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# Control Design for Trajectory Tracking and Stabilization of Sensor LOS in an Inertially Stabilized Platform

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***Abstract*—Optical sensors are often mounted on moving platforms to aid in a variety of tasks like data collection, surveillance and navigation. This necessitates the precise control of the inertial orientation of the optical sensor line-of-sight (LOS) towards a desired stationary or mobile target. A two-axes gimbal assembly is considered to achieve this control objective which can be decomposed into two parts - stabilization and tracking. A novel state space model is proposed based on the dynamics of a two-axes gimbal system. Using a suitable change of variables, this state space model is transformed into an LTI system. Feedback linearization based control laws are proposed that achieve the desired objectives of stabilization and tracking. The effectiveness of these control laws are demonstrated via simulation in MATLAB based on a typical model of a two-axes gimbal system.**

***Index Terms*—feedback linearization, two-axes gimbal, output control**

## I. Introduction

Optical sensors such as IR, radar and camera are often mounted on moving platforms like ground vehicles, aircraft or marine vessels to collect data, conduct surveillance or aid in navigation. Such uses can be widely observed in applications spanning military reconnaissance, agricultural monitoring, or guided weapons systems. In these applications it is imperative to precisely control the inertial orientation of the optical sensor line-of-sight (LOS) towards a desired stationary or mobile target. An inertially stabilized platform (ISP), achieved through gimbal assemblies, is an appropriate mechanism to achieve this desired control [1], [2].

An ISP comprises of three modules: an electromechanical assembly interfacing the optical sensor with the platform body, a control system orienting the sensor in the desired direction, and auxiliary equipment computing the target location [3]. The focus of this paper lies in the design of the control system for the ISP, which typically consists of two subsystems - the inner stabilization loop and the outer tracking loop [4]. The objective of the stabilization loop is to maintain the inertial orientation of the optical sensor in order to obtain jitter-free high quality data. The inner loop obtains this objective by controlling the angular rates of the sensor LOS. Meanwhile, the objective of the outer tracking loop is to orient the sensor LOS towards the desired target. Based on the desired target location, the outer loop provides desired rate commands to the inner loop. This cascading allows the optical sensor LOS to be oriented towards the desired target and collect high quality jitter-free data.

To achieve LOS stabilization and tracking, the gimbal system must counteract all torque disturbances while orienting the sensor LOS towards the desired target. The torque disturbances originate from three primary sources: platform body motion, cross-coupling disturbances, and gimbal system mass unbalance. This necessitates a precise mathematical model for the system. In the literature [5]–[7], equations of motion for each gimbal axis have been derived with varied assumptions on the mass unbalance and the symmetry of the system using either Newton's laws or Lagrange equations.

Until recently, LOS stabilization and tracking have been accomplished using classical control methods, often variations of the well-established PID controller design [8], [9]. Nevertheless, the use of modern controllers like LQG/LTR controller, adaptive PID controllers, and sliding mode control have been explored in the literature [10]–[14]. However, there has been limited research dealing with the control objectives of stabilization and tracking in an ISP using the methods of nonlinear analysis and control.

This paper introduces a feedback control approach for LOS stabilization and tracking. Unlike the conventional approach of employing separate controllers for the inner and outer loops, this work introduces a unified control law. First, a control law is introduced to achieve stabilization. This is followed by another control law that enables seamless LOS tracking without the need of an inner loop for stabilization. The main contributions of this paper can be surmised in the following points:

1) First, a novel state space model for the two-axes gimbal system is developed by making an appropriate choice for the state variables, assuming the gimbal to be symmetric and having no mass unbalance.
2) The choice of state variables renders the dynamical system as a set of two decoupled Brunovsky canonical

subsystems (see Chapter 13 [15]), which allows a transformation into a linear dynamical system using a suitable change of variables.

3) A feedback control law is then designed to facilitate exponential convergence to a desired angular velocity trajectory. Stabilization is obtained as a consequence of this result. Additionally, a feedback control law is designed that achieves exponential convergence to a desired LOS trajectory, thus achieving LOS tracking.
4) The efficacy of these control laws is then demonstrated on a gimbal system model using MATLAB simulation.

This paper is organized as follows: The comprehensive model of a two-axes gimbal system along with the control objectives are provided in Section II. In Section III, a novel state space model is proposed, and a change of variables is considered which transforms the nonlinear state space model into a linear system. The nonlinear state feedback control laws for the stabilization and tracking of sensor LOS are proposed in Section IV. The simulation of the proposed control laws has been implemented on MATLAB based on the model of a typical two-axes gimbal system in Section V. Finally in Section VI, the conclusions are drawn and possible future directions are cited.

## II. Two-axes Gimbal Dynamics

Consider a two-axes yaw-pitch gimbal system as shown in Figure 1. The outer gimbal is the yaw gimbal while the inner gimbal is the pitch gimbal. The optical sensor is placed on the inner gimbal. A rate gyro is placed on the platform to measure its angular velocity with respect to an inertial frame of reference.

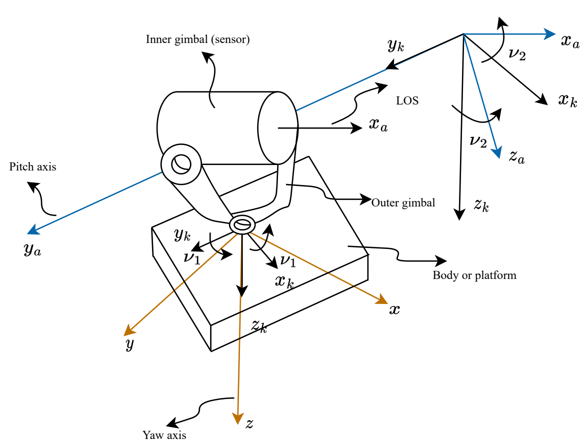



Fig. 1: Two-axes gimbal system

Three coordinate frames are introduced as follows: frame $B$ fixed to the platform body with axes $(x, y, z)$, frame $K$ fixed to the yaw (outer) gimbal with axes $(x_k, y_k, z_k)$ and frame $A$ fixed to the pitch (inner) gimbal with axes $(x_a, y_a, z_a)$. The $x_a$ axis coincides with the sensor optical axis. The center of rotation is assumed to be at the common origin of all the coordinate frames, *i.e.* the gimbals are considered to be rigid bodies with no mass unbalance.

The inertial angular velocity vectors of frames $B, K$, and $A$, respectively are as follows:

$$\vec{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \vec{\omega}_K = \begin{bmatrix} p_k \\ q_k \\ r_k \end{bmatrix}; \vec{\omega}_A = \begin{bmatrix} p_a \\ q_a \\ r_a \end{bmatrix} \tag{1}$$

Further, let $\theta_q(t) := \int_0^t q_a(\tau)d\tau$ and $\theta_r(t) := \int_0^t r_a(\tau)d\tau$ represent the elevation and azimuth angles of the sensor LOS respectively with respect to an inertial frame of reference. Here onwards, all the angular velocity terms and $\theta_q, \theta_r$ are used with an implicit dependence on time.

Now, let $\nu_1$ ($\nu_2$) denote the angle of rotation about the $z$-axis ($y_k$-axis) to carry body-fixed frame $B$ (yaw gimbal frame $K$) into coincidence with the yaw gimbal frame $K$ (pitch gimbal frame $A$). As a result, the coordinate transformation matrices from frame $B$ to $K$ and $K$ to $A$ are given by:

$$R_{KB} = \begin{bmatrix} \cos\nu_1 & \sin\nu_1 & 0 \\ -\sin\nu_1 & \cos\nu_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$R_{AK} = \begin{bmatrix} \cos\nu_2 & 0 & -\sin\nu_2 \\ 0 & 1 & 0 \\ \sin\nu_2 & 0 & \cos\nu_2 \end{bmatrix} \tag{3}$$

Utilizing the angular velocity vectors given in (1), along with the coordinate transformation matrices defined above, the following angular velocity relations can be obtained:

$$\vec{\omega}_K - R_{KB}\vec{\omega}_B = \begin{bmatrix} 0 \\ 0 \\ \dot{\nu}_1 \end{bmatrix}, \text{ and } \vec{\omega}_A - R_{AK}\vec{\omega}_K = \begin{bmatrix} 0 \\ \dot{\nu}_2 \\ 0 \end{bmatrix}. \tag{4}$$

Simplifying the above equation, the inertial angular velocity of frames $K$ and $A$ can be written in terms of the inertial angular velocity of frames $B$ and $K$ respectively as

$$\begin{aligned} p_k &= p\cos\nu_1 + q\sin\nu_1, & p_a &= p_k\cos\nu_2 - r_k\sin\nu_2, \\ q_k &= -p\sin\nu_1 + q\cos\nu_1, & q_a &= q_k + \dot{\nu}_2, \\ r_k &= r + \dot{\nu}_1, & r_a &= p_k\sin\nu_2 + r_k\cos\nu_2. \end{aligned} \tag{5}$$

Now, consider the inertia matrices of the pitch and yaw gimbal as

$$\text{Inner gimbal}: J_A = \begin{bmatrix} J_{ax} & D_{xy} & D_{xz} \\ D_{xy} & J_{ay} & D_{yz} \\ D_{xz} & D_{yz} & J_{az} \end{bmatrix} \tag{6}$$

$$\text{Outer gimbal}: J_K = \begin{bmatrix} J_{kx} & d_{xy} & d_{xz} \\ d_{xy} & J_{ky} & d_{yz} \\ d_{xz} & d_{yz} & J_{kz} \end{bmatrix} \tag{7}$$

where the diagonal terms represent the moments of inertia while the off diagonal terms represent the products of inertia.

Two equations of motion can then be obtained for the two gimbal axes as shown in [5]. Assuming the total external torque about the pitch gimbal $y_a$-axis to be $T_y$, the equation of motion for the pitch gimbal can be derived as

$$J_{ay}\dot{q_a} = T_y + T_{D_y}, \tag{8}$$

where $T_{D_y}$ represents all the inertia disturbances that arise along the $y_a$-axis of the gimbal system. These inertia disturbance terms are functions of the inertia matrices, the angular velocity vector of frame $B$ and the angles $\nu_1$ and $\nu_2$.

A similar equation can be written for the yaw gimbal motion given by

$$J_k \dot{r}_k = T_z + T_{D_z}, \tag{9}$$

where $J_k$ is an inertia term dependent on the angle of rotation $\nu_2$, $T_z$ is the total external torque about the yaw gimbal $z_k$-axis, and $T_{D_z}$ contains all the inertia disturbances due to the design of the gimbal system.

Due to the inherent characteristics of the gimbal system and the coupling between the yaw and pitch gimbals, the system is susceptible to various inertia disturbances. However, system design typically aims to minimize these disturbances. One effective approach is to design a symmetric gimbal system without any mass unbalance. The symmetric assumptions in this work are entailed by the following conditions:

$$\begin{aligned} &D_{xy} = D_{xz} = D_{yz} = 0,\ J_{ax} = J_{az}, \\ &d_{xy} = d_{xz} = d_{yz} = 0,\ J_{kx} + J_{ax} = J_{ky}. \end{aligned} \tag{10}$$

Under the assumptions (10) we have $J_k = J_{kz} + J_{az}$, and the equations of motion for the pitch and yaw gimbals can be written as

$$J_{ay}\dot{q}_a = T_y, \text{ and } J_k \dot{r}_k = T_z - J_{ay} p_k q_a. \tag{11}$$

While the assumptions in the gimbal system design (10) remove most of the identified noise sources in the system, it is possible for design errors to persist resulting in noise entering the system through the dynamic equations in (11). Note that the external torque is provided primarily by the motors placed in each gimbal axis.

With an understanding of the gimbal system's dynamics, the control objectives of stabilization and tracking can be mapped onto the system variables in the following manner:

1) **Stabilization -** The primary objective of LOS stabilization hinges on the isolation of torque disturbances affecting the pitch and yaw axes. This is achieved by driving the angular velocities $q_a$ and $r_a$ to zero.
2) **Tracking -** The objective of tracking refers to the sensor LOS oriented towards the desired target, while rejecting the torque disturbances. This is obtained by the sensor LOS elevation ($\theta_q$) and azimuth ($\theta_r$) angles following a desired target trajectory.

## III. State Space Modelling

Given the dynamic equations of a two-axes gimbal system (11), consider a state space model described by a state vector $\mathbf{x} \in \mathcal{X} := S^1 \times \mathbb{R} \times S^1 \times \mathbb{R}$ and a control vector $\mathbf{u} \in \mathbb{R}^2$ given by

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \nu_2 \\ \dot{\nu}_2 \\ \nu_1 \\ \dot{\nu}_1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} T_y \\ T_z \end{bmatrix}. \tag{12}$$

Clearly, $\dot{x}_1 = x_2$ and $\dot{x}_3 = x_4$. From (5), $x_2 = \dot{\nu}_2 = q_a - q_k$, and using (11), we have

$$\dot{x}_2 = \frac{u_1}{J_{ay}} + f_1(t, \mathbf{x}), \tag{13}$$

where $f_1(t,\mathbf{x}) = \dot{p}\sin x_3 + x_4 p \cos x_3 - \dot{q}\cos x_3 + x_4 q \sin x_3$ is obtained by differentiating (5). Again from (5), $x_4 = \dot{\nu}_1 = r_k - r$ and using (11), we have

$$\dot{x}_4 = \frac{u_2}{J_k} + f_2(t, \mathbf{x}). \tag{14}$$

where $f_2(t,\mathbf{x}) = -\dot{r} - \frac{J_{ay}}{J_k} p_k q_a = -\dot{r} - \frac{J_{ay}}{J_k}(p\cos x_3 + q\sin x_3)(-p\sin x_3 + q\cos x_3 + x_2)$ is obtained by differentiating (5). Thus the state space model is represented by the dynamics equations

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \frac{u_1}{J_{ay}} + f_1(t,\mathbf{x}) \\ x_4 \\ \frac{u_2}{J_k} + f_2(t,\mathbf{x}) \end{bmatrix}. \tag{15}$$

In this state space representation of the system, the body fixed angular velocities of the platform, being measured from the rate gyro placed on the platform, are assumed to be known quantities and are thus considered as functions of time. Thus, the state dynamics can be written as a function of time, states and controls.

*Remark* 1. Under the assumptions outlined in (10), significant reductions in inertial disturbances are evident. However, the products of inertia terms may not be entirely eliminated due to potential design errors. One effective way to circumvent this issue is by modifying the functions $f_1$ and $f_2$ to accommodate the inertia disturbance terms that arise due to products of inertia on the RHS of the dynamics equations (13) and (14). However, this has been avoided to keep the expressions concise as it does not aid in any conceptual development. In the case that the products of inertia terms are relatively insignificant to the moments of inertia, which is often the scenario, these disturbances can be treated as noise, akin to other sources of mechanical noise in the system. In Section V, the results of the proposed control laws are demonstrated in the presence of noise.

Next, a change of variables is studied that transforms the nonlinear state space model (15) into a linear time-invariant (LTI) system.

**Lemma 1.** *The controls*

$$\begin{aligned} u_1 &= J_{ay}[v_1 - f_1(t,\mathbf{x})] \\ u_2 &= J_k[v_2 - f_2(t,\mathbf{x})] \end{aligned} \tag{16}$$

*are well defined and transform the state space model considered in* (15) *into a linear dynamical system given by*

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{v}, \tag{17}$$

*where*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \ \textit{and } \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{18}$$

*Proof.* Consider the controls $\mathbf{u}$ as proposed in (16). Using these inputs in the state space model, we have the resulting dynamics given by

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \frac{J_{ay}[v_1 - f_1(t,\mathbf{x})]}{J_{ay}} + f_1(t,x) \\ x_4 \\ \frac{J_k[v_2 - f_2(t,\mathbf{x})]}{J_k} + f_2(t,x) \end{bmatrix}, \qquad (19)$$
$$= \begin{bmatrix} x_2 \\ v_1 \\ x_4 \\ v_2 \end{bmatrix}.$$

Thus, the resulting dynamical system can be written as an LTI system with the controls $\mathbf{v}$ and the $A$, $B$ matrices specified by (18). Similarly, taking the inverse of the transformation considered in (16) results in

$$\begin{aligned} v_1 &= \frac{u_1}{J_{ay}} + f_1(t,\mathbf{x}), \\ v_2 &= \frac{u_2}{J_k} + f_2(t,\mathbf{x}). \end{aligned} \qquad (20)$$

Applying these controls to the LTI system (17), results in the original state space system (15).

□

*Remark* 2. As the original state space model (15) is proved equivalent to an LTI system (17) in Lemma 1, any behaviour that can be obtained in the LTI system can be reproduced in the original model only via a change of variables (16).

## IV. Feedback Control Law

In this section, feedback control laws are proposed to achieve the control objectives of stabilization and tracking. To this end, first consider the definition of two functions.

**Definition 1.** Consider the functions $g_1, g_2 : \mathbb{R}^+ \times \mathcal{X} \to \mathbb{R}$ defined by

$$\begin{aligned} g_1(t,\mathbf{x}) &= -\dot{p}\sin x_3 - x_4 p\cos x_3 \\ &\quad + \dot{q}\cos x_3 - x_4 q \sin x_3, \text{ and} \\ g_2(t,\mathbf{x}) &= (\dot{p}\cos x_3 - x_4 p \sin x_3 + \dot{q}\sin x_3 \\ &\quad + x_4 q\cos x_3)\sin x_1 + (p\cos x_3 \\ &\quad + q\sin x_3)x_2\cos x_1 - x_2 r\sin x_1 \\ &\quad - x_2 x_4 \sin x_1 + \dot{r}\cos x_1. \end{aligned} \qquad (21)$$

Here onwards the usage of the terms $g_1$ and $g_2$ indicate the functions defined in (21) with an implicit dependence on time and the states.

The next result provides a control law that enables the sensor pitch ($q_a$) and yaw ($r_a$) angular velocities to follow any twice differentiable desired rate trajectories $q_a^d(t)$ and $r_a^d(t)$ respectively.

**Theorem 1.** *Consider the LTI system given by* (17), (18). *Let $q_a^d(t)$ and $r_a^d(t)$ be once differentiable desired trajectories and let $c_1$, $c_2$ be positive real quantities, then the controls*

$$\begin{aligned} v_1 &= -g_1 + \dot{q}_a^d + c_1(q_a^d - q_a), \\ v_2 &= \frac{-g_2 + \dot{r}_a^d + c_2(r_a^d - r_a)}{\cos x_1}, \end{aligned} \qquad (22)$$

*where $g_1$ and $g_2$ are as introduced in Definition 1, enable the sensor angular velocities $q_a$ and $r_a$ to converge exponentially to the desired rate trajectories $q_a^d$ and $r_a^d$, with decay rates $-c_1$ and $-c_2$ respectively.*

*Proof.* Consider the linear state space model (17), (18), then the angular velocities $q_a$ and $r_a$ can be written as functions of time and states from equation (5) as follows:

$$\begin{aligned} q_a &= -p\sin x_3 + q\cos x_3 + x_2, \\ r_a &= p\cos x_3 \sin x_1 + q\sin x_3 \sin x_1 \\ &\quad + r\cos x_1 + x_4\cos x_1. \end{aligned} \qquad (23)$$

Differentiating $q_a$ and $r_a$ we get

$$\dot{q}_a = g_1(t,\mathbf{x}) + v_1, \text{ and } \dot{r}_a = g_2(t,\mathbf{x}) + v_2\cos x_1. \qquad (24)$$

Plugging in the controls proposed in (22), we get the output dynamics as

$$\begin{aligned} \dot{q}_a &= \dot{q}_a^d + c_1(q_a^d - q_a), \\ \dot{r}_a &= \dot{r}_a^d + c_2(r_a^d - r_a). \end{aligned} \qquad (25)$$

Then the errors of the actual angular velocity with the desired trajectories $e_q := q_a^d - q_a$ and $e_r := r_a^d - r_a$ follow the following first order dynamics:

$$\begin{aligned} \dot{e}_q + c_1 e_q &= 0, \\ \dot{e}_r + c_2 e_r &= 0. \end{aligned} \qquad (26)$$

By any choice of the control parameters $c_1$ and $c_2$ as positive real values leads to the error dynamics being exponentially stable. Thus, the angular velocities $q_a$ and $r_a$ converge to the desired rate trajectories exponentially fast. □

*Remark* 3. As shown in Theorem 1, these control parameters denote the decay rate of the angular velocity to the desired trajectory. Thus, the choice for these parameters need not be arbitrary and can be made as per the required decay rate considering a trade-off on the demand on the control input.

An immediate corollary following Theorem 1 is that the control law necessary for obtaining LOS stabilization is a special case of the controls given in (22).

**Corollary 1.** *Consider the LTI system given by* (17), (18). *Let $c_1$ and $c_2$ be positive real quantities, then the controls*

$$\begin{aligned} v_1 &= -g_1 - c_1 q_a, \\ v_2 &= \frac{-g_2 - c_2 r_a}{\cos x_1}, \end{aligned} \qquad (27)$$

*where $g_1$ and $g_2$ are as introduced in Definition 1, achieve LOS stabilization by driving the sensor angular velocities $q_a$ and $r_a$ to zero at an exponential rate of $-c_1$ and $-c_2$ respectively.*

*Proof.* Substituting the desired trajectories $q_a^d(t)$ and $r_a^d(t)$ as identically equal to zero in Theorem 1 proves this result. □

Having achieved LOS stabilization, the next result proposes a control law that deals with LOS tracking.

**Theorem 2.** *Consider the LTI system given by* (17), (18). *Let $\theta_q^d(t)$ and $\theta_r^d(t)$ be twice-differentiable desired trajectory of the elevation ($\theta_q$) and the azimuth ($\theta_r$) angles of the sensor*

*LOS. Let $c_1$, $c_2$, $c_3$, $c_4$ be positive real quantities, then consider the controls*

$$
\begin{aligned}
v_1 &= -g_1(t,\mathbf{x}) + \ddot{\theta}_q^d + c_1(\dot{\theta}_q^d - \dot{\theta}_q) + c_2(\theta_q^d - \theta_q),\\
v_2 &= \frac{1}{\cos x_1}\Big(-g_2(t,\mathbf{x}) + \ddot{\theta}_r^d \\
&\qquad + c_3(\dot{\theta}_r^d - \dot{\theta}_r) + c_4(\theta_r^d - \theta_r)\Big),
\end{aligned} \tag{28}
$$

*where $g_1$ and $g_2$ are as introduced in Definition 1. Then the elevation ($\theta_q$) and azimuth ($\theta_r$) angles of the sensor LOS converge exponentially to the desired trajectories $\theta_q^d(t)$ and $\theta_r^d(t)$ respectively.*

*Proof.* Consider the linear state space model (17), (18). The double derivatives of the elevation and azimuth angles of the sensor LOS can be written as $\ddot{\theta}_q = \dot{q}_a$ and $\ddot{\theta}_r = \dot{r}_a$. From the proof of Theorem 1, it is clear that $\dot{q}_a = g_1 + v_1$ and $\dot{r}_a = g_2 + v_2 \cos x_1$. Now plugging in the controls proposed in (28), we get the second order dynamics of the outputs as

$$
\begin{aligned}
\ddot{\theta}_q &= \ddot{\theta}_q^d + c_1(\dot{\theta}_q^d - \dot{\theta}_q) + c_2(\theta_q^d - \theta_q), \text{ and}\\
\ddot{\theta}_r &= \ddot{\theta}_r^d + c_3(\dot{\theta}_r^d - \dot{\theta}_r) + c_4(\theta_r^d - \theta_r).
\end{aligned} \tag{29}
$$

Denote $e_q := \theta_q^d - \theta_q$ and $e_r := \theta_r^d - \theta_r$. Then from the equations above, it is evident that the errors of $\theta_q$ and $\theta_r$ from their respective desired trajectories follow a second order dynamics given by

$$
\begin{aligned}
\ddot{e}_q + c_1\dot{e}_q + c_2 e_q &= 0\\
\ddot{e}_r + c_3\dot{e}_r + c_4 e_r &= 0
\end{aligned} \tag{30}
$$

By a suitable choice of the control parameters $c_1, c_2, c_3$ and $c_4$, the decay rate of the errors can be desirably controlled. Thus, this control law drives the sensor LOS towards its desired trajectory and can do so fast enough by making a large enough choice of the control parameters. □

*Remark* 4. As mentioned in Remark 2, the behaviour observed in the LTI system (17) using the controls (22) or (28), can be reproduced in the original state space model (15) by using controls $\mathbf{u}$ as given by (16). Further, the choice of control parameters can be made by considering the desired decay rate (as discussed in Remark 3).

*Remark* 5. Note that, the control $v_2$ cannot directly affect $r_a$ or consequently $\theta_r$. This control is achieved through the control of $r_k$ which then affects $r_a$ through a gain of $\cos x_1$. Hence, in both the stabilizing (22), (27) and the tracking (28) controls, there is a $\cos x_1$ term in the denominator for the expression of $v_2$. Thus, as the loop gain ($\cos x_1$) approaches zero, the demand on the control $v_2$ blows up which appears analytically as the cosine term in the denominator.

## V. Simulations

The proposed state space model along with the feedback control laws have been implemented in MATLAB R2023a. The two-axes gimbal system considered is assumed with the following moments of inertia: $J_{ax} = 0.003$, $J_{ay} = 0.008$, $J_{az} = 0.003$, $J_{kx} = 0.003$, $J_{ky} = 0.006$, and $J_{kz} = 0.0003$. The products of inertia terms are considered zero in line with assumptions (10). The roll, pitch and yaw angular velocities of the platform are assumed to be sinusoidally varying curves given by $p(t) = 0.1\sin(\frac{\pi}{15}t)$, $q(t) = 0.1\sin(\frac{\pi}{20}t)$, and $r(t) = 0.2\sin(\frac{\pi}{15}t)$. This can be seen in Figure 2. The cosine term in the denominator of the control $v_2$ has been dealt with a saturation block by restricting its value beyond a threshold around zero.

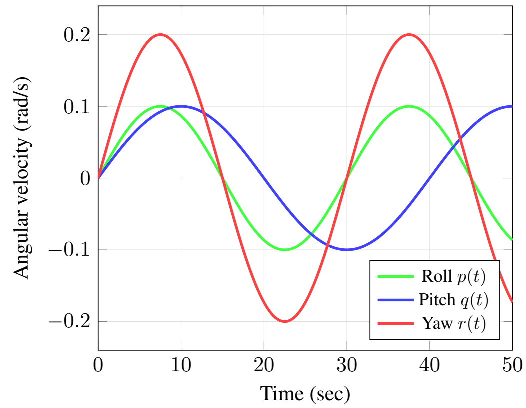
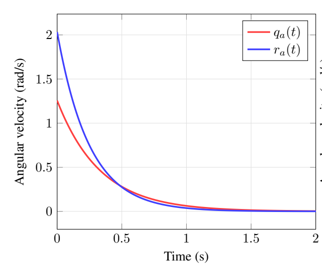
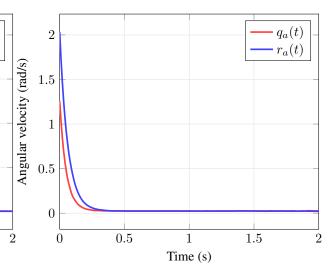



Fig. 2: Platform body motion

*Example* 1. As already discussed, to achieve stabilization of sensor LOS, the output angular velocities of the pitch channel, $q_a$ and of the yaw channel, $r_a$ are driven to zero. These responses can be seen in Figure 3a. The control parameters used for this case are $c_1 = 3$ and $c_2 = 4$. Another case with the presence of noise has been considered. Using the same control parameters however did not yield a stabilizable result. But, by cranking up the control parameters (effectively increasing decay rates) to $c_1 = 20$ and $c_2 = 16$, stabilization of sensor LOS can be achieved as shown in Figure 3b.



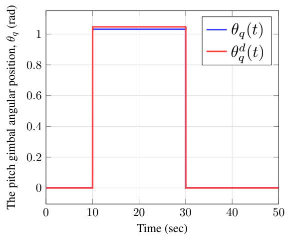

(a) Angular velocities without noise



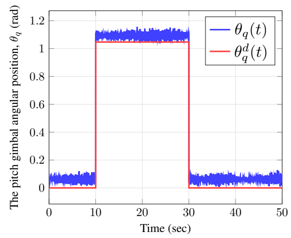

(b) Angular velocities with noise

Fig. 3: Angular velocity behaviour during stabilization

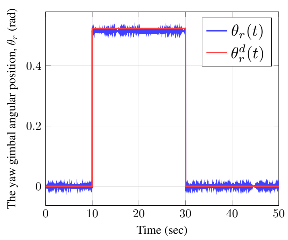
*Example* 2. Consider a situation where the desired trajectories for the azimuth and elevation angles are given by step functions. The aim of the azimuth and elevation angles is to reach a step value of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ for a period of $20s$ before dropping back to the initial value. The behaviour of the sensor LOS orientation can be observed in Figures 4a and 4c in comparison with the desired. A case considering mechanical noise for the both the angles are shown in Figure 4b and 4d. These responses have been obtained by a use of the control parameters: $c_1 = 6$, $c_2 = 8, c_3 = 9$, and $c_4 = 10$. A comparison with the classical PID control has been done in the online supplementary version of the paper [16].



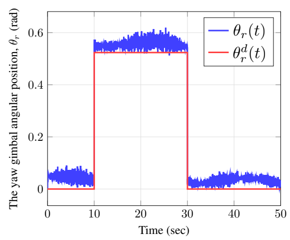

(a) Elevation angle ($\theta_q$) without noise




(b) Elevation angle ($\theta_q$) with noise



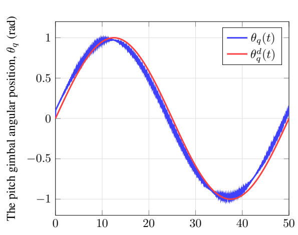

(c) Azimuth angle ($\theta_r$) without noise



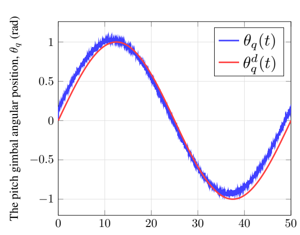

(d) Azimuth angle ($\theta_r$) with noise

Fig. 4: Sensor LOS tracking a step signal

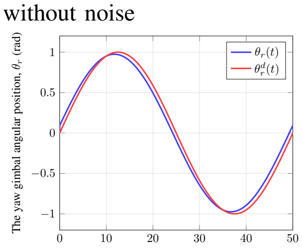
*Example* 3. Consider a situation where the desired trajectories for the azimuth and elevation angles are given by sinusoidal functions. This desired trajectory is defined as $\sin(\frac{\pi}{25}t)$. The behaviour of the sensor LOS orientation can be observed in Figures 5a and 5c in comparison with the desired. A case considering mechanical noise for the both the angles are shown in Figure 5b and 5d. These responses have been obtained by a use of the control parameters: $c_1 = 8,\ c_2 = 10, c_3 = 6$, and $c_4 = 8$.



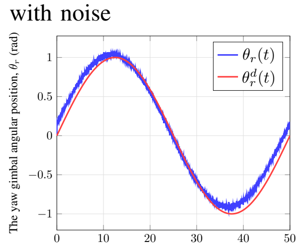

(a) Elevation angle ($\theta_q$) without noise




(b) Elevation angle ($\theta_q$) with noise




(c) Azimuth angle ($\theta_r$) without noise




(d) Azimuth angle ($\theta_r$) with noise

Fig. 5: Sensor LOS tracking a sinusoidal signal

## VI. Conclusion

This paper considers a two-axes gimbal system with symmetry and no mass unbalance. A novel state space model is proposed to capture the system dynamics in a symmetric setting. Using a suitable change of variables, the nonlinear state space model is transformed into a linear time-invariant system. This transformation can also be attained even under asymmetric conditions, and symmetry has been assumed only to keep the expressions concise. Control laws are proposed for the transformed system that enable sensor LOS stabilization and tracking. Further, it has been shown that the control objectives are achieved exponentially fast.

## References


[1] J. DEBRUIN, "Control systems for mobile satcom antennas," *IEEE Control Systems Magazine*, vol. 28, no. 1, pp. 86–101, 2008.

[2] J. Negro, S. F. Griffin, B. L. Kelchner, and V. Beazel, "Inertial stable platforms for precision pointing of optical systems in aerospace applications," *AIAA Journal*, vol. 61, no. 8, p. 3234–3246, 2023.

[3] M. K. MASTEN, "Inertially stabilized platforms for optical imaging systems," *IEEE Control Systems Magazine*, vol. 28, no. 1, pp. 47–64, 2008.

[4] J. HILKERT, "Inertially stabilized platform technology concepts and principles," *IEEE Control Systems Magazine*, vol. 28, no. 1, pp. 26–46, 2008.

[5] B. Ekstrand, "Equations of motion for a two-axes gimbal system," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 37, no. 3, pp. 1083–1091, 2001.

[6] M. Abdo, A. R. Vali, A. Toloei, and M. R. Arvan, "Research on the cross-coupling of a two axes gimbal system with dynamic unbalance," *International Journal of Advanced Robotic Systems*, vol. 10, no. 10, p. 357, 2013.

[7] M. Lungu, R. Lungu, C. Efrim, and O. Boubaker, "Backstepping control of magnetically suspended double-gimbal control moment gyroscope," in *2020 24th International Conference on System Theory, Control and Computing (ICSTCC)*, 2020, pp. 154–159.

[8] M. H. Ahmad, K. Osman, M. F. M. Zakeri, and S. I. Samsudin, "Mathematical modelling and pid controller design for two dof gimbal system," in *2021 IEEE 17th International Colloquium on Signal Processing & Its Applications (CSPA)*, 2021, pp. 138–143.

[9] S. Aggarwal, A. Joshi, and S. Kamal, "Line of sight stabilization of two-dimensional gimbal platform," in *2017 IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI)*, 2017, pp. 2553–2558.

[10] K.-j. Seong, H.-g. Kang, B.-y. Yeo, and H.-p. Lee, "The stabilization loop design for a two-axis gimbal system using lqg/ltr controller," in *2006 SICE-ICASE International Joint Conference*, 2006, pp. 755–759.

[11] M. M. Abdo, A. R. Vali, A. R. Toloei, and M. R. Arvan, "Stabilization loop of a two axes gimbal system using self-tuning pid type fuzzy controller," *ISA Transactions*, vol. 53, no. 2, p. 591–602, 2014.

[12] S. Dey, T. K. Kumar, S. Ashok, and S. K. Shome, "Design of adaptive network based fuzzy inference pid control methodology for 3 degree of freedom gimbal stabilized platform," *2023 International Conference on Power, Instrumentation, Energy and Control (PIECON)*, 2023.

[13] X. Zhou, Y. Shi, L. Li, R. Yu, and L. Zhao, "A high precision compound control scheme based on non-singular terminal sliding mode and extended state observer for an aerial inertially stabilized platform," *International Journal of Control, Automation and Systems*, vol. 18, no. 6, p. 1498–1509, 2020.

[14] H. Li, S. Yang, and Y. Le, "Torque ripple minimization of low-speed gimbal servo system using parameter-optimized eso," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 11, no. 2, pp. 2094–2103, 2023.

[15] H. K. Khalil, *Feedback Linearization*. Prentice Hall, 2002.

[16] A. Agasti, A. Hazarika, and B. Bhikkaji, "Control design for trajectory tracking and stabilization of sensor los in an inertially stabilized platform," 2023, https://arxiv.org/abs/2311.01859.