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Model Predictive Control based Target Defense with Attacker Trajectory Prediction

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Abstract— **This paper considers a target attack defense scenario where the attacker aims to reach the target while avoiding the defender, and the defender wants to protect the target by intercepting the attacker. While most of the prior works assume the target to be known to the defender, in our setting the defender has access to only the current states of the attacker at any instant, and is not aware of the target coordinates. A novel approach is proposed in this work to leverage past data of the attacker states to construct a future trajectory of the attacker. The defender deploys a model predictive control scheme to minimize the discrepancy between its own future trajectory and the predicted future trajectory of the attacker. Simulation results show that use of estimated future trajectories helps in more effective protection of the target compared to when only current state of the attacker is used. The effectiveness of the proposed approach is also highlighted in the presence of obstacles.**

I. Introduction

Games of conflict between two or more agents, known as pursuit-evasion games continue to receive a lot of attention due to their relevance in applications such as predator-prey models in biology [1], combat scenarios in aerospace [2], mobile robotics [3] and defense [4]. Starting from the seminal work by Isaacs in 1965 [5], many variations of this class of problems have been examined in the framework of differential games; examples include target-attacker-defender (TAD) problems [6], [7] , multiplayer problems [8]–[11], games with bounded rationality [12], [13], cooperative defense problems [14]–[16], range limited pursuit-evasion [17], [18], games over heterogeneous dimensions [19]–[21], and perimeter defense problems [22], [23]. An interesting survey of the recent developments related to these topics is presented in [2].

In most of the above settings, each player solves a continuous-time optimal control problem, and the analytical methods, largely based on Hamilton-Jacobi-Issacs (HJI) equation, suffer from the curse of dimensionality [24]. Thus, various geometrical approaches based on intersection of isochrones have been studied and used extensively by researchers [24]–[26]. These approaches are complex in nature and rely heavily on intuition and brute force geometry.

In contrast with continuous-time optimal control, model predictive control (MPC) is a promising online optimisation technique that facilitates feedback implementation of optimal control using finite prediction horizon. It has been applied in solving pursuit-evasion problems in many recent works [27]–[33] due to its intrinsic ability to deal with constraints, disturbances, obstacles and stability. Most of these works have assumed that complete information of the opponent is available which includes its states, and the control strategy used by the opponent.

A closely related work [34] assumes the $x-y$ coordinates of the opponent to be known and the orientation is estimated by the other player. It is argued that the performance of a player does not improve considerably even if the estimated heading is used. In [31], a target defense scenario is considered where the defender is trying to track a reference which is set to be a convex combination of the attacker's and target's position. Meanwhile, the attacker is trying to reach the target while avoiding the defender by treating it as a dynamic obstacle. This work assumes that the target of the attacker is known to the defender. However, it is possible that in a large city or establishment, the attacker's target may not be fixed, or the defender may not be exactly aware of it. Authors in [33] employ an inverse optimal control (IOC) based technique to estimate the opponents cost function by observing its trajectories for some time. This work uses an offline approach to study a large number of trajectories using Monte-Carlo simulations. This is a fundamental limitation since availability of such data is difficult in real-time target defense settings.

In view of these potential limitations, we present an approach to estimate the attacker's future trajectory online by leveraging its recent past trajectory information. The estimated future trajectory of the attacker is then used in a MPC based TAD formulation of the defender. The proposed approach does not require the defender to have the knowledge of the target coordinates and attacker's control strategy. We demonstrate through simulations that having the estimates of the attacker's future trajectory helps in more effective protection of the target compared to the case when only current states are used. Furthermore, we highlight the effectiveness of the proposed scheme in the presence of static obstacles in the paths of both players. The remainder of the paper is organized as follows. The formulation of TAD problem is presented in Section II, followed by the controller design using MPC in Section III. The algorithm to estimate attacker's future trajectory is presented in Section IV, while the simulation results and relevant discussions are given in Section V. Section VI concludes the paper.

II. Problem Formulation

This section presents the TAD setting. One of the agents is an attacker (A), while the other agent is a Defender (D).

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The Defender aims to reach a static target (T), which is represented by its Cartesian position $p_T = (x_T, y_T)$. Agents A and D are represented by unicycle models, relevant in many applications, such as non-holonomic mobile robots. Following the standard models presented in [35], [36], the states of agent $i \in \{A, D\}$, denoted by $z_i = [x_i, y_i, \theta_i]^\top$, evolve according to the dynamics

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i, \\ \dot{y}_i &= v_i \sin\theta_i, \\ \dot{\theta}_i &= \omega_i, \end{aligned} \tag{1}$$

where (x_i, y_i) denote the Cartesian position, θ_i denotes the orientation, and the control inputs are the linear and angular speeds denoted by v_i and w_i, respectively. We assume that the position of both agents as well as the target are contained in a subset of $\mathbb{R}^2$.

The defender aims to pursue the attacker and protect the target, while the attacker wants to reach the target quickly while avoiding the defender. The game terminates if any of the following conditions are satisfied.

1) The distance between the attacker and defender reduces below a threshold value denoted by R_{AD}, i.e,
$$\sqrt{(x_A - x_D)^2 + (y_A - y_D)^2} < R_{AD}.$$
2) The distance between the attacker and target reduces below a threshold value denoted by R_{AT}, i.e,
$$\sqrt{(x_T - x_A)^2 + (y_T - y_A)^2} < R_{AT}.$$
3) The defender prevents the attacker from reaching the target for a sufficiently long duration.

III. Model Predictive Control Strategies

MPC is an online optimization technique that computes the optimal sequence of control inputs that minimize a cost function over a finite prediction horizon subject to constraints that include system dynamics, as well as constraints on input and state variables [37]. The system is excited with the first element of the sequence while the rest are discarded. At every decision instant, the prediction horizon is shifted one step and the process is repeated to obtain the new optimal control sequence. The weighing matrices for terminal cost and running state and control costs are tuned to stabilize the system and improve performance. We now formulate the MPC problem for the attacker and the defender.

A. MPC Formulation for the Attacker

Let $u_{A,[1:N]} := \{u_A(1), u_A(2), \ldots, u_A(N)\}$ represent the sequence of control inputs of the attacker over a prediction horizon of length N. We assume that the attacker can observe its own current states and the current position of the defender, denoted by $(\bar{x}_D, \bar{y}_D)$. The attacker solves the following finite horizon optimal control problem:

$$\min_{u_{A,[1:N]}, z_{A,[1:N]}} \quad J_A(u_{A,[1:N]}, z_{A,[1:N]}; p_T) \tag{2}$$
$$\text{s.t.} \quad z_A(k+1) = f\left(z_A(k), u_A(k)\right), \tag{3}$$
$$z_{A_{\min}} \leq z_A(k) \leq z_{A_{\max}}, \tag{4}$$
$$u_{A_{\min}} \leq u_A(k) \leq u_{A_{\max}}, \tag{5}$$
$$\sqrt{(x_A(k+1) - \bar{x}_D)^2 + (y_A(k+1) - \bar{y}_D)^2} \geq R_{AD}, \tag{6}$$

where the constraints hold for all $k \in [N] := \{1, 2, \ldots, N\}$. The cost function is given by

$$\begin{aligned} J_A(u_{A,[1:N]}, z_{A,[1:N]}; p_T) &= \|p_A(N) - p_T\|^2_{Q_{Na}} \\ &+ \sum_{k=0}^{N-1} \|p_A(k) - p_T\|^2_{Q_a} + \|u_A(k)\|^2_{R_a}, \end{aligned} \tag{7}$$

where Q_{Na}, Q_a and R_a are positive definite matrices of appropriate dimensions and $\|w\|_Q := w^\top Q w$, $p_A = (x_A, y_A)$ denotes the Cartesian coordinates of the attacker which is a subset of the state vector z_A and p_T is the position of the static target. The constraint given by (3) represents the discretized version of the dynamics stated in (1). The limits on state and control variables are expressed in equations (4) and (5) respectively. The constraint (6) requires the attacker to avoid the defender assuming that the defender will remain in its current position over the prediction horizon. This is a reasonable assumption for the attacker since it is focused on reaching the target and does not bother about the defender until it is close enough to cause any harm.

B. MPC Formulation for the Defender

Two cases are presented for defender's control strategy using NMPC. In the first case, the defender is only aware of the attacker's current state while in the second case, the defender has an estimate of attacker's future trajectory. We start with the first setting.

Let $u_{D,[1:N]} := \{u_D(1), u_D(2), \ldots, u_D(N)\}$ represent the sequence of control inputs of the defender over a prediction horizon of length N. We assume that the defender can observe its own current states and the current state of the attacker (z_A) at the sampling instant. The defender solves the following finite horizon optimal control problem:

$$\min_{u_{D,[1:N]}, z_{D,[1:N]}} \quad J_D(u_{D,[1:N]}, z_{D,[1:N]}; z_A) \tag{8}$$
$$\text{s.t.} \quad z_D(k+1) = f\left(z_D(k), u_D(k)\right), \tag{9}$$
$$z_{D_{\min}} \leq z_D(k) \leq z_{D_{\max}}, \tag{10}$$
$$u_{D_{\min}} \leq u_D(k) \leq u_{D_{\max}}, \tag{11}$$

where the constraints hold for all $k \in [N]$. The cost function is given by

$$\begin{aligned} J_D(u_{D,[1:N]}, z_{D,[1:N]}; z_A) &= \|z_D(N) - z_A\|^2_{Q_{Nd}} \\ &+ \sum_{k=0}^{N-1} \|z_D(k) - z_A\|^2_{Q_d} + \|u_D(k)\|^2_{R_d}, \end{aligned} \tag{12}$$

where Q_{Nd}, Q_d and R_d are positive definite matrices of appropriate dimensions. As before, the constraint given by (9) represents discretized version of the equation (1). The limits on the state and control variable are expressed in equations (10) and (11) respectively.

We now consider the second setting where we assume that the defender is aware of the predicted state trajectory of the attacker over the horizon, which is denoted by $\hat{z}_{A,[1:N]}$. The defender's cost function makes use of the predicted trajectory of the attacker, and is defined as

$$\hat{J}_D(u_{D,[1:N]}, z_{D,[1:N]}; \hat{z}_{A,[1:N]}) = \|z_D(N) - \hat{z}_A(N)\|^2_{Q_{Nd}} + \sum_{k=0}^{N-1} \|z_D(k) - \hat{z}_A(k)\|^2_{Q_d} + \|u_D(k)\|^2_{R_d}. \quad (13)$$

The above cost function is minimized subject to the same set of constraints as given in (9)-(11).

Remark 1: The above MPC formulations allow us to tackle presence of obstacles in the environment on which the agents operate. Specifically, if an obstacle, approximated as a point mass, is present at coordinates $p_O = (x_O, y_O)$, then the following set of constraints

$$\sqrt{(x_i(k+1) - x_O)^2 + (y_i(k+1) - y_O)^2} \geq R_O, k \in [N],$$

may be added to the MPC problem for $i \in \{A, D\}$ where R_O represents the safe distance.

IV. Attacker Trajectory Prediction using Past Samples

We now discuss our approach for estimating the future state trajectory of the attacker based on past data. At a given time t, the defender observes the current state of the attacker, and computes the change in attacker states from their values k time steps earlier for $k \in [N]$. Formally, let $\beta^k \in \mathbb{R}^3$ denote the difference between the current state of the attacker and the state of the attacker k steps prior to the current time. For each $k \in [N]$, the defender collects N_s number of such samples or scenarios denoted by $\beta^k_{[1:N_s]} = \{\beta^k_1, \ldots, \beta^k_{N_s}\}$ from past N_s time points, and computes the sample mean as

$$\bar{\beta}^k := \frac{1}{N_s} \sum_{j=1}^{N_s} \beta^k_j. \quad (14)$$

The sample mean $\bar{\beta}^k$ is now used to predict the state of the attacker k step ahead as

$$\hat{z}_A(t+k) = z_A(t) + \bar{\beta}^k \quad \forall k \in [N], \quad (15)$$

where $z_A(t)$ denotes the state of the attacker at current time instant t. The sequence of predicted states $\{\hat{z}_A(t+1), \ldots, \hat{z}_A(t+N)\}$ results in the predicted trajectory $\hat{z}_{A,[1:N]}$ which is then used by the defender in its MPC cost function (13) at time t.

V. Results and Discussion

In this section, the performance of the pursuit strategy of the defender under the proposed trajectory prediction scheme is compared with baseline strategies via simulations. The simulations are carried out in MATLAB environment. An open source software Interior point optimizer (IPOPT) is interfaced to solve the MPC problems defined in Sections III-A and III-B. The following three cases are compared.

1) Defender predicts the future trajectory of the attacker as described in the previous section.
2) Defender has access to the complete MPC solution of the attacker which includes its future trajectory. While this assumption is impractical, it serves as a baseline against which the performance of the proposed scheme is compared.
3) Defender is only aware of the current state of the attacker, and optimizes the cost function (12). This setting was considered in [34] where the authors argued that in pursuit-evasion settings, if the defender uses only the current information regarding the attacker states, it achieves comparable performance compared to when it uses future trajectory of the attacker in the MPC cost function.

The thresholds R_{AD} and R_{AT} are chosen to be 0.3m and 0.1m, respectively. The safe distance R_O for obstacle avoidance is assumed to be 0.2m. The sampling time is chosen as 0.05 seconds and the prediction horizon is set as $N = 15$. The states of both the players are constrained in the range $[-10, -10, -\infty]^\top$ to $[10, 10, \infty]^\top$. The orientations are considered to be unconstrained in our simulations. The constraints on the control input are as chosen as follows:

$$u_{D_{\min}} = [0, -10\pi]^\top, \qquad u_{D_{\max}} = [7, 10\pi]^\top,$$
$$u_{A_{\min}} = [0, -10\pi]^\top, \qquad u_{A_{\max}} = [5, 10\pi]^\top,$$

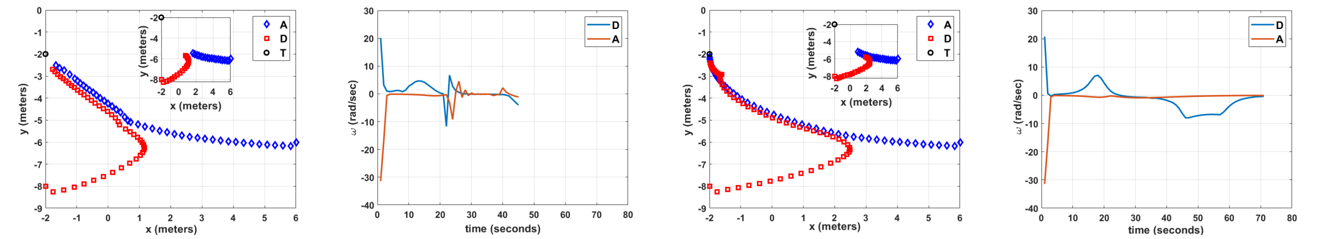
i.e., the defender is allowed to have a larger longitudinal speed compared to the attacker. The weighing matrices are tuned and set as follows:

$$R_a = \texttt{diag}[0.1, 0.01], Q_a = \texttt{diag}[1, 0.1], Q_{Na} = 100Q_a,$$
$$R_d = \texttt{diag}[0.1, 0.01], Q_d = \texttt{diag}[1, 1, 0.1], Q_{Nd} = 100Q_d.$$

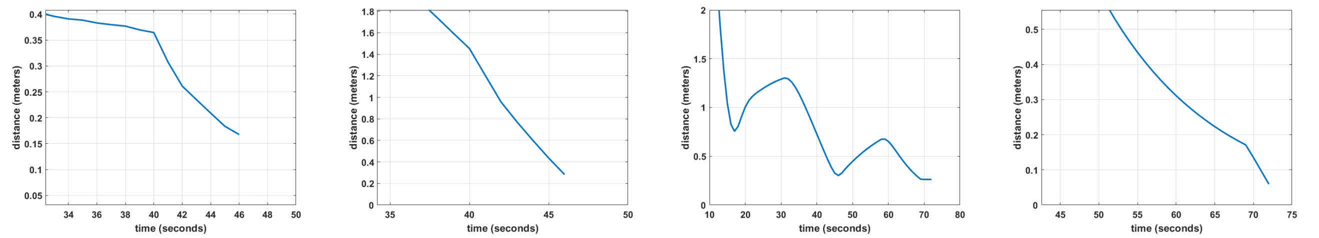
The weight on the angular speed is set to be higher than the weight on the linear speed which reflects that longitudinal motion is often easier compared to lateral motion. Similarly, weights on the position are kept larger compared to the weights on orientation. A large weight on the terminal cost is imposed as well.

The initial state of the defender and the attacker are set to be $[-2, -8, -\pi/4]^\top$ and $[6, -6, -\pi/4]^\top$ respectively. The target location x_T is set at $[-2, -2]^\top$. The target is present North of the defender, and the attacker must move towards it to reach the target. The target location is unknown to the defender, which is strategically beneficial for the attacker. Thus, knowledge of the attacker's trajectory or the target in could potentially result in more efficient capture of the attacker. The simulations terminate in three possible ways as discussed in Section II. The simulation stops at 80 seconds unless the attacker has reached the target or the defender has reached the attacker prior to it.

Figure 1 shows the trajectory of both the attacker and the defender in the Cartesian plane as well as the respective angular velocities of both players under two conditions: (i) defender is aware of the future trajectory of the attacker obtained by solving the MPC problem for the attacker (left panel), and (ii) defender is only aware of the current state of the attacker (right panel). In the first case, the defender is able to neutralize the attacker before it reaches the target as

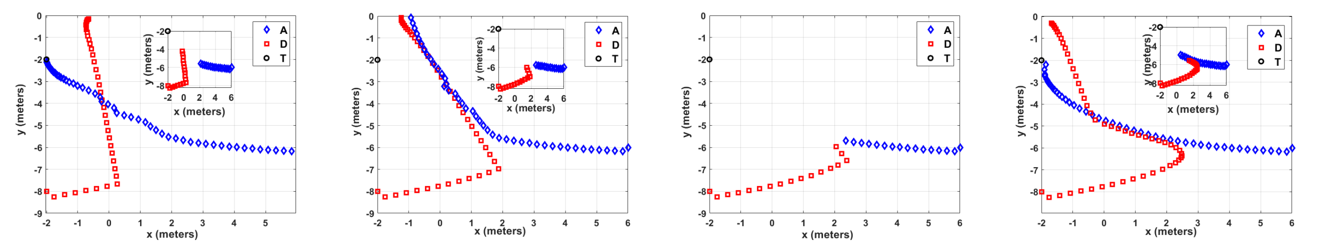


(a) Defender has access to attacker's MPC solution trajectory (b) Angular speed of both players for case (a) (c) Defender has access to only attacker's current states (d) Angular speed of both players for case (c)

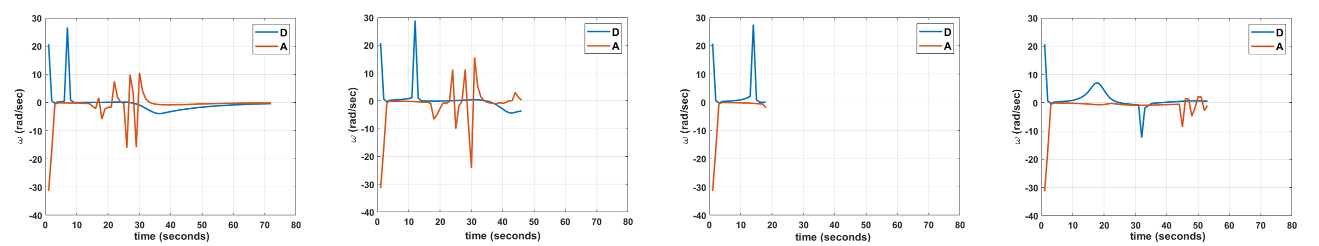


(e) Distance between attacker and defender for case (a) (f) Distance between attacker and target for case (a) (g) Distance between attacker and defender for case (c) (h) Distance between attacker and target for case (c)

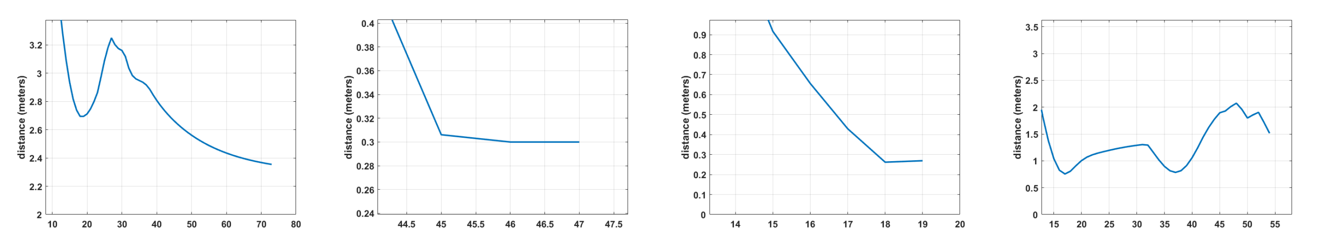
Fig. 1: Target defence with knowledge of attacker's MPC trajectory versus only current state information. The knowledge of the attacker's future trajectory over the horizon (its MPC solution) if available, helps the defender to neutralise the attacker before it reaches the target. The defender fails to protect the target when it is only aware of current states of the attacker.



(a) Trajectory for $N_s = 5$ (b) Trajectory for $N_s = 10$ (c) Trajectory for $N_s = 12$ (d) Trajectory for $N_s = 20$

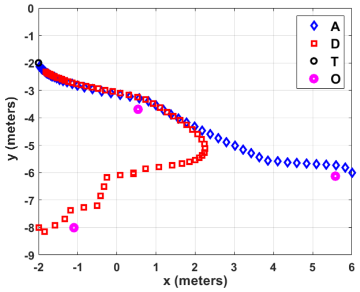


(e) Angular speed for $N_s = 5$ (f) Angular speed for $N_s = 10$ (g) Angular speed for $N_s = 12$ (h) Angular speed for $N_s = 20$

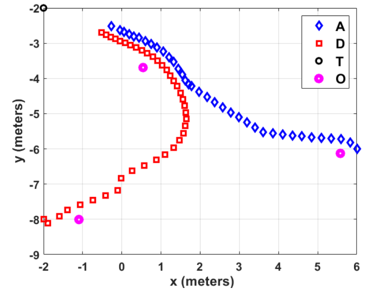


(i) Distance between attacker and defender for $N_s = 5$ (j) Distance between attacker and defender for $N_s = 10$ (k) Distance between attacker and defender for $N_s = 12$ (l) Distance between attacker and defender for $N_s = 20$

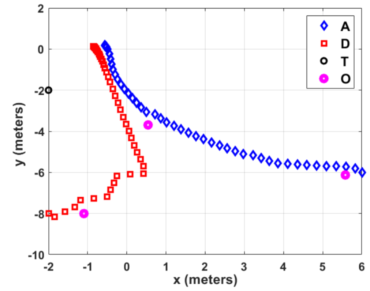
Fig. 2: Trajectory and angular velocity of the players when the defender uses the predicted mean trajectory of the attacker with past sample data of different lengths N_s. The defender is successful for $N_s = 10$ and $N_s = 12$ while it does not suceed when N_s is either too small or too large.



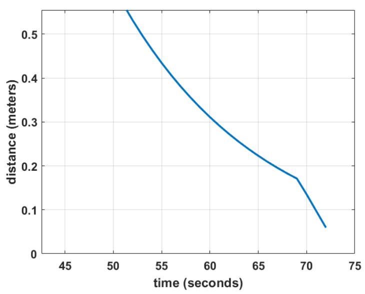
(a) Current State of Attacker



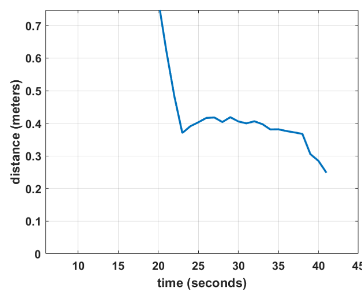
(b) Future State of Attacker



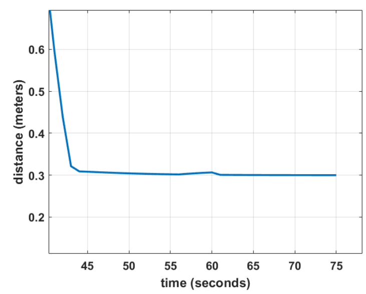
(c) Predicted State of Attacker, $N_s = 10$



(d) Distance between attacker and target for case (a)



(e) Distance between attacker and defender for case (b)



(f) Distance between attacker and defender for case (c)

Fig. 3: Comparison of trajectories of attacker and defender in presence of obstacles. When the defender is only aware of the current state of attacker (left), the attacker reaches the target. The defender wins when it uses the future state of the attacker given by MPC solution of attacker (middle) and predicted future state of attacker using past data with $N_s = 10$ (right).

shown in Figure 1a in approximately 45 seconds. However, the defender fails to protect the target if it follows the attacker using its current states as shown in Figure 1c. The simulation terminates in 70 seconds when the attacker reaches the target. The angular velocity figures (Figures 1b and 1d) show that in the first case, the defender turns towards the target before the attacker does, leading to successful interception. In the second case, the defender turns too late, and ends up following the attacker without being able to catch up. This case study shows that knowledge of the attacker's future trajectory plays a significant role for the defender to achieve its desired objective even when the target is unknown. The instantaneous distances between the attacker and the defender for the two cases are shown in Figure 1e and 1g respectively, while the distances between the attacker and target are shown in Figure 1f and 1h respectively. It is clear from the figure in the first case (defender having access to complete future trajectory), that the distance between attacker and defender is below the threshold at the end of the game while the attacker has not yet crossed the threshold distance to hit the target. However, in the second case (defender having access to only current states), the attacker crosses the threshold distance to reach the target while the defender is very close to the threshold for successful interception.

We now examine the performance of the proposed strategy which uses predicted future trajectory of the attacker using the approach presented in Section IV. The accuracy of prediction is highly sensitive to the length of the past data sample N_s. The trajectories of both players for different values of N_s is shown in Figure 2. For a very small sample length i.e $N_s = 5$, the defender takes a wrong path ahead of the attacker, thus allowing it to reach the target successfully as shown in Figure 2a. This is because limited past data is not sufficiently rich to yield reasonable prediction of future trajectory. When a large sample length is chosen ($N_s = 20$), the defender follows the attacker for some time but then diverts from the right path enabling the attacker to reach the target. In this case, the predicted trajectory is dominated by the shape of attacker trajectory farther away from the current time-instant. In both of these cases, the attacker is able to reach the target. When $N_s = 10$ and 12, the performance of the defender is found to be excellent. When $N_s = 10$, Figure 2b shows that the defender dominates the space containing the target, thus forcing the attacker to divert from the target. When $N_s = 12$, the defender promptly takes a turn at the appropriate position and manages to quickly capture the attacker. The instantaneous distance between the attacker and the defender in each of the cases presented in Figure 2i-2l shows that the defender is able to hit the attacker within the threshold distance only when the sample data lengths are chosen as $N_s = 10$ and $N_s = 12$.

The performance of the proposed approach is also examined in the presence of obstacles. Figure 3 shows the result for the proposed approach at $N_s = 10$ in presence of obstacles in the path of both players. It is seen that the defender is successful in protecting the target even when obstacles are placed in the path of both the players.

VI. Conclusion

This paper proposes a new approach to improve the performance of the defender in TAD game when the target position is unknown. This approach is used to estimate the mean trajectory of the attacker which is further used by the defender in its MPC formulation. It is shown via simulations that the proposed approach helps in improved capture and target defence when target as compared to a few past works that make use of only current states of the attacker. Despite its simplicity, this work offers hope that learning the attacker's trajectory could lead to improved performance for the defender. We believe that the trajectory prediction could be considerably improved in future by employing tools from learning theory such as reinforcement learning or probabilistic approaches such as stochastic MPC. Future work will thus explore alternative approaches to estimate unknown target position and improve attacker's trajectory prediction, and extend the current work to include multiple attackers and defenders.

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