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Stability of distributed pump configuration for cooling systems

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Abstract— **Hydronic Heating, Ventilation, and Air Conditioning (HVAC) systems, where water is used as a media for cooling energy transport, are often used in large buildings. Distributed Air Handling Units (AHUs) condition the air for the cooling and ventilation needs in the building by controlling the chilled water flow. A distributed pump setup, where local pump controllers control the exhaust air temperature, is considered. Commissioning of HVAC is important for the operation of the HVAC and is the focus of this paper. Specifically, a method for local pump controller design, that enables individual operation of the local control loops, as well as operation of the fully connected system. This controller design is expected to fulfill the need for flexibility when setting the building into operation, and thereby ensure better building performance in the end. The theoretical findings are supported by numerical studies of a chilled water HVAC system.**

I. Introduction

With the increased temperature due to climate changes and with the increased middle class in developing countries, it is expected that cooling will be used in more and more buildings. Already now the energy consumption in the building sector is substantial, with just below 12.000 GW in 2016 used for cooling alone [1]. Half of this is used in commercial buildings. Commissioning of Heating, Ventilation, and Air Conditioning (HVAC) systems is often not done to the extent needed, and energy efficiency can often be improved by proper commissioning [2]. Commissioning has been a problem in the building sector for a long time. In [3] it is argued that commissioning is extremely important for the building to be operated properly. However, it is often not done due to primary cost.

Part of building commissioning is controller design and tuning. An overview of control methods in HVAC control systems with a special focus on MPC is given in [4]. In this paper we will focus on commissioning of a hydronic network of cooling systems, where distributed pumps are used for circulating chilled water from the chillers to Air Handling Units (AHU). Such a structure is proposed in [5] and automatic balancing of the system is considered in [6]. In the later each distributed pump has its own controller controlling local outlet air temperatures of AHU's. Stability of the distributed pump setup with local controllers is not treated in [5] nor in [6]. Establishing system stability will be the focus of this work.

Here, we will pursue a solution where each control loop of AHU branches of the system is designed separately, and at the same time is robust to the interconnection created by the hydraulic network. With the anticipated structure, the control setup can be tested and set into operation individually, which is expected to be more appealing for the industry. Distributed control of hydraulic networks with pressure control objectives has already been treated in [8] and [9], where it is shown that the distributed PI control is globally asymptotically stable under some constraints on the network topology. The difference between these works and the results presented in the following is that the local controllers will control the outlet air temperature of the water to air heat exchangers of the AHU, as opposite to the local pressure controls in [8] and [9].

Our starting point is the new type of hydronic system presented in [6] for carrying the cooling load from the chillers to the AHUs. The idea of the mechanical design is to include only the necessary components leading to reduced installation costs. In the proposed system, flows through the AHUs and chillers are controlled by distributed pumps placed at the AHUs. Therefore, control valves at the AHUs and dedicated chiller pumps are not present in the system. The removal of the valves and thereby the pressure losses over these can potentially lead to savings on the pump operation. This paper considers the design of the local outlet air temperature controllers at the AHUs, whereas the design and tuning of the room control are not in focus. It is assumed that a temperature setpoint for each local outlet air temperature controller is available. The main result of the paper is three design procedures for the pump controllers. The design procedures utilize results from robust control via Linear Matrix Inequalities (LMIs) to design a set of separate controllers that is robust to the interconnected imposed by the hydraulic network.

The rest of the paper proceeds as follows. Section II presents the considered hydronic system and the distributed control setup, together with a derivation of the underlying model of the system. The design of the distributed controllers and stability analysis is treated in Section III. Numerical studies of the stability and simulation results from a nonlinear model of the considered hydronic system are presented in Section IV.

Nomenclature: We use $\triangleq$ to denote "defined by" and let $sign(x) = x/|x|$ with $sign(0) = 0$ denote the sign of x. For an $n \times m$ matrix $M = M_{n\times m}$ we let M' denote the transposed, $(M)_{ij}$ denote the ij'th entry, and

$$Diag(M_{n\times m}, N_{l\times k}) \triangleq \begin{bmatrix} M_{n\times m} & 0_{n\times k} \\ 0_{l\times m} & N_{l\times k} \end{bmatrix} \qquad (1)$$

with $0_{n\times m}$ the $n \times m$ zero matrix, and $Diag(M, N, K) \triangleq$

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$Diag(Diag(M,N),K)$ etc. We let $1_{n\times m}$ denote the $n\times m$ matrix with only ones, set $1_n \triangleq 1_{n\times 1}$, and let e_i denote the i'th unit vector. For a set S we let χ_S denote the indicator function on S. Finally, inequalities between vectors and scalars are to be understood coordinate-wise.

II. Cooling Systems

We consider a hydronic cooling system as sketched in Fig. 1. This type of hydronic system is also considered in [6], where a simple model is derived. This model is described in this section and will in the following be used for the proposed controller design procedure.

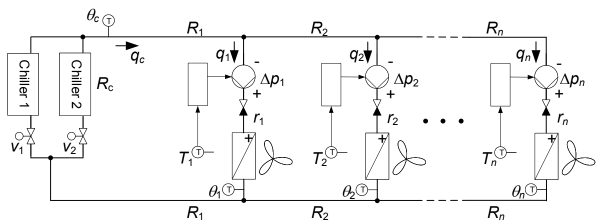

Fig. 1. Sketch of the hydronic cooling system considered in this work.

The system in Fig. 1 consists of two chillers that provide cold water to n air handling units (AHUs) each containing a water to air heat exchanger (WAHE) which is sketched in Fig. 2.

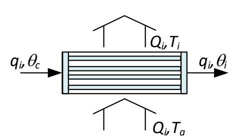

Fig. 2. A sketch of the i'th water to air heat exchanger (WAHE).

The exogenous system inputs are the supply water temperature θ_c provided by the chillers, the outdoor air temperature T_a, and the n air flows $Q_i,\quad i=1,2,\dots,n$ through the n WAHE. The outdoor temperature T_a and the airflow Q_i are expected to change slowly compared to the control dynamics and hence can be assumed constant in the controller design. The water in-flow $q_i \geq 0$ to the i'th AHU transfers the heat to the chillers via the hydraulic network shown in Fig. 1. Note that q_i is positive due to the non-return valve just after the pump. The dynamics of the hydraulic network are fast and stable (it only contains passive components) and will in the sequel be neglected. The water in-flow q_i is determined by the operation of the pumps and the structure of the network. The relation between the pressure (difference) of the i'th pump, Δp_i, and the water in-flows of the cooling system is found by setting up the pressure loop equations for each loop of the network, ending up with the following relation

$$\Delta p_i = r_i q_i^2 + R_c\left(\sum_{j=1}^{n} q_j\right)^2 + \sum_{k=1}^{i} 2R_k\left(\sum_{j=k}^{n} q_j\right)^2 \quad (2)$$

where $r_i > 0$ is the combined hydraulic resistance of the WAHE and the non-return valve of the i'th AHU and $R_k > 0$ is the resistance of the k'th pipe segment. The i'th pump operation is control by the i'th pump speed ω_i and relates the pressure Δp_i and water in-flow q_i as follows

$$\Delta p_i = -a_i q_i^2 + b_i\omega_i^2\ ,\ q_i \geq 0 \quad (3)$$

with $a_i > 0\ i=1,\dots,n$ and $b_i > 0\ i=1,\dots,n$. The constants a_i and b_i describe the pump operation and are determined by the type and size of the pump. The pump speed $\omega_i \geq 0$ is the control input to the system. Combining the network model (2) and the pump model (3) leads to the following relation between the control input ω_i and the flows of the AHU's

$$\omega_i^2 = \frac{r_i + a_i}{b_i} q_i^2 + \frac{R_c}{b_i}\left(\sum_{j=1}^{n} q_j\right)^2 + \sum_{k=1}^{i} 2\frac{R_k}{b_i}\left(\sum_{j=k}^{n} q_j\right)^2 . \quad (4)$$

A compact version of (4) can be found in (30).

The system output to be controlled consists of the n air temperatures T_i produced by the WAHEs. In this work, the objective is to control each of the air temperatures T_i to some given constant reference value T_i^* by adjusting the water in-flow q_i using the pump speed ω_i.

Using a finite volume approximation with one water volume V_w (blue area of the sketch in Fig. 2) and one air volume V_a (white area of the sketch in Fig. 2), the following dynamic model for the i'th WAHE can be obtained

$$C_w V_{w,i}\dot{\theta}_i = C_w q_i(\theta_c - \theta_i) - L_i(\theta_i - T_i) \quad (5a)$$

$$C_a V_{a,i}\dot{T}_i = C_a Q_i(T_a - T_i) + L_i(\theta_i - T_i) \quad (5b)$$

with θ_i the return water temperature produced by the i'th WAHE and L_i the heat transfer constant describing the energy transfer between water and air. The constants C_w and C_a are the specific heat capacities of water and air respectively, while the constants $V_{w,i}$ and $V_{w,i}$ are the volumes of water and air inside the WAHE respectively.

It is important to note that the WAHE dynamics in (5), do not contain any explicit information of the control inputs. Indeed, it is through the i'th in-flow, q_i, that the i'th AHU may be accessed. Moreover, the i'th control action, ω_i, affects not only the i'th AHU but also all the other AHUs through the relation in (4), which is illustrated in Fig. 3, where $\mathcal{C}_i$ are controllers and AHU$_i$ are the AHUs.

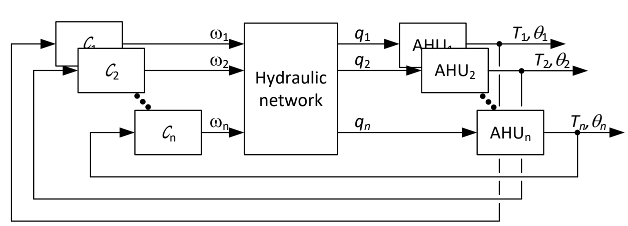

Fig. 3. The control interconnection.

Remark 1 *The measurement of the air temperature typically imposes a delay time due to the placement of the sensor. That is, the measured air temperature $\tilde{T}_i(t) = T_i(t - \delta t)$, where δt depends on the air flow Q_i. The relation is $\delta t = k_d/Q_i$, where k_d depends on the sensor position and the design of the air dock. However, in this work, the airflow Q_i and the sensor position is assumed to lead to a negligible delay δt.*

III. Distributed controllers

For the distributed pump system under consideration, it is desirable to be able to commission the control loops one by one. Therefore, we propose a controller set-up with integral action where each AHU is stabilized independent of the others using (4) with $q_j = 0,\ j \neq i$. In details, extend the i'th WAHE system (5) with one integral state

$$C_w V_{w,i} \dot{\theta}_i = C_w q_i(\theta_c - \theta_i) - L_i(\theta_i - T_i) \tag{6a}$$

$$C_a V_{a,i} \dot{T}_i = C_a Q_i(T_a - T_i) + L_i(\theta_i - T_i) \tag{6b}$$

$$\dot{\zeta}_i = T_i - T_i^* \tag{6c}$$

and let the exogenous inputs T_a and Q_i be constant. From the reference value, T_i^*, we then obtain the steady state values

$$\theta_i^* = \frac{-C_a Q_i(T_a - T_i^*) + L_i T_i^*}{L_i} \tag{7}$$

$$q_i^* = \frac{L_i(\theta_i^* - T_i^*)}{C_w(\theta_c - \theta_i^*)} \tag{8}$$

The values for exogenous inputs T_a and Q_i are found from the design condition for the cooling system, i.e., depending on building heating load and air exchange requirements. The relation between the air exchange requirement and the heating load of the building defines the airflow temperature reference T_i^*. Finally, the chilled water supply temperature θ_c is defined by the sizing of the heat exchanges. Hence, the parameters for the operating point are defined by the design of the room's cooling system. Now linearizing (6) at $(\theta_i^*, T_i^*, q_i^*)$ we get

$$\dot{x}_i = A_i x_i + B_i u_i \tag{9}$$

with $u_i = q_i - q_i^*$

$$A_i = \begin{bmatrix} -\left(\frac{q_i^*}{V_{w,i}} + \frac{L_i}{C_w V_{w,i}}\right) & \frac{L_i}{C_w V_{w,i}} & 0 \\ \frac{L_i}{C_a V_{a,i}} & -\left(\frac{Q_i}{V_{a,i}} + \frac{L_i}{C_a V_{a,i}}\right) & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{10}$$

$$B_i = \begin{bmatrix} \frac{\theta_c - \theta_i^*}{V_{w,i}} \\ 0 \\ 0 \end{bmatrix}, \qquad x_i = \begin{bmatrix} \theta_i - \theta_i^* \\ T_i - T_i^* \\ \zeta_i \end{bmatrix} \tag{11}$$

It is remarke that the pairs (A_i, B_i) are all controllable for any relevant system parameters since the determinant of the controllability matrix is

$$\left(\frac{\theta_c - \theta_i^*}{V_{w,i}}\right)^3 \left(\frac{L_i}{C_a V_{a,i}}\right)^2.$$

Note also that if the extra integral state $\dot{\xi}_i = \theta_i - \theta_i^*$ is introduced the controllability matrix will not have full rank. Therefore, even though it would be desiable, it is not possible to control both the outlet air temperature T_i and the return water temperature θ_i to a desired reference.

The outlet air temperature T_i is the control objective hence measured. Moreover, the return temperature θ_i can easily be measured in this type of hydronic system. The last state ζ in (11) is the internal state of the controller hence known. With the chosen heat exchange model these measurements lead to full state information. It then follows that we may apply state-feedback

$$u_i = K_i x_i \tag{12}$$

chosen such that the convergence $T_i - T_i^* \to 0$ as $t \to \infty$ happens in accordance with some predefined specification (e.g., rise-time and overshoot). Finally, since $q_j = 0$ for $j \neq i$ is assumed while stabilizing the ith AHU, the relation (4) transform to

$$\omega_i^2 = \left(\frac{r_i + a_i}{b_i} + \frac{R_c}{b_i} + \sum_{k=1}^{i} 2\frac{R_k}{b_i}\right) q_i^2 \triangleq \hat{\alpha}_i q_i^2 \tag{13}$$

which yields $\omega_i = \sqrt{\hat{\alpha}_i} q_i \triangleq \bar{\alpha}_i q_i$ since $sign(q_i) = sign(\omega_i)$ by physical considerations. The stabilizing control input to the i'th AHU (with $q_j = 0,\ j \neq i$) is therefore

$$\omega_i = \bar{\alpha}_i K_i x_i + \bar{\alpha}_i q_i^* \tag{14}$$

It is remarked that the control gain K_i should be robust with respect to (the constant) exogenous inputs T_a and Q_i. That is, one should choose K_i such that it stabilizes (9) for any relevant values of T_a and Q_i. By controllability and since the exogenous inputs change slowly compared to the system dynamics, this is indeed doable.

It is of course important to verify that when the above control design is applied to the true system with interconnection given by (5) and (4), then it does not destabilize the system. To that end, we write (6) compactly as (the affine bilinear system)

$$\dot{z}_i = F_i z_i + M_i z_i q_i + G_i q_i + E_i \tag{15}$$

with $M_i = Diag(-1/V_{w,i}, 0, 0)$

$$F_i = \begin{bmatrix} -\frac{L_i}{C_w V_{w,i}} & \frac{L_i}{C_w V_{w,i}} & 0 \\ \frac{L_i}{C_a V_{a,i}} & -\left(\frac{Q_i}{V_{a,i}} + \frac{L_i}{C_a V_{a,i}}\right) & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{16}$$

$$G_i = \begin{bmatrix} \frac{\theta_c}{V_{w,i}} \\ 0 \\ 0 \end{bmatrix}, \qquad E_i = \begin{bmatrix} 0 \\ \frac{Q_i T_a}{V_{a,i}} \\ -T_i^* \end{bmatrix}, \qquad z_i = \begin{bmatrix} \theta_i \\ T_i \\ \zeta_i \end{bmatrix} \tag{17}$$

Note that the right-hand side of (15) is zero when evaluated at

$$q_i = q_i^*, \quad z_i = z_i^* \triangleq \begin{bmatrix} \theta_i^* & T_i^* & 0 \end{bmatrix}'$$

and that if the control (14) is to be applied to (15), then q_i should be replaced with $\omega_i/\bar{\alpha}_i$.

The full system dynamics may then be represented as

$$\dot{z} = Fz + Gq + E + H(z)q \tag{18a}$$

$$= Fz + G\bar{\Lambda}^{-1}\omega + E + H(z)\bar{\Lambda}^{-1}\omega \tag{18b}$$

with

$$F = Diag(F_1, \ldots, F_n) \in \mathbb{R}^{3n\times 3n} \quad (19)$$

$$G = Diag(G_1, \ldots, G_n) \in \mathbb{R}^{3n\times n} \quad (20)$$

$$H(z) = Diag(M_1 z_1, \ldots, M_n z_n) \in \mathbb{R}^{3n\times n} \quad (21)$$

$$\bar{\Lambda} = Diag(\bar{\alpha}_1, \ldots, \bar{\alpha}_n) \in \mathbb{R}^{n\times n} \quad (22)$$

$$E = \begin{bmatrix} E_1 \\ \vdots \\ E_n \end{bmatrix},\ z = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix},\ q = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix},\ \omega = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} \quad (23)$$

To simplify the equations below we chose to make minor abuse of notation and write

$$H(q) = Diag(M_1 q_1, \ldots, M_n q_n) \in \mathbb{R}^{3n\times 3n} \quad (24)$$

which correspond to the derivative of $H(z)q$ with respect to z.

As above, note that the right-hand side of (18) is zero when evaluated at

$$q = q^* \triangleq \begin{bmatrix} q_1^* & \cdots & q_n^* \end{bmatrix}',\quad z = z^* \triangleq \begin{bmatrix} z_1^* & \cdots & z_n^* \end{bmatrix}'$$

Moreover, let $\bar{\phi}$ denote the control law generated by the control laws (14), that is

$$\omega = \bar{\phi}(z) \triangleq \bar{\Lambda}(K(z - z^*) + q^*) \quad (25)$$

with $K = Diag(K_1, \ldots, K_n) \in \mathbb{R}^{n\times 3n}$, so

$$\omega_i = \bar{\phi}_i(z) = \bar{\alpha}_i K_i (z_i - z_i^*) + \bar{\alpha}_i q_i^*$$

as in (14). Then the equilibrium point z^* for the closed loop system

$$\begin{aligned} \dot{z} &= Fz + G\bar{\Lambda}^{-1}\bar{\phi}(z) + E + H(z)\bar{\Lambda}^{-1}\bar{\phi}(z) && (26a) \\ &= Fz + G(K(z - z^*) + q^*) + E \\ &\qquad + H(z)(K(z - z^*) + q^*) && (26b) \end{aligned}$$

is stable since the linearized system (at z^*) is

$$\begin{aligned} \dot{x} &= (F + GK + H(z^*)K + H(q^*))x && (27a) \\ &= (F + H(q^*) + (G + H(z^*))K)x && (27b) \\ &\triangleq (\mathcal{A} + \mathcal{B}K)x && (27c) \end{aligned}$$

with $x = z - z^*$ and

$$\mathcal{A} = F + H(q^*) = Diag(A_1, \ldots, A_n) \in \mathbb{R}^{3n\times 3n} \quad (28)$$

$$\mathcal{B} = G + H(z^*) = Diag(B_1, \ldots, B_n) \in \mathbb{R}^{3n\times n}. \quad (29)$$

Note that both (24) and (21) has been used in the above.

The relation (4) can also be written compactly as

$$\omega_i^2 = q'(\Lambda_i + \Psi_i + \Gamma_i)q \triangleq q' S_i q \quad (30)$$

with

$$(\Lambda_i)_{jl} = \begin{cases} \alpha_i \triangleq \frac{r_i + a_i}{b_i} & j = l = i \\ 0 & \text{otherwise} \end{cases}$$

$$\Psi_i = \frac{R_c}{b_i} 1_{n\times n} \triangleq \psi_i 1_{n\times n}$$

$$\Gamma_i = \sum_{k=1}^{i} 2\frac{R_k}{b_i} Diag(0_{(k-1)\times(k-1)}, 1_{(n-k+1)\times(n-k+1)})$$

Clearly S_i is symmetric and positive semi-definite ($S_i \geq 0$).

The relation between all the control inputs and the flows of the AHU's can now be written compactly as

$$\omega = f(q) = \begin{bmatrix} \sqrt{q' S_1 q} \\ \vdots \\ \sqrt{q' S_n q} \end{bmatrix} \quad (31)$$

The Jacobian matrix $\partial f(q)$ of f at q is therefore

$$\partial f(q) = \begin{bmatrix} \rho_1(q) S_1 q & \cdots & \rho_n(q) S_n q \end{bmatrix}' \quad (32)$$

with $\rho_i(q) = \frac{1}{\sqrt{q' S_i q}}$. Now let $q > 0$ be any water inflow such that $\partial f(q)$ is nonsingular. By the inverse function theorem f has a (local) inverse $g \triangleq f^{-1}$. From this, we may combine (18) and (31) to obtain the following complete system description

$$\dot{z} = \Phi(z, \omega) = Fz + Gg(\omega) + E + H(z)g(\omega) \quad (33)$$

Note that the above control design used a coordinate change having ith coordinate function $\omega_i = f_i(q) = \bar{\alpha}_i q_i$ (and therefore $q_i = g_i(\omega) = \omega_i/\bar{\alpha}_i$) with $\bar{\alpha}_i$ define just below (13).

We now investigate whether the control law given by (25) stabilizes (33). First, recognize that the control law (25) is obtained under the assumption that only one of the branches is active at the time. Due to the interaction between the branch flows, described in (30) subsequently in (31), the steady-state value of the speed must be different in the case with one active branch compared to the case where all branches are active. Let the operating point for the speed in the case where all branches are active be denoted by $\omega^* \triangleq f(q^*)$, hence $\Phi(z^*, \omega^*) = 0$ so (z^*, ω^*) is a steady-state value for (33). However, $\bar{\phi}(z^*) \neq \omega^*$ in general, so $\bar{\phi}$ do not qualify as a state feedback control law for (33). We therefore modify $\bar{\phi}$ with an offset as

$$\omega = \phi(z) \triangleq \bar{\phi}(z) - \bar{\Lambda}q^* + \omega^*.$$

so $\Phi(z^*, \phi(z^*)) = \Phi(z^*, \omega^*) = 0$ and ϕ qualify as a state feedback control law for (33). Now consider the closed loop system

$$\dot{z} = \Phi(z, \phi(z)) = Fz + Gh(z) + E + H(z)h(z) \quad (34)$$

with $h(z) = g(\phi(z))$. To check the stability property of the equilibrium point z^* we linearize (34) at z^* and obtain

$$\begin{aligned} \dot{x} &= (F + G\partial g(\omega^*)\bar{\Lambda}K \\ &\qquad + H(z^*)\partial g(\omega^*)\bar{\Lambda}K + H(q^*))x && (35a) \\ &= (F + H(q^*) + (G + H(z^*))\partial g(\omega^*)\bar{\Lambda}K)x && (35b) \\ &\triangleq (\mathcal{A} + \bar{\mathcal{B}}K)x && (35c) \end{aligned}$$

with[1] $\partial g(\omega^*) = \partial f(q^*)^{-1}$ the Jacobian matrix of g at ω^*, and $\bar{\mathcal{B}} = \mathcal{B}\partial g(\omega^*)\bar{\Lambda}$. Compared to (27) one sees that the control gain K now is replaced by $\partial g(\omega^*)\bar{\Lambda}K$.

[1] As a passing remark we mention that from numerical examples it seems like ∂g is an L-matrix.

In conclusion, we see that the control gain K from the control law (25) needs to be chosen such that both the closed loop system (27) and (35) are stable. To obtain this we propose the following three stabilization procedures.

Stabilization procedure 1: First consider the Lyapunov equation for the set of disconnected controllers

$$\mathcal{P}(\mathcal{A}+\mathcal{B}K)+(\mathcal{A}+\mathcal{B}K)'\mathcal{P}+\varepsilon\mathcal{P}<0 \tag{36}$$

with $\varepsilon \geq 0$ a tuning parameter, $\mathcal{P} = Diag(\mathcal{P}_1,\ldots,\mathcal{P}_n)$ and $\mathcal{P}_i > 0$. Now let $\mathcal{Q} = \mathcal{P}^{-1}$, so $\mathcal{Q} = Diag(\mathcal{Q}_1,\ldots,\mathcal{Q}_n)$ with $\mathcal{Q}_i = \mathcal{P}_i^{-1}$. Using the change of variables $\mathcal{Q} = \mathcal{P}^{-1}$ we obtain

$$\mathcal{A}\mathcal{Q}+\mathcal{Q}\mathcal{A}'+\mathcal{B}Y+Y'\mathcal{B}'+\varepsilon\mathcal{Q}<0 \tag{37}$$

with $Y = K\mathcal{Q}$. Note that $Y = Diag(Y_1,\ldots,Y_n)$ with $Y_i = K_i\mathcal{Q}_i$. Hence we may find a block diagonal control gain matrix K stabilizing (27), by first solving the LMI (37) subject to the constraints

$$\mathcal{Q} = Diag(\mathcal{Q}_1,\ldots,\mathcal{Q}_n), \quad \mathcal{Q}_i > 0 \tag{38a}$$

$$Y = Diag(Y_1,\ldots,Y_n) \tag{38b}$$

and then set $K = Y\mathcal{P} = Diag(Y_1\mathcal{P}_1,\ldots,Y_n\mathcal{P}_n)$ which guarantees stability of the closed loop system (27). Due to the diagonal structure of K one could now check for stability of the closed loop system (35), however, we propose the following which also guarantee robustness towards parameter uncertainties.Therefore, secondly to verify that the closed loop system with the hydraulic interconnection (35) is stable, one should verify the LMI (in the variables $\mathcal{R} > 0$ and $\tau > 0$)

$$\begin{bmatrix} \mathcal{R}\mathcal{A}+\mathcal{A}'\mathcal{R}+\tau K'K & \mathcal{R}\bar{\mathcal{B}} \\ \bar{\mathcal{B}}'\mathcal{R} & -\tau I \end{bmatrix} < 0 \tag{39}$$

which guarantee's stability of $\mathcal{A}+\bar{\mathcal{B}}\Delta K$ for all Δ with $\|\Delta\|_2 \leq 1$, hence in particular for $\Delta = I$.

Stabilization procedure 2: This procedure will produce a control gain K stabilizing the closed loop systems (27) and (35) simultaneous. To obtain this we simply solve (37) and (39) simultaneously. In more details, apply Schur complement to (39) and set $\mathcal{R} = \mathcal{P}$ and $K = Y\mathcal{P}$, then use the change of variables $\mathcal{Q} = \mathcal{P}^{-1}$ and apply Schur complement again to obtain

$$\begin{bmatrix} \mathcal{A}\mathcal{Q}+\mathcal{Q}\mathcal{A}'+\vartheta\bar{\mathcal{B}}'\bar{\mathcal{B}} & Y' \\ Y & -\vartheta I \end{bmatrix} < 0 \tag{40}$$

with $\vartheta = 1/\tau$. It follows that the control gain $K = Y\mathcal{P} = Y\mathcal{Q}^{-1}$ obtained by solving the LMI's (37) and (40) subject to the constraints $\varepsilon, \vartheta \geq 0$ and (38) will stabilizing the closed loop systems (27) and (35) simultaneous.

Stabilization procedure 3: If the above two procedures turn out to be too conservative one can solve (37) and

$$\mathcal{A}\mathcal{Q}+\mathcal{Q}\mathcal{A}'+\bar{\mathcal{B}}Y+Y'\bar{\mathcal{B}}'+\tilde{\varepsilon}\mathcal{Q}<0 \tag{41}$$

simultaneously. As with procedure 2, this will yield a control gain stabilizing both (27) and (35). However, no robustness guarantees are given when applying this procedure.

Note that procedures 1 and 2 both guarantee robustness since both use (39). However, procedure 2 is conservative compared to procedure 1 since the variable $\mathcal{Q}$ is used in both LMI's when using procedure 2.

The synthesis procedures and robustness analysis are illustrated in the next section.

IV. Numerical Studies

To exemplify the above results, numerical tests are presented in the following. The tests are done using a simulation model developed based on (6) and (4) describing the nonlinear behavior of the system. The model has four AHU's with the outlet air temperature controlled by local controllers designed using the procedures described in Section III. The parameters for the hydraulic network connecting the AHU's and the AHU's are given in Table I.

For comparison, local controllers are designed for each of the AHU's neglecting the influence of the hydraulic network from the adjacent AHU's. Standard pole placement is used for this design with the closed loop poles {-0.04, -0.02+i·0.02, -0.02-i·0.02} for each AHU loop. These closed loop poles lead to the desired response for each separate control loop. However, introducing the influence between AHU's from the hydraulic network (meaning changing the input matrix from $\mathcal{B}$ to $\bar{\mathcal{B}}$) leads to the following poles {5.2630, 5.9125, 5.9870, 2.6053, 0.0007, 0.0005, 0.0004, 0.0004, -0.0312, -0.0312, -0.0312, -0.0312} for the closed system, which clearly is unstable. This verifies the need for a more involved design procedure.

All three design procedures in Section III have been solved using YALMIP [7] with Matlab, and lead to stable designs for both the separated systems (input matrix $\mathcal{B}$) and the connected system (with input $\bar{\mathcal{B}}$). Results with design procedure 2 are presented in the following. With this procedure, it was easier to tune the system to a desired slow response. The response with a step in the temperature reference from 293.15 [K] (20 ^oC) to 294.15 [K] (21 ^oC) is shown in Fig. 4.

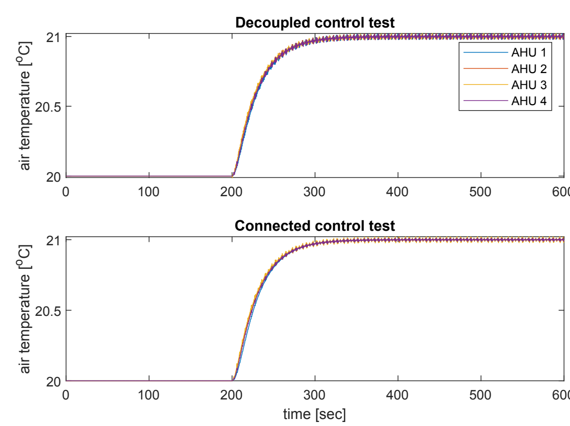

Fig. 4. Controller response to a 1 degree change in the air supply reference for all 4 AHU's. The top plot shows the responses when each AHU is running separately, and the lower plot shows the responses when all AHU's are running at the same time and therefore experience the influence from adjacent AHU's.

TABLE I

MODEL PARAMETERS USED FOR SIMULATION AND ANALYSIS.

Parameter	Sym	$i = 1$	$i = 2$	$i = 3$	$i = 4$	Unit
Heat transfer between water and air in AHU	B	$24.00 \cdot 10^3$	$14.00 \cdot 10^3$	$12.00 \cdot 10^3$	$20.00 \cdot 10^3$	[W·K^{-1}]
Specific heat coefficient for water	C_w	$4.17 \cdot 10^6$				[J·m^{-3}·K^{-1}]
Specific heat coefficient for air	C_a	891.80				[J·m^{-3}·K^{-1}]
Nominal air temperature reference	T_i^*	293.15				[K]
Nominal ambient temperature	T_a	308.15				[K]
Nominal supply water temperature	θ_c	283.15				[K]
Water volume in AHU	V_w	$230.00 \cdot 10^{-3}$	$134.00 \cdot 10^{-3}$	$115.00 \cdot 10^{-3}$	$192.00 \cdot 10^{-3}$	[m^3]
Air volume in AHU	V_a	9.00	5.20	4.50	7.50	[m^3]
Nominal water flow through AHU	q	$5.75 \cdot 10^{-3}$	$3.36 \cdot 10^{-3}$	$2.89 \cdot 10^{-3}$	$4.81 \cdot 10^{-3}$	[m^3/s]
Nominal air flow through AHU	Q	8.97	5.23	4.49	7.48	[m^3/s]
Hydraulic resistance of the chilled water source	R_c	$8.85 \cdot 10^3$				[pa/(m^3 /s)]
Hydraulic resistance of pipelines to branch	R	$8.85 \cdot 10^3$	$20.45 \cdot 10^3$	$42.23 \cdot 10^3$	$108.26 \cdot 10^3$	[pa/(m^3/s)]
Hydraulic resistance of branch	r	$151.23 \cdot 10^3$	$442.59 \cdot 10^3$	$599.11 \cdot 10^3$	$216.51 \cdot 10^3$	[pa/(m^3/s)]
Pump constant	a	$453.69 \cdot 10^3$	$2212.96 \cdot 10^3$	$4193.79 \cdot 10^3$	$1948.61 \cdot 10^3$	[-]
Pump constant	b	30.00	50.00	70.00	90.00	[-]

The top plot in Fig. 4 shows the responses when each AHU is running separately, whereas the lower plot shows the responses when all AHU's are running at the same time and therefore experience the influence from adjacent AHU's.

It should be noted that for the tested system the numerical values of τ (design procedure 1) and ϑ (design procedure 2) are close to zero, though YALMIP indicates a primal and dual feasibility. These small values mean that we cannot claim robustness for the test system when using these design procedures. However, as mentioned above, stability is obtained.

V. CONCLUSION

This work considers the design of local AHU controllers in a distributed pump setup for HVAC systems. A robust design procedure via LMIs is presented that ensures robust operation and at the same time opens up for stepwise test and commissioning procedures. Such stepwise procedures are expected to enable better building commissioning and therefore better building performance when taken into use.

Further research includes restricting the robustness formulation to enable better numerical properties and include performance requirements in the design. Moreover, as an alternative to the designed state feedback control using standard PI control is interesting, as the commissioning staff typically is familiar with this type of controller. Finally, only the case where all controllers are active and the cases with only one active controller are considered. In future research, we will relax this.

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