be made less costly from a communication perspective (typically sacrificing though convergence rates, i.e., the number of local steps required to achieve convergence). ra-NRC enables exploring convergence rates vs. bandwidth usage per packet; since it is robust against packet losses, and can be used off-the-shelf in asynchronous broadcast-based communication protocols, this optimization scheme is natural for investigating OFDM tuning in underwater communication scenarios.

III. Objective Function

To optimize the utilization of the communication channel, one shall define an opportune cost function. We propose one in this contribution that captures the impact of various communication parameters on performance. We note that quantifying accurately the effects of the individual parameters on the actual bit rates given the environmental conditions is of paramount importance to the final purpose of selecting effectively the desired channel properties.

Therefore our goal is first to create a model that is parameterized on both the sea conditions s and the OFDM parameters σ (number of blocks – OFDM symbols – per packet m, number of orthogonal frequencies per OFDM symbol N, and how many bits each data carrier contains $\log_2 M$, where M is the modulation order), so that it returns the expected data exchange rates per packet given those conditions. We use that model to formulate an optimization problem for which we choose the σ, given s, that induces the most effective communication between agents. For the sake of mathematical precision, we let s be the multi-dimensional and global vector field of the sea conditions in the area of interest. We assume that the agents can sample, at the beginning of their mission, the random field s and thus measure the local sea conditions s_i. In other words, letting $\mathcal{N}$ be the set of agents, the set $\{s_i\}_{i\in\mathcal{N}}$ will indicate the sampled sea conditions around the various agents. Each vector s_i, will be for simplicity assumed time-constant, that cannot be adjusted at will, and whose value depends on the geographical location of agent i.

Given this formalism, we assume that communication performance of agent i with its out-neighbor j may be modeled as a function $R_{ij}(s, \sigma)$. Note that underwater acoustic communication channels typically are asymmetric; this means that in general $R_{ij}(s, \sigma) \neq R_{ji}(s, \sigma)$. Given this, we let the network-wise objective function be the sum of the various R_{ij}'s. So, letting $\mathcal{N}$ be the set of agents and $\mathcal{N}_i$ be the out-neighbors of i, maximizing the network performance means minimizing the negative sum of the per-agent performances. Given though that estimating the whole s distributedly would introduce an additional round of computations (and thus delay before deciding the OFDM parameters that shall be used), we actually take the approach for which agents solve

$$\arg\min_{\sigma\in\mathcal{X}} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}_i} \big(- R_{ij}(s_i, \sigma)\big), \tag{1}$$

i.e., when building the local costs they consider just the sea conditions that they can directly sample. We moreover note that considering the cost function above means implicitly promoting OFDM parameters viable for a fully connected network. If full connectivity is not required, one may though easily modify (1) so to give more weight to more important links.

In any case, the decision variable σ, representing the set of OFDM parameters the agents should reach consensus on, is constrained (motivating the $\sigma \in \mathcal{X}$ in (1)). Indeed, the cyclic prefix fraction p_c must be large enough to absorb inter-symbol interference (ISI); at the same time it typically does not exceed one. Increasing p_c may mitigate ISI-induced performance losses due to delay spread in the channel. We assume that τ, the delay spread, spans all the paths that are at least as strong as L dB relative to the strongest path, implying that the cyclic prefix fraction p_c should satisfy the constraint

$$\tau\Delta f \leq p_c \leq 1 \,, \tag{2}$$

where Δf is the carrier spacing, which also determines the prefix-less OFDM symbol duration as $1/\Delta f$. As for the number of symbols per packet m and data carriers per symbol $R_N N$, they must both be positive, giving $m \geq 1$ and $R_N N \geq 1$. If the carriers are closely spaced in frequency, the OFDM symbol becomes more sensitive to inter-carrier interference (ICI). To mitigate ICI, a minimum frequency spacing constraint must be enforced. Specifically, we must ensure that the frequency spacing between any two adjacent carriers is at least equal to k times the maximum Doppler shift, where k is a design parameter. Higher k means better robustness to Doppler spread - a condition that may be expressed as $\Delta f \geq k\nu$ or, equivalently, $B/N \geq k\nu$ where B is the system bandwidth and N is the number of carriers in one symbol.

Instead, as for the link performance $R_{ij}(s_i, \sigma)$, the dependence on the water body conditions s_i intuitively is motivated by the fact that agents communicating in an isolated lake with no boats nor big fish in it will likely have an easier life than agents in an heavily trafficked and noisy harbor. We also note that the effect of s (and thus the sampled versions s_i) is mostly on the speed of sound c, which can be effectively modeled by the Medwin equation [18], simplified in this work with

$$\begin{aligned} c(T, D, S) =& 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 \\ &+ (1.34 - 0.01T)(S - 35) + 0.16D \end{aligned} \tag{3}$$

where

- T is the local water temperature in $^\circ C$;
- S is the local salinity in parts per thousand;
- D is the local depth in meters.

Considering then the OFDM modulation mechanisms described in Section II-A, we note that the number of uncoded bits that can be transmitted over one symbol duration t_s can be modelled as

$$R_c R_N N \log_2 M, \tag{4}$$

where R_c represents the coding rate and indicates the number of uncoded bits that can fit coded bits, and R_N, represents the ratio of carriers that carry data and clarifies what percentage of carriers are data carriers. Additionally, N represents the number of carriers, and M represents the modulation order, where $\log_2 M$ indicates the number of bits in one carrier. Furthermore, if we denote p_c as the cyclic prefix fraction, the data rate (excluding the overhead time t_{oh}) can be calculated using the expression

$$\frac{R_c R_N N \log_2 M}{(1+p_c)N/B} = \frac{B R_c R_N N \log_2 M}{(1+p_c)N} \; . \tag{5}$$

In (5), the numerator represents the number of data bits that can be transmitted in one symbol, while the denominator represents the symbol duration, i.e., the time required to transmit one symbol. Equation (5) is thus our model expressing the expected data rate for a single, isolated OFDM symbol. To calculate the data rate for a packet, we shall then take into account the number of symbols per packet m, the overhead time t_{oh} and the transmission delay $t_d = r/c$, where r is the range. Considering these factors leads to the data rate for the packets model

$$R_p = \frac{m B R_c R_N N \log_2 M}{m(1+p_c)N + B(t_{\text{oh}} + t_d)} \; , \tag{6}$$

where the numerator represents the number of data bits that can be transmitted in one packet. The denominator in (6) represents the packet duration, i.e., the time required to transmit one packet in addition to overhead time and propagation time which are both required to data transmission for each packet. We note that both sea conditions and distance among the agents affect the transmission delay $t_d = r/c$. r above is the distance between the two specific agents implicitly involved in (6), and $c(T, S, D)$ is the speed of sound underwater, which can be calculated using formula (3).

TABLE I: Glossary of parameters used in our underwater communication modeling.

Variable	***Meaning***	***Type***
σ	OFDM parameters	Optimized
m	Symbols per packet	
N	Subcarriers	
M	Modulation order	
s	Sea condition	Estimated
T	Temperature	
D	Depth	
S	Salinity	
τ	Delay spread	
ν	Doppler spread	
P_n	Noise power	
r	Range	
p_c	Cyclic prefix fraction	Design
P_{tx}	Transmit power	
t_{oh}	Overhead per packet	
B	Bandwidth	
L	Energy spread threshold	
k	Relative Doppler margin	
p_l^{t}	Target packet loss	
R_N	Fraction of data carriers	
R_c	Coding rate	
t_s	Symbol duration	Derived as $(1+p_c)/\Delta f$
Δf	Carrier spacing	Derived as B/N

Our aim is to maximize the sum of the per-link outputs predicted by model (6). Considering that maximizing $\log(f)$ is equivalent to minimizing $-f$, the proposed reward function is reworded to a cost function to be minimized as

$$\begin{aligned} J_0 = -\log R_p &= -\log m - \log R_c - \log B \\ &\quad - \log(R_N N) - \log(\log_2 M) \\ &\quad + \log\Big(m(1+p_c)N + B(t_{\text{oh}} + t_d)\Big) \end{aligned} \tag{7}$$

with respect to $(m, N, M, p_c) \in \mathcal{X}$.
Equation (7) can be shown to be non-convex everywhere in $\mathcal{X}$, because the determinant of its Hessian, and therefore the product of its eigenvalues, is negative. Noticing that J_0 is concave in the cyclic prefix fraction p_c, p_c was then held fixed, changing the constraints to

$$N - \tau\frac{B}{p_c} \geq 0 \qquad\qquad N - \frac{B}{k\nu} \leq 0 \; . \tag{8}$$

Then, with $q = \frac{t_{\text{oh}} B}{1+p_c}$, the Hessian of J_0 with p_c fixed is given by

$$H(J_0) = \begin{pmatrix} \frac{1}{m^2} - \frac{1}{\left(m+\frac{q}{N}\right)^2} & \frac{q}{(mN+q)^2} & 0 \\ \frac{q}{(mN+q)^2} & \frac{1}{N^2} - \frac{1}{\left(N+\frac{q}{m}\right)^2} & 0 \\ 0 & 0 & \frac{\log M + 1}{(M \log M)^2} \end{pmatrix}. \tag{9}$$

Note that the Hessian in (9) is block diagonal, where m and N form one block, and M is alone in its block and positive for $M > 1$. Then, convexity depends on whether the (m, N) block is positive definite. With a little algebra, the corresponding characteristic equation can be written as

$$\begin{aligned} 0 = \lambda^2 &- \left(\frac{1}{m^2} - \frac{1}{(m+q/N)^2} + \frac{1}{N^2} - \frac{1}{(N+q/m)^2}\right)\lambda \\ &+ \left(\frac{1}{m^2 N^2} + \frac{m^2 N^2}{(mN+q)^4} - \frac{2}{(mN+q)^2} - \frac{q^2}{(mN+q)^4}\right). \end{aligned}$$

Because $m + q/N > m$ and $N + q/m > N$, the coefficient of λ is necessarily negative, which in turn is necessary for the existence of only positive roots. It can then be shown that the constant term is nonnegative for all positive values of the variables in question. Summarizing, given the models and assumptions above, the proposed final distributed optimization can be formulated as

$$\begin{aligned} \arg\min_{m,N,M} \; & \sum_{i \in \mathcal{N}} \big(-J_{0,i}(m_i, M_i, N_i)\big) \\ \text{subject to } \; & m_i \geq 1 \\ & M_i \geq 2 \\ & M_i \leq 64, \\ & N_i - \tau\frac{B}{p_c} \geq 0, \\ & N_i - \frac{B}{k\nu} \leq 0 \\ & p_{l,i} \leq p_l^{\text{t}} \; , \end{aligned} \tag{10}$$

where $p_{l,i}$, the packet loss ratio for node i, should not exceed a given target packet loss ratio p_l^{t}.

Finally, the model in (10) can be reformulated as in

$$\arg\min_{m,N,M} \sum_{i\in\mathcal{N}} \left(-J_{0,i} + \frac{1}{t} \sum_{\kappa=1}^{4} \psi_\kappa(s_i, \sigma) \right) \tag{11}$$

with

- $\psi_1(s_i,\sigma) = -\log(-(1-m_i))$
- $\psi_2(s_i,\sigma) = -\log(-(\tau \frac{B}{p_c} - N_i)) - \log(-(N_i - \frac{B}{k\nu}))$
- $\psi_3(s_i,\sigma) = -\log(-(2-M_i)) - \log(-(M_i - 64))$
- $\psi_4(s_i,\sigma) = -\log\Big(-0.2 m_i R_N N_i \log_2(M_i) \cdot$
  $\cdot \exp\Big(-\frac{3}{2(M_i-1)} \frac{P_{rx}}{LP_{rx}+P_n}\Big)^{\frac{1}{R_c}} - p_l^{\mathrm{t}}\Big)$.

We note that the nonincreasing factor t^{-1} in (11) controls the weight of the constraints, which are represented as barrier functions [19]. The barrier function ψ_4 represents the constraint on the packet loss ratio, which is based on the BER model used in [20]. When the optimization algorithm terminates, the obtained $\{m, N, M\}$ are rounded to the nearest feasible point associated with the lowest cost. We also note that after completing the optimization process, the agents round the found m, M, N variables to the closest feasible values. This, together with the fact of using only one Lagrangian multiplier for coping with all the ψ_k's instead of one per ψ_k, constitute a simplification whose consequences we plan to study in our future works.

For the sake of completion, we present then in Table I all the parameters used in our underwater communication model. The optimized type indicates an optimization variable, the estimated type denotes an unknown variable that must be estimated or measured, the design type designates a user specified variable, and the derived type indicates that the variable is calculated from at least one optimization variable. Derived variables have their formula given.

IV. Adapting the Communication Parameters via a Multi-Agent Approach

Problem (11) models the expected global data rate obtainable when choosing a given set of OFDM parameters $\sigma(m, N, M)$ given a set of environmental conditions $c(T, S, D)$. To solve it in a distributed fashion we take an asynchronous distributed Newton-Raphson Consensus (ra-NRC) approach, since it enables lossy asynchronous broadcast updates [16] and, more importantly, it enables inspecting the effect of varying communication complexity on the overall suitability of the proposed approach.

Underwater acoustic communication has high latency, so we want to investigate <u>not</u> how many local optimization steps are required to arrive at the optimum, but rather <u>how much time in minutes</u> it takes to find it. Informally, 10 optimization steps involving communicating 10000 bits per step, may take more time than 100 steps each requiring 100 bits to communicate (only in first approximation, though) the final assessment requires taking in consideration which protocol is used for performing this optimization. Thus the picture is more sophisticated than just verifying which optimization strategy has the lowest "number of iterations" · "number of bits per iteration" product. More details will follow in Sections V and VI.

We thus summarize in Algorithm 1 the pseudo-code of such a ra-NRC approach. There, $x_i = \sigma_i$ is the vector of to-be-optimized OFDM variables on node i, and f_i is J_0 of (7) at node i. A more thorough explanation of the algorithm can be found in [16].

Algorithm 1 ra-NRC

Require: x^0, ϵ, c
Initialization (atomic)
$x_i \leftarrow x^0$
$y_i \leftarrow 0, g_i \leftarrow 0, g_i^{\mathrm{old}} \leftarrow 0,$
$z_i \leftarrow I_n, h_i \leftarrow I_n, h_i^{\mathrm{old}} \leftarrow I_n,$
$\sigma_{i,y} \leftarrow 0, \sigma_{i,z} \leftarrow 0$
$\rho_{i,y}^{(j)} \leftarrow 0, \rho i, z^{(j)} \leftarrow 0$
$\mathrm{flag}_{\mathrm{reception},i} \leftarrow 0, \mathrm{flag}_{\mathrm{update},i} \leftarrow 0$
$\mathrm{flag}_{\mathrm{transmission},i} \leftarrow 1$
Data Transmission (atomic)
if $\mathrm{flag}_{\mathrm{transmission},i}$=1 **then**
  transmitter_node_ID $\leftarrow i$
  $y_i \leftarrow \frac{1}{|N_i^{\mathrm{OUT}}|+1} y_i$
  $z_i \leftarrow \frac{1}{|N_i^{\mathrm{OUT}}|+1} z_i$
  $\sigma_{i,y} \leftarrow \sigma_{i,y} + y_i$
  $\sigma_{i,z} \leftarrow \sigma_{i,z} + z_i$
  Broadcast: transmit_node_ID, $\sigma_{i,y}, \sigma_{i,z}$
  $\mathrm{flag}_{\mathrm{transmission},i} \leftarrow 0$
end if
Data Reception (atomic)
if $\mathrm{flag}_{\mathrm{reception},i}$=1 **then**
  $j \leftarrow$ transmitter_node_ID, $(j \in N_i^{IN})$
  $y_i \leftarrow y_i + \sigma_{j,y} - \rho_{i,y}^{(j)}$
  $z_i \leftarrow z_i + \sigma_{j,z} - \rho_{i,z}^{(j)}$
  $\rho_{i,y}^{(j)} \leftarrow \sigma_{i,y}, \quad \forall j \in N_i$
  $\rho_{i,z}^{(j)} \leftarrow \sigma_{i,z}, \quad \forall j \in N_i$
  $\mathrm{flag}_{\mathrm{reception},i} \leftarrow 0$
  $\mathrm{flag}_{\mathrm{update},i} \leftarrow 1$ (optional)
end if
Estimate Update (atomic)
if $\mathrm{flag}_{\mathrm{update},i}$=1 **then**
  $x_i \leftarrow (1-\epsilon)x_i + \epsilon z_i^{-1} y_i$
  $g_i^{old} \leftarrow g_i$, $h_i^{old} \leftarrow h_i$
  $h_i \leftarrow \nabla^2 f_i(x_i)$
  $g_i \leftarrow h_i x_i - \nabla f_i(x_i)$
  $y_i \leftarrow y_i + g_i - g_i^{\mathrm{old}}$
  $z_i \leftarrow z_i + h_i - h_i^{\mathrm{old}}$
  $\mathrm{flag}_{\mathrm{update},i} \leftarrow 0$
  $\mathrm{flag}_{\mathrm{transmission},i} \leftarrow 1$ (optional)
end if

When Algorithm 1 converges for a given factor t used in the barrier function of (11), and t is below a threshold, the algorithm is restarted, using the point to which it converged as x^0, and multiplying t by a constant μ. Such a procedure implements the outer Newton step of the barrier method as described in [19].

Algorithm 1 can be modified opportunely to implement either a Jacobi approach (by letting $h_i \leftarrow \text{tr}\left(\nabla^2 f_i(x_i)\right)$, which requires only n parameters for the Newton direction) or even a Gradient descent (basically by ignoring all the part of the code relative to the local Hessians h_i). Summarizing, the same NRC algorithm may be implemented so to have higher or lower communication requirements, and consequently a faster or slower per-step convergence rate.

V. Comparing the different versions of the optimization algorithm

We investigate which version of the ra-NRC can arrive at the same final OFDM performance in the shortest time. As the proposed OFDM parameter selection approach must be executed before the actual mission begins, minimizing its duration is an important performance metric.

To compare the NR, Jacobi and GD versions of the original ra-NRC algorithm above we consider the network in Figure 1.

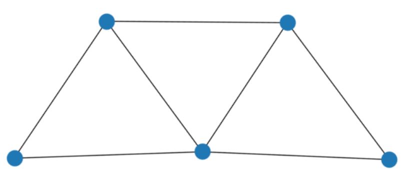

Fig. 1: Network topology for performing numerical performance comparison between original ra-NRC, two other modified varieties using Jacobi and Gradient descent

We also consider the same null initial conditions for all the agents, and the cost function defined by the specified conditions in Table IV. Reaching a 10^{-2}-ball around the global optimum took then 2100 iterations for the NR, 2500 for the Jacobi, and 4000 for the GD. Correspondingly, the required time for these three versions of the optimization algorithm took 1363, 1742, and 2788 seconds respectively (the time calculation for such OFDM data transmissions being explained in the appendix). This makes ra-NRC with Newton-Raphson the best option for OFDM parameter adjustment, and shows the impact of the choice of optimization algorithm on the performance of the proposed tuning process.

VI. Impact of the used communication protocol over the selection of the optimization algorithm

We note that which communication protocol is used has also an impact on selecting the best optimization algorithm. To explain this concept, consider that the estimates of the time taken (in seconds) by the optimization algorithm above were made assuming to use again an OFDM scheme (in other words, agents initially use OFDM with a pre-fixed set of parameters to select a novel set, and then switch to the new set of parameters when converging to a consensus). We may though implement such a first optimization round via other protocols, e.g., JANUS [21]. Using another protocol will lead to a different time (in seconds) for performing the same optimization process. Performing the identical explained scenario in V assuming JANUS as the communication protocol, would lead us to spend 8610, 7250, and 11600 seconds for performing optimization using NR, Jacobi, and GD (again the time calculation for the specific case of JANUS data transmissions is in the appendix).

TABLE II: Time required to execute the optimization algorithm. Comparison between the OFDM vs. JANUS cases.

Modulation	Iterations	Time/iteration (s)	Total runtime (s)
OFDM			
GD	4000	0.697	2788
J	2500	0.697	1742
NR	2100	0.697	1463
JANUS			
GD	4000	2.9	11600
J	2500	2.9	7250
NR	2100	4.1	8610

Comparing the required times of OFDM vs. JANUS (a summary of these results being in Table II) highlights how a NR approach seems best for the first, and Jacobi for the second. The key issue here is that different communication protocols have different payloads per packet. For example, every time an agent sends a JANUS packet, it sends also a preamble and postamble (information that serves the purpose of explaining which packet type the packet is, which type of information it contains, etc.). This difference in such overheads among different communication protocols may then distort the results as above. In short, thus, the indication from Section V that NR seems to be the best suitable strategy for our purposes should be considered as limited to the case of using OFDM for the initial optimization process.

VII. Assessing the improvements brought by the scheme

We then assess the improvements that running the proposed optimization algorithm may bring to the performance of the final communication scheme. Although a thorough evaluating the impact of real-life conditions would require several field experiments, a numerical comparison can still offer valuable insights into the potential benefits of the proposed feedback approach. For the purpose we randomly selected a scenario with 6 agents, and initial conditions & parameters as in Tables III and IV. Initially, as per (4) the chosen OFDM parameters allowed for the transmission of approximately 2255 uncoded bits per packet. However, the OFDM parameters chosen after the optimization round enable exchanging 6205 bits per packet.

TABLE III: Summary of the environmental conditions considered for the assessments in Section VII.

	agent1	agent2	agent3	agent4	agent5	agent6
Temperature (C)	13	11	10	10	11	9
Salinity (ppt)	35	40	37	41	39	40
Depth (m)	20	40	30	35	15	25

TABLE IV: Summary of the initial OFDM parameters considered for the assessments in Section VII.

variables	values	variables	values
R_N	1	τ(s)	0.025
B (Hz)	4000	R_c	0.5
k	2	ν (Hz)	1
p_c	0.25	step-size	0.005
t_{oh} [s]	10^{-3}	p_l^{t}	0.1

VIII. On the aid that weather forecasts may provide

One may expect that the improvement shown in the previous section may be mitigated by the availability of accurate weather predictions. Indeed one may think that having accurate predictions of water temperature, the locations of where the agents will be (and thus their inter-ranges), and the depth of the water column where they are located, may enable choosing near-optimal OFDM parameters already before running the optimization algorithm. Intuition would then say that in this case the proposed online OFDM parameters selection approach is worthless. However, these environmental parameters - the unique ones that one may forecast accurately - have a negligible (for our purposes) influence on communication performance. This can be seen via the following Monte Carlo experiment: generate a series of random networks and associated sea conditions s, for each of such a scenario find the global optimum defined by Problem (11), and estimate the communication efficiency of the so-identified optimum via (4).

Each dot in Figure 2 represents one of the so-identified optima: each panel is a scatter plot showing how the decision variables depend on the inputs. Interestingly, the optimal parameters tend to be independent of temperature, range, and depth. For example, while temperature and salinity affect sound speed, they will not significantly impact underwater communication performance. This observation is particularly significant given that the physical properties of water are among the parameters that are more easily predicted based on historical data. In other words, the optima (and thus the communication performance) are mainly unaffected by that parameters that are easily predictable, and are thus mainly affected by the unpredictable ones. This makes the on-site tuning of the parameters a valuable step.

IX. Conclusions

We introduced a novel multi-agent approach for autonomously adjusting OFDM parameters in underwater acoustic communication. Our approach utilizes distributed optimization over environmental information that is collected

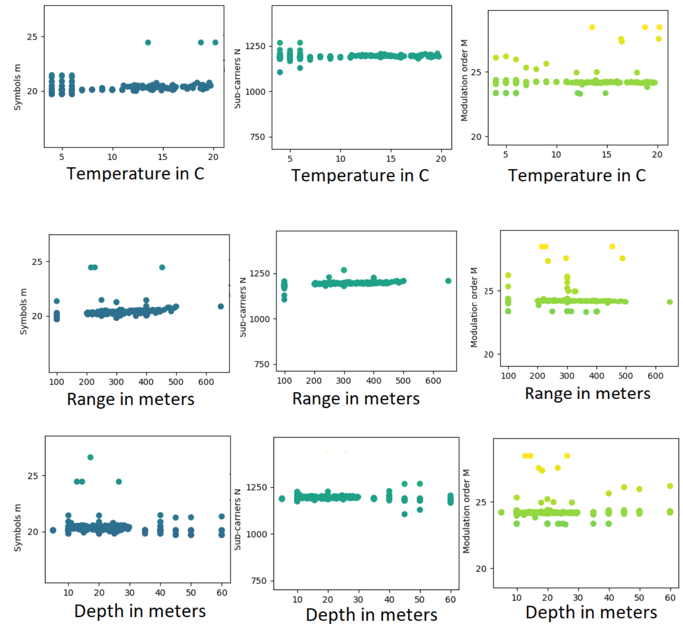

Fig. 2: Highlighting how temperature, range, and depth (in order from top to bottom) tend to do not influence the results from the optimization process (from left to right: Symbols m, number of sub-carrier N, and modulation order M).

from different agents and used in a collaborative fashion to enhance overall communication performance among all agents. Since this method can be utilized prior to start of the agents' missions or can be repeated after a radical change in the environment conditions, the communication channel optimization will not add any communication overhead to the main mission.

To achieve this, we developed an objective function that projects the effect of environmental parameters on communication performance, and proposed to optimize it in a distributed fashion via an off-the-shelf optimization method that is resilient to packet loss and supports asynchronous broadcast communication. We then checked, using the possibility of varying the communication requirements of the algorithm, which variant best serves our purpose of tuning OFDM parameters online.

Furthermore, we analyzed the fact that different communication protocols may actually associate with different best optimization methods – in other words, we found that each protocol has its optimal optimization algorithm.

We then examined how much the most easily predictable environmental parameters affect the sought optimal OFDM parameters, and found that they have little impact on the communication performance. This analysis led us to conclude that on-site environmental evaluation (i.e., optimizing the parameters after the mission started) is likely to lead to benefits. Since the proposed feedback approach can adjust communication parameters automatically without requiring human intervention, it can be suitable for autonomous underwater systems requiring acoustic communication.

In future work, we seek to develop improved objective functions that consider other environmental parameters, and lead to more accurate estimates. Additionally, developing methods for performing step-size selection during the optimization process and improving the optimization performance could enhance the proposed approach in this contribution and many other applications.

APPENDIX
OFDM AND JANUS DATA TRANSMISSION TIME CALCULATION

Communicating an OFDM packet requires time for sending and decoding the information, time that may be estimated to be equal to

$$t = t_d + p_t + \text{crc16}, \tag{12}$$

where t_d is the propagation delay, p_t is the packet duration, and crc16 is the additional duration due to a cyclic redundancy check, which is used to detect errors in received packets. We assume that we can fit sufficiently many bits in a single OFDM symbol to accommodate six 32-bit floats and any other necessary data, such as a sender identifier. Similarly, the time it takes to communicate a JANUS packet may be estimated as

$$t = t_d + p_t + \text{cargo} + \text{crc16}, \tag{13}$$

with the parameters above having been measured in one of our experimental campaigns as

- propagation delay $t_d = r/c$ (ex. $\frac{300\text{ m}}{1500\text{ m/s}} = 0.2$ s)
- packet time $p_t = t_{\text{oh}} + \text{time\_per\_bit} \cdot n^{o}_{\text{bits}}$
- cargo = 80 bits/s
- Cyclic redundancy check, crc16: 0.2 s
- single precision: 32 bits
- time per coded bit: 12.5 ms.

We note that exchanging a JANUS packet requires exchanging the following data:

- ID $\rightarrow$ 8 bits;
- $\sigma_{y,i}$ 3 single floats $\rightarrow$ 96 bits;
- $\sigma_{z,i}$ 6 single floats $\rightarrow$ 192 bits for hessian. Symmetry allows transmitting only the lower triangular part.

All the mentioned information can be summarized in Table V.

TABLE V: Time required for exchanging packets enough to execute a local optimization step using OFDM and JANUS.

OFDM	alg		time/iteration (second)
	GD	$t_d + p_t + \text{crc16}$	0.697
	J	$t_d + p_t(\text{diag}) + \text{crc16}$	0.697
	NR	$t_d + p_t(\text{matrix}) + \text{crc16}$	0.697
JANUS			
	GD	$t_d + p_t + \text{cargo} + \text{crc16}$	2.9
	J	$t_d + p_t + \text{cargo} + \text{crc16}$	2.9
	NR	$t_d + p_t + 2 \cdot \text{cargo} + \text{crc16}$	4.1

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