2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

# An Active Disturbance Rejection Model Predictive Controller for Constrained Over-actuated Systems

Erica Salvato[1], Gianfranco Fenu[1], Felice Andrea Pellegrino[1] and Thomas Parisini[2]

*Abstract*— **This paper focuses on the control of multi-input multi-output (MIMO) over-actuated systems with unknown output disturbances and partially unknown dynamics. Our proposed solution integrates model predictive control (MPC) and active disturbance rejection control (ADRC) methodologies, offering a unified solution tailored to the specific demands of over-actuated constrained systems. We demonstrate the effectiveness of the proposed approach through comprehensive simulation results and also provide proof of the intervals that guarantee the convergence, feasibility, and BIBO stability of the method. Notably, our approach outperforms conventional output-feedback MPC, resulting in better performance in terms of noise reduction and reference tracking accuracy.**

*Index Terms*— **Receding Horizon Control, Unmodelled Disturbance Compensation, Robust Control**

## I. Introduction

Constrained control systems with uncertain or partially unknown dynamics, potentially coupled with disturbances and noise, represent a significant and recurring challenge in the field of control engineering. These complex systems find extensive application in a diverse array of practical domains, encompassing disciplines such as robotics, industrial automation, aerospace and automotive control.

Model-predictive control (MPC) provides a flexible and powerful tool for handling these kinds of systems. It explicitly considers the constraints on the control inputs, the output, and state variables and leverages predictive modeling to optimize control inputs at each time step with respect to a specific cost function while adhering to the system's constraints. However, MPC assumes that a model of the system dynamics is known. When the model is imprecise, or disturbances are difficult to model—as is often the case in real-world scenarios—the MPC performance can be significantly impacted.

Alternatively, active disturbance rejection control (ADRC) [1] is a control tool characterized by the ability to mitigate the effects of unmodeled dynamics, disturbances, and noise on system dynamics, based on a unified global disturbance term that encapsulates all these unknown quantities [2].

Properly combining the MPC predictive power with the disturbance-rejection capabilities of ADRC may allow to optimally manage constrained dynamical systems affected by unmodelled dynamics or noises, as already shown in the case of torque or speed control of induction machines [3], [4], tracking control of wheeled robot [5], and visual servoing applications of underwater vehicle manipulator systems [6].

A common feature of all of the mentioned systems, however, is that the number of control inputs $m$ is equal to the number of control outputs $p$. In the present paper, instead, we are interested in solving a similar problem involving MIMO over-actuated constrained systems (i.e., such that $m > p$), subject to unknown output disturbances and/or having partially known dynamics. Briefly, we address the control problem of a plant whose dynamics follow:

$$\dot{y} = f\left(y,\, w\right) + Mu \tag{1}$$

where $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and $w \in \mathbb{R}^p$ are the control input, the output, and an unknown disturbance, respectively; while $f(\cdot)$ is a function describing the unknown or unmodelled dynamics. Let $M \in \mathbb{R}^{p\times m} = M_0 + \Delta\, M$, where $M_0$ is the best available estimate of the matrix $M$ and $\Delta\, M$ is the associated uncertainty. The plant is input-affine, *over-actuated*, i.e., $m > p$, and it is subject to the following constraints:

$$u \in \mathbb{U} \subset \mathbb{R}^m,\; y \in \mathbb{Y} \subset \mathbb{R}^p, \tag{2}$$

and to an unknown and bounded disturbance $w \in \mathbb{W} \subset \mathbb{R}^p$. We seek a control strategy for tracking an output reference $y_{\text{ref}} \in \mathbb{Y}$, satisfying the system constraints, while also mitigating the disturbance effects on the measured output. Our proposed solution combines MPC with a properly modified ADRC approach able to deal with over-actuated systems. Recently in [7], the authors present an ADRC-based control strategy for an over-actuated vehicle, exploiting the Moore-Penrose pseudo-inverse matrix to spread the virtual control action to the effective actuators. However, their control approach cannot cope with possible constraints related to control inputs, controlled variables, or state variables. In contrast, our proposal considers the constraints on the physical system's inputs and outputs while mapping those constraints into properly tuned limitations for the virtual control actions of the squared system obtained by applying the ADRC strategy.

*This work has been partially supported by the Italian Ministry for Research in the framework of the 2017 Program for Research Projects of National Interest (PRIN), Grant no. 2017YKXYXJ.

**This study was carried out within the PNRR research activities of the consortium iNEST (Interconnected North-Est Innovation Ecosystem) funded by the European Union Next-GenerationEU (Piano Nazionale di Ripresa e Resilienza (PNRR) – Missione 4 Componente 2, Investimento 1.5 – D.D. 1058 23/06/2022, ECS_00000043). This manuscript reflects only the Authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

[1]Department of Engineering and Architecture, University of Trieste, Italy. {fenu,fapellegrino}@units.it, erica.salvato@dia.units.it

[2] Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, UK, with the Department of Engineering and Architecture, University of Trieste, 34127 Trieste, Italy, and also with the KIOS Research and Innovation Center of Excellence, University of Cyprus, CY-1678 Nicosia, Cyprus(e-mail: t.parisini@imperial.ac.uk)

### A. Motivating example

As a motivation for our research, consider the problem of beam trajectory control in *synchrotrons*, i.e., cyclic particle accelerators in which the accelerating particle beam travels along a fixed closed trajectory.

A fundamental component of such a facility is the storage ring—a high-vacuum steel tube—comprising a fixed number of beam position monitors (BPMs) and a fixed number of correction magnets. BPMs are used for real-time monitoring, providing measurements of the horizontal and vertical positions of the particle beam at different points in its orbit. Simultaneously, the correction magnets generate magnetic fields that deflect the electrons within the beam from their existing orbit to align them with the desired trajectory.

Denoting by $u \in \mathbb{R}^m$ the currents driving the corrector magnets, and by $y \in \mathbb{R}^p$ the monitored beam positions, an interesting and particular case is the one with $m > p$, i.e., the number of corrector magnets in the facility is greater than the number of BPMs. This design choice allows greater flexibility and more degrees of freedom in control. For example, by doubling the number of correctors near each BPM, and imposing one narrow-band and one wide-band corrector at each location, the resulting over-actuation can be conveniently exploited to improve the control efficacy of the overall structure.

It is usual to describe the steady-state relationship between a generic current variation $\Delta u$ and the corresponding orbit modification $\Delta y$, by introducing the orbit response matrix $R \in \mathbb{R}^{p\times m}$ [8]:

$$\Delta y = R\,\Delta u, \tag{3}$$

where the generic entry $R_{ij}$ could be evaluated using a model [9], [10], or, more often, estimated based on experimental data [11], [12].

Let's denote by $\ell$ the time required by the $j$-th BPM to correctly perform data acquisition, transmission, and processing, and only consider the low-frequency dynamics of the corrector magnets. The dynamics of the general $i, j$-th correction channel, i.e., the one including the $i$-th BPM and the $j$-th corrector magnet, can be easily modeled as follows [13]:

$$T_{ij}(s) = R_{ij}\,\frac{a_j}{s + a_j}\,e^{-\ell\, s}, \tag{4}$$

where $a_j$ is the bandwidth of the $j$-th corrector magnet. Then, assuming that the delay $\ell$ is negligible, a state-space linear model for the whole orbit control system takes the form of:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + d \end{cases}, \tag{5}$$

where $x \in \mathbb{R}^n$ is the state corresponding to the correction channels ($n = m$), $d$ represents both measurement noise and the disturbances, $B = -A$, $C = R$, while $A = \text{diag}\{-\lambda_1, -\lambda_2, \ldots, -\lambda_n\}$ describes the low-pass dynamics of the correction channels, where each $\lambda_i$, $i = 1, 2, \ldots, n$, depends on the cut-off frequency value of the corresponding corrector magnet.

This orbit control system is usually an ill-conditioned MIMO system challenging to control due to the presence of small singular values in the $R$ matrix [14]. Moreover, the orbit response matrix is usually affected by uncertainty, i.e., $R = R_0 + \Delta\, R$. Therefore, the corresponding orbit control strategy needs to operate far enough from the physical constraints of the corrector magnets[1] and must be able to overcome possible modelling errors to avoid subsequent stability problems. In addition, the complete absence of a model characterizing the time behavior of $d$ makes it difficult to reduce its effect on the system output, thus not guaranteeing a steady electron beam. The inherent difficulties in controlling such systems are well-known and lead to the use of model-free approaches [15], [16], as well as of iterative, learning-based approaches, such as [17].

### B. Notation

In the reminder of the paper, we denote by $S^{(1)}$ the $\{1\}$-inverse of a matrix $S$, i.e., the matrix satisfying $SS^{(1)}S = S$ [18], by $\ker(S)$ the kernel of the matrix $S$, and by $I_l$ the identity matrix of size $l \times l$.

## II. Active disturbance rejection controller

Typically, active disturbance rejection controllers [19] allow tracking a reference $y_{\text{ref}}$ on a system subject to unknown disturbances and of which only a partial model knowledge is available.

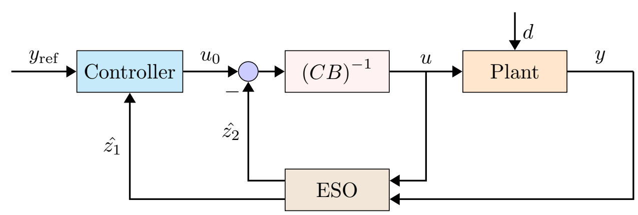


Fig. 1. General ADRC schematic.

Consider a dynamical system as in (5), with $C = C_0 + \Delta\, C$, assume $m = p$ and $d \in \mathcal{C}^2$, with the derivatives of $d$ unknown but bounded. Equation (1) leads to the following:

$$\dot{y} = z_2 + C_0 Bu, \tag{6}$$

where $z_2 = f\left(x,\, \dot{d}\right) = CAx + \Delta\, CBu + \dot{d}$ is a virtual state that plays the role of all unknown, or unmodeled, dynamical components.

The main idea of [19] consists of using an estimate of $z_2$ to decouple the known dynamics of the system from both the actual disturbance acting on it and the neglected dynamics. The rate of change of $z_2$ is unknown but bounded owing the assumption on $d$ and according to (2). By considering the extended state version of (6):

$$\begin{aligned} \dot{z}_1 &= C_0 Bu + z_2 \\ \dot{z}_2 &= w \\ y &= z_1, \end{aligned} \tag{7}$$

[1] otherwise, huge control signals in the low-gain directions lead to correction magnets saturation.

where $z = \left[z_1^\top,\ z_2^\top\right]^\top \in \mathbb{R}^{2p}$ is the extended state vector, and $w$ is the unknown bounded rate of change of the neglected dynamics $z_2$, an extended state observer (ESO) able to provide punctual estimates $\hat{z}_1$ and $\hat{z}_2$ of $z_1$ and $z_2$, respectively, takes the form of:

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + C_0 B u + \beta_1 (y - \hat{z}_1) \\ \dot{\hat{z}}_2 &= \beta_2 (y - \hat{z}_1) \\ \hat{y} &= \hat{z}_1, \end{aligned} \tag{8}$$

where $\beta_1$ and $\beta_2$ are the observer gain matrices.

Imposing $C_0 B u = u_0 - \hat{z}_2$, as $\hat{z} - z \to 0$, from (7), we get:

$$\dot{z}_1 = u_0 \tag{9}$$

$$y = z_1, \tag{10}$$

and we can compute the virtual control input $u_0$ leading the system output (10) towards a desired reference, e.g., $y \to y_{\text{ref}}$.

In particular, if $C_0 B$ is a *non-singular* $m \times m$ matrix, then the resulting plant control input is:

$$u = (C_0 B)^{-1} (u_0 - \hat{z}_2). \tag{11}$$

A general ADRC scheme is presented in Figure 1. Notice that no assumptions related to the control approach to be used are made. Therefore, depending on the task at hand, any type of control approach can be chosen. However, as previously observed, in the case of non-square $C_0 B$ matrix, e.g., when $m > p$, (11) cannot be applied in order to obtain the plant control input vector from $u_0$ and $\hat{z}_2$. This particular case, which is the one we are interested in solving, requires further technicalities, to be addressed in the following section.

## III. PROPOSED SOLUTION

To apply the ADRC approach to a dynamical system described by the equations (5) (or the more general expression (1) ), characterized by over-actuation ($m > p$), we need first of all to establish a suitable method for replacing (11). This replacement must guarantee that the mapping operation from the $p$-dimensional vector $u_0 - \hat{z}_2$ to the $m$-dimensional vector $u$ results in input values that meet the physical constraints of the plant, ultimately requiring suitable conditions on $u_0$.

Subsequently, given our objective of solving a tracking problem with a reference output value denoted by $y_{\text{ref}}$, all the necessary components are in place to formulate an MPC strategy that operates on the variable $\hat{z}_1$. This strategy is capable of generating $u_0$ in such a manner that ensures that both $u$ belongs to the $\mathbb{U}$ set, and $y$ belongs to the $\mathbb{Y}$ set.

A schematic representation of the proposed ADRMPC (Active Disturbance Rejection Model Predictive Control) reference tracking control strategy is provided in Figure 2.

In practice, denoting by $T$ the plant sampling time, every $T$ time units, the plant output $y$ and control input $u$ are provided to the ESO, which returns $\hat{z}_2$ in order to punctually compensate for disturbances and unknown dynamics, and $\hat{z}_1$, which estimates the noise-cleaned output. The latter becomes the input for the MPC, yielding $u_0$ as the output. As a final

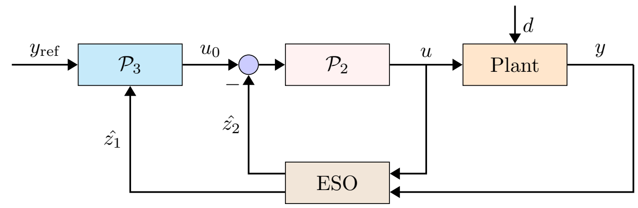


Fig. 2. General schematic of the proposed Active Disturbance Rejection MPC approach.

step, the resultant $p$-dimensional signal $u_0 - \hat{z}_2$ is mapped into an $m$-dimensional vector $u$, i.e., the actual plant input. The procedure is repeated over time and may be employed for tracking non-constant references as well.

In the following, we first expand upon the ADRC theory, customizing it to address the specific challenges posed by over-actuated systems. We then introduce our Active Disturbance Rejection Model Predictive Control (ADRMPC) approach, providing evidence of robustness and stability. We also provide the conditions to guarantee feasibility.

### A. ADRC for over-actuated systems

When $m > p$, Equation (11) cannot be applied.

Following Corollary 2 of [18], page 53, the solutions of $C_0 B u = u_0 - \hat{z}_2$, when $(C_0 B)^{(1)} C_0 B \neq I_m$, may be written as:

$$u = (C_0 B)^{(1)} (u_0 - \hat{z}_2) + (I_m - (C_0 B)^{(1)} C_0 B)\lambda, \tag{12}$$

for any arbitrary $\lambda \in \mathbb{R}^m$. To ease the notation we define $L = (I_m - (C_0 B)^{(1)} C_0 B)$.

Then, given $u_0 \in \mathbb{U}_0$ and $z_2 \in \mathbb{Z}_2$, the plant control input $u$ can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min l(\lambda) \\ &s.t. : (12),\ u \in \mathbb{U}, ; \end{aligned} \tag{$\mathcal{P}_2$}$$

where $l(\cdot)$ is a suitably designed cost function, e.g., $l(\cdot)$ could be designed in order to properly map the $p$-dimensional signal $u_0 - \hat{z}_2$ into an $m$-dimensional plant input $u$ by optimizing some performance criterion.

It is worth noticing that the constraints imposition on $u$ in ($\mathcal{P}_2$) allow us to determine the $u_0$ constraints, to be imposed on the MPC, such that ($\mathcal{P}_2$) always admits solution.

*Assumption 1:* $\mathbb{U}$ is a box-shaped set including the origin:

$$\mathbb{U} = \{u \in \mathbb{R}^m :\ \underline{u} \leq u \leq \bar{u}\}, \tag{13}$$

where $\underline{u}$ and $\bar{u}$ denote the vectors containing the lower and the upper bounds of each component of $u$ and such that $0 \in \mathbb{U}$.

*Assumption 2:* $\hat{z}_2 \in \mathbb{Z}_2$, where $\mathbb{Z}_2$ is a box-shaped set:

$$\mathbb{Z}_2 = \{\hat{z}_2 \in \mathbb{R}^p :\ \underline{z}_2 \leq \hat{z}_2 \leq \bar{z}_2\}. \tag{14}$$

where $\underline{z}_2$ and $\bar{z}_2$ denote the vectors containing the lower and the upper bounds of each component of $\hat{z}_2$.

We further denote by $z_{\text{avg}} = \dfrac{\underline{z}_2 + \bar{z}_2}{2}$ the average vector of $\hat{z}_2$, and by $z_\delta = \dfrac{\bar{z}_2 - \underline{z}_2}{2}$ the mid-range of the admissible values of $\hat{z}_2$.

*Theorem 1:* Let $\mathbb{U}$ and $\mathbb{Z}_2$ be defined as in (13) and (14), respectively. Denote by $cb^{(i)}$ the $i$-th row of $(C_0B)^{(1)}$, and by $\underline{u}^{(i)}$ and $\bar{u}^{(i)}$ the $i$-th component of $\underline{u}$ and $\bar{u}$, respectively. Then by imposing:

$$\begin{aligned} cb^{(i)}u_0 &\le \bar{u}^{(i)} + cb^{(i)}\xi^+ + cb^{(i)}z_{\text{avg}} \\ -cb^{(i)}u_0 &\le -\underline{u}^{(i)} - cb^{(i)}\xi^- - cb^{(i)}z_{\text{avg}}, \end{aligned} \tag{15}$$

where $\xi^+$ and $\xi^-$ are chosen according to:

$$\begin{aligned} \xi^+ &= \underset{\xi^+\in\{-z_\delta, z_\delta\}}{\arg\min} \; \bar{u}^{(i)} + cb^{(i)}\xi^+ + cb^{(i)}z_{\text{avg}}, \\ \xi^- &= \underset{\xi^-\in\{-z_\delta, z_\delta\}}{\arg\min} \; -\underline{u}^{(i)} - cb^{(i)}\xi^- - cb^{(i)}z_{\text{avg}}, \end{aligned} \tag{16}$$

$(\mathcal{P}_2)$ always admits at least one feasible solution, regardless of the chosen $\lambda$.

*Proof:* We denote by $\ker(C_0B) = \{u \in \mathbb{R}^m : C_0Bu = 0\}$ the kernel of the linear map $C_0B : \mathbb{R}^m \to \mathbb{R}^p$, where 0 is the origin in $\mathbb{R}^p$. Since $L\lambda \in \ker(C_0B)$, then $C_0B\, L\lambda = 0$.

On the other hand, imposing $u \in \mathbb{U}$ in $(\mathcal{P}_2)$ we are defining a polytope

$$\mathbb{L} = \Big\{\lambda \in \mathbb{R}^m : \\ \underline{u} - (C_0B)^{(1)}(u_0 - \hat{z}_2) \le L\lambda \le \bar{u} - (C_0B)^{(1)}(u_0 - \hat{z}_2)\Big\}.$$

Then, if

$$\begin{cases} \underline{u} - (C_0B)^{(1)}(u_0 - \hat{z}_2) \le 0 \\ \bar{u} - (C_0B)^{(1)}(u_0 - \hat{z}_2) \ge 0 \end{cases} \tag{17}$$

are component-wise satisfied (i.e., (15) holds), then $\mathbb{L}$ is centered in $-(C_0B)^{(1)}(u_0 - \hat{z}_2)$ and contains the origin. Therefore, $\forall\lambda \in \mathbb{R}^m$, $\mathbb{L} \cap \ker(C_0B)$ contains at least the origin, and $(\mathcal{P}_2)$ always admits solution regardless of the chosen $\lambda$. ∎

By imposing (15), we guarantee a solution for $(\mathcal{P}_2)$, and, practically, we constrain $u_0$ in a polytope $\mathbb{U}_0$, which can be used as MPC constraint.

### B. Active disturbance rejection MPC

We can now formalize the ADRMPC approach.

Given a plant whose dynamics follows (5), with $m > p$, and such that $(C_0B)^{(1)} C_0B \ne I_m$. We assume that $Cf(x)$ and the unknown disturbance $d \in \mathbb{D}$ are bounded. Then, an ESO can be designed as in (8).

We recall that $z_1 = y$ and that $y \in \mathbb{Y}$, then $z_1 \in \mathbb{Y}$.

*Assumption 3:* $\mathbb{Y}$ is a symmetric, box-shaped set including the origin:

$$\mathbb{Y} = \{z_1 \in \mathbb{R}^p : \underline{z}_1 \le z_1 \le \bar{z}_1\}, \tag{18}$$

where $\underline{z}_1$ and $\bar{z}_1$ denote the vectors containing the lower and the upper bound of each component of $z_1$, respectively, and $\underline{z}_1 = -\bar{z}_1$.

*Assumption 4:* $u_0 \in \mathbb{U}_0$, with $\mathbb{U}_0$ a box-shaped set including the origin, defined by (15):

$$\mathbb{U}_0 = \{u_0 \in \mathbb{R}^p : \underline{u}_0 \le u_0 \le \bar{u}_0\}. \tag{19}$$

where $\underline{u}_0$ and $\bar{u}_0$ denote the vectors containing the lower and the upper bound of each component of $u_0$, and such that $0 \in \mathbb{U}_0$.

The overall output-feedback problem of finding an input such that $y \in \mathbb{Y}$ and $y \to y_{\text{ref}}$ can be solved by repeating the following steps at each $k$-th time instant:

1) solve the following receding horizon control problem involving the ZOH-discretization of (9)

$$\begin{aligned} &\min_{u_0(\cdot)} \sum_{i=0}^{H-1} g\left(z_1(i), u_0(i)\right) + g_H\left(z_1(H) - y_{\text{ref}}\right), \\ &s.t.: z_1(k+1) = I_p z_1(k) + I_p u_0(k) \\ &\qquad z_1(k) \in \mathbb{Y} \\ &\qquad u_0(k) \in \mathbb{U}_0 \\ &\qquad z_1(0) = z_0 \in \mathbb{Y} \\ &\qquad z_1(H) \in \mathbb{Z}_H \end{aligned} \tag{$\mathcal{P}_3$}$$

where $H$ is the control horizon, and $g(z_1, u_0)$ and $g_H(z_1 - y_{\text{ref}})$ two convex cost functions, and $\mathbb{Z}_H$ is a $\mu$-ball of $\infty$-norm centered in $y_{\text{ref}}$, i.e., $\mathbb{Z}_H = \{z_1 \;\; \|z_1 - y_{\text{ref}}\|_\infty \le \mu\}$.

2) apply the first control input of the sequence of $H$ steps obtained with the MPC.

3) solve $(\mathcal{P}_2)$ to convert the control input $(u_0 - \hat{z}_2) \in \mathbb{R}^p$ into an $u \in \mathbb{U} \subset \mathbb{R}^m$.

If Assumption 4 holds, the solution of the step 1 allows to formulate a feasible $(\mathcal{P}_2)$ in accordance with Theorem 1.

Notice that, since the dynamical system on which the MPC operates is a fully actuated and decoupled system, by properly choosing the horizon $H$ we can ensure the feasibility of $(\mathcal{P}_3)$.

We define by $\Delta h$ the vector containing in each $i$-th component $\Delta h^{(i)}$ the least number of steps required to reach the $i$-th component of $y_{\text{ref}}$ (i.e., $y_{\text{ref}}^{(i)}$) starting from the farthest admissible initial value of the $i$-th component of the state $z_1^{(i)}$:

$$\Delta h^{(i)} = \frac{\max\left\{\left|\underline{z}_1^{(i)} - y_{\text{ref}}^{(i)}\right|, \left|\bar{z}_1^{(i)} - y_{\text{ref}}^{(i)}\right|\right\}}{\min\left\{\left|\underline{u}_0^{(i)}\right|, \left|\bar{u}_0^{(i)}\right|\right\}}. \tag{20}$$

Then, by choosing $H$ in $\mathcal{P}_3$ such that: $H \ge \|\Delta h\|_\infty$ holds (i.e., imposing a horizon greater than the maximum value among the $\Delta h^{(i)}$), $(\mathcal{P}_3)$ is feasible.

*Remark 1:* The BIBO stability of the ADRMPC approach is contingent upon the stability of the ESO system. Indeed, when considering the ADRMPC closed-loop system, it can be represented by the following state-space equation:

$$\begin{bmatrix}\dot{y} \\ \dot{e}\end{bmatrix} = \begin{bmatrix}0 & -I_pE_z \\ 0 & A_z - \beta C_z\end{bmatrix}\begin{bmatrix}y \\ e\end{bmatrix} + \begin{bmatrix}0 \\ E_z\end{bmatrix} w + \begin{bmatrix}I_p \\ 0\end{bmatrix} u_0. \tag{21}$$

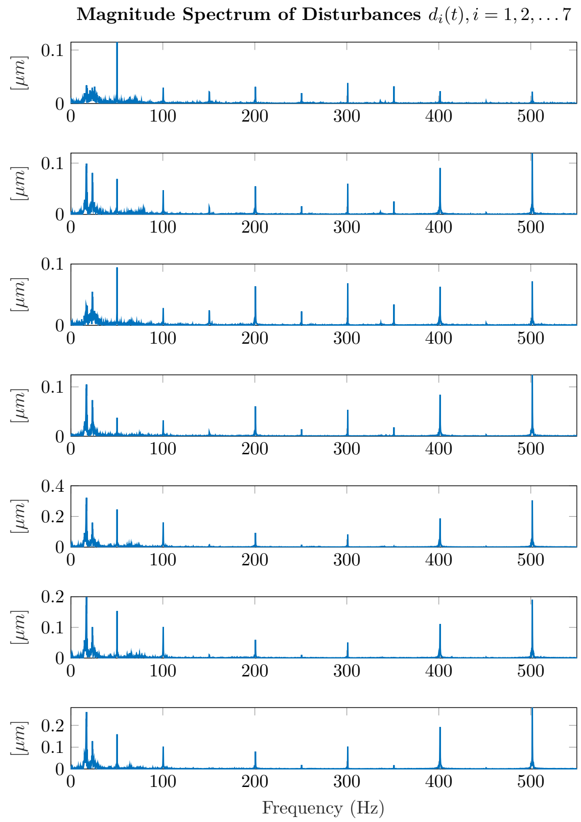


Fig. 3. Magnitude spectrum of the applied disturbance during the performed simulations.

where $e = z - \hat{z}$, $A_z = \begin{bmatrix} 0 & I_p \\ 0 & 0 \end{bmatrix}$, $E_z = \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, and $C_z = \begin{bmatrix} I_p & 0 \end{bmatrix}$. Clearly, the ADRMPC closed-loop system is BIBO stable provided that the matrix $A_z - \beta C_z$ is Hurwitz.

## IV. Implementation

To evaluate the proposed approach, we conducted simulations using the experimental setup described in Section Section I-A, where $m = 15$ and $p = 7$. Each corrector magnet was assigned a cutoff frequency of $f = 500\,\mathrm{Hz}$. As a result, the continuous-time model (5) was defined with $A = 2\pi f I_m$. The $R$ matrix used in the simulations and the code for replicating experimental results can be found in https://github.com/EricaSalvato/ADRMPC.git. The applied disturbance $d$ exhibited frequency characteristics shown in Figure 3. Additionally, we assumed $\underline{u}_i = -\bar{u}_i = -10\,\mathrm{A}$ for all $i = 1, 2, \ldots, m$ and $\underline{y}_i = -\bar{y}_i = -2\,\mathrm{mm}$ for all $i = 1, 2, \ldots, p$. We run a simulation of $5\,\mathrm{s}$ performing the proposed ADRMPC with a sampling-time $T = 1 \times 10^{-4}\,\mathrm{s}$, $\beta_1 = 1.9136$, $\beta_2 = 9.1544 \cdot 10^3$. The control strategy $(\mathcal{P}_3)$ was designed to minimize a quadratic

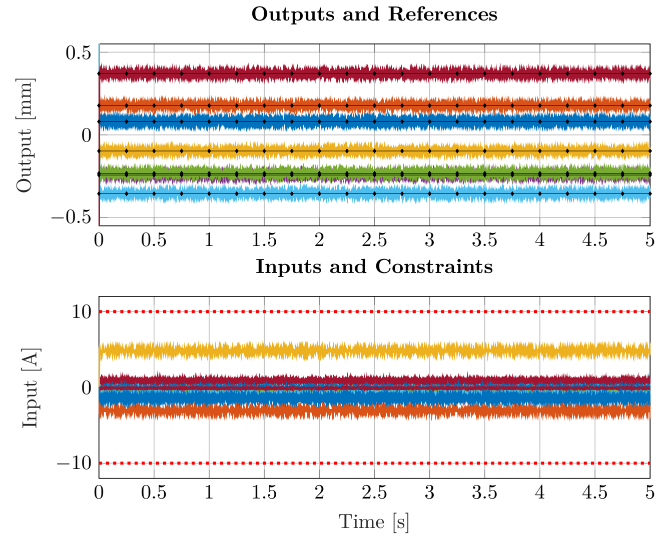


Fig. 4. (a) the 7 outputs in the ADRMPC simulation with respect to their references values (thin black marked lines); (b) trend of all the 15 input provided by the ADRMPC during the simulation with respect to their constraints (dotted lines)

cost function:

$$J = \sum_{i=0}^{H-1} \left( z_1^\top(i) \frac{1}{2} Q z_1(i) + u_0^\top(i) \frac{1}{2} R u_0(i) \right) + \left(z_1(H) - y_{\text{ref}}\right)^\top Q_H \left(z_1(H) - y_{\text{ref}}\right), \tag{22}$$

where $H = 4$, $Q = I_p$, $R = 10^{-3} I_p$, and $Q_H$ is the solution of the discrete algebraic Riccati equation corresponding to the discrete-time version of (9) with quadratic cost $\sum_{i=0}^{\infty} z_1^\top(i) \frac{1}{2} Q z_1(i) + u_0^\top(i) \frac{1}{2} R u_0(i)$. The objective was to achieve both reference tracking and constraint satisfaction. In $(\mathcal{P}_2)$, we sought to minimize $\|\lambda\|_\infty$ .

The simulation results, as depicted in Figure 4, clearly demonstrate the effectiveness of ADRMPC in reference tracking while adhering to input and output constraints. In Figure 5, the capability of ADRMPC in reducing disturbances, particularly in the $0 - 200\mathrm{Hz}$ frequency band, is highlighted. To assess the performance of ADRMPC in comparison to a standard MPC output-feedback controller [20], simulations were run for different values of the MPC time horizon $H \in 4, 8, 12, 16, 20$. Results, presented in Table IV, are based on three performance indices:

- The reduction of the standard deviation of the output error $\text{std}(y - y_{\text{ref}})$ with respect to the standard deviation of the disturbance $\text{std}(d)$ during the simulation

$$\text{STD}_{\text{red}} = \frac{\text{std}(y - y_{\text{ref}})}{\text{std}(d)}$$

- The reduction of the maximum value of the output error $\max(|y - y_{\text{ref}}|)$ with respect to the maximum value of the disturbance $\max(|d|)$ during the simulation

$$\max(\mathrm{d})_{\text{red}} = \frac{\max(|y - y_{\text{ref}}|)}{\max(|d|)}$$

TABLE I

PERFORMANCE INDICES VALUES OF ADRMPC (BLUE) VS. MPC (WHITE)

| $\mathrm{STD}_{\mathrm{red}}$ | | | | | $\max(\mathrm{d})_{\mathrm{red}}$ | | | | | $\|\|y-y_{\mathrm{ref}}\|\|_1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $H=4$ | $H=8$ | $H=12$ | $H=16$ | $H=20$ | $H=4$ | $H=8$ | $H=12$ | $H=16$ | $H=20$ | $H=4$ | $H=8$ | $H=12$ | $H=16$ | $H=20$ |
| 0.5290 | 0.5297 | 0.5301 | 0.5317 | 0.5340 | 0.5814 | 0.5767 | 0.5774 | 0.5818 | 0.5865 | 127.6176 | 128.7167 | 130.4902 | 132.5130 | 134.5785 |
| 0.8951 | 0.8951 | 0.8951 | 0.8951 | 0.8951 | 0.9059 | 0.9059 | 0.9059 | 0.9059 | 0.9059 | 266.5091 | 266.5109 | 266.5096 | 266.5046 | 266.5162 |

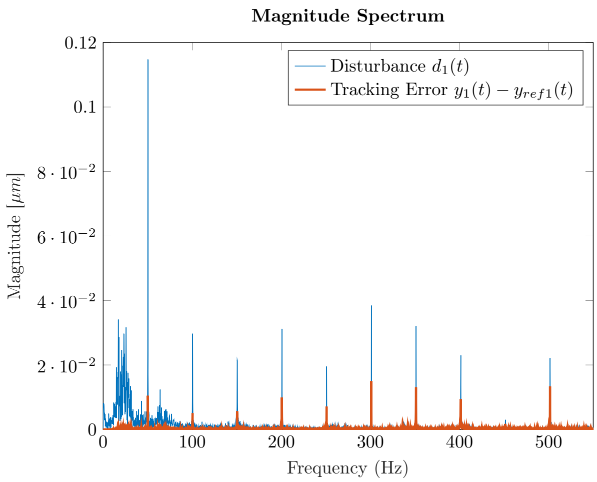


Fig. 5. Magnitude spectrum of the first component of the output error (orange) with respect to the magnitude spectrum of the first component of the disturbance (blue) during the simulation.

- The norm-1 of the output error $||y - y_{\mathrm{ref}}||_1$ during the simulation.

In all three indices, better performance is indicated by lower values. We can observe that ADRMPC outperforms the standard output-feedback MPC in terms of all three indices and for the different horizon values adopted during the simulations.

## V. CONCLUSIONS

This study proposes a closed-loop control solution for MIMO over-actuated systems with uncertain output disturbances and partially known dynamics that integrates model predictive control (MPC) and active disturbance rejection control (ADRC) methodologies. The proposed approach successfully ensures BIBO stability while simultaneously satisfying the plant's physical output and input constraints. The effectiveness of the proposed approach has been tested through extensive simulations performed on a simplified version of a synchrotron. Notably, results highlight that our approach outperforms conventional output-feedback MPC techniques, demonstrating superior noise reduction and reference tracking accuracy. Further works could explore extending this methodology to real-world implementations and integrating adaptive elements to enhance adaptability to varying system conditions and disturbances.

## REFERENCES

[1] H. Feng and B.-Z. Guo, "Active disturbance rejection control: Old and new results," *Annual Reviews in Control*, vol. 44, pp. 238–248, 2017.

[2] Y. Huang and W. Xue, "Active disturbance rejection control: Methodology and theoretical analysis," *ISA transactions*, vol. 53, no. 4, pp. 963–976, 2014.

[3] L. Yan, F. Wang, M. Dou, Z. Zhang, R. Kennel, and J. Rodríguez, "Active disturbance-rejection-based speed control in model predictive control for induction machines," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 4, pp. 2574–2584, 2020.

[4] L. Yan, F. Wang, M. Dou, Z. Zhang, R. Kennel, and J. Rodríguez, "Active disturbance-rejection-based speed control in model predictive control for induction machines," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 4, pp. 2574–2584, 2019.

[5] H. Yang, M. Guo, Y. Xia, and Z. Sun, "Dual closed-loop tracking control for wheeled mobile robots via active disturbance rejection control and model predictive control," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 1, pp. 80–99, 2020.

[6] J. Gao, X. Liang, Y. Chen, L. Zhang, and S. Jia, "Hierarchical image-based visual serving of underwater vehicle manipulator systems based on model predictive control and active disturbance rejection control," *Ocean Engineering*, vol. 229, p. 108814, 2021.

[7] H. Zhou, G. Yuan, K. Liu, and L. Ma, "Active Disturbance Rejection Control for an Over-Actuated Vehicle on SO (3)," in *2023 2nd Conference on Fully Actuated System Theory and Applications (CFASTA)*. IEEE, 2023, pp. 745–750.

[8] M. G. Minty and F. Zimmermann, "Beam techniques - beam control and manipulation," in *US Particle Accelerator School, University of Chicago and Argonne National Laboratory*, June 14–25 1999, sLAC-R-621.

[9] C. J. Bocchetta, "Review of orbit control," in *Proc. of EPAC98, the sixth European Particle Accelerator Conference*, I. of Physics Publishing, Ed., Bristol and Philadelphia, 22-26 June 1998.

[10] W. Corbett, M. Lee, and V. Ziemann, "A fast model calibration procedure for storage rings," in *Proceedings of International Conference on Particle Accelerators*, 1993, pp. 108–110 vol.1.

[11] D. Bulfone, "Overview of fast beam position feedback systems," in *Proc. of Eur. Particle Accelerator Conf.® EPAC*, vol. 8, 2008, p. 1021Ä1025.

[12] M. Aiba and M. Böge, "Local orbit response matrix measurement at sls," in *Proc. 6th Int. Particle Accelerator Conf. (IPAC'15)*. JACoW Publishing, May 2015, paper TUPJE044, pp. 1713–1715, https://doi.org/10.18429/JACoW-IPAC2015-TUPJE044. [Online]. Available: https://jacow.org/IPAC2015/papers/TUPJE044.pdf

[13] S. H. Mirza, R. Singh, P. Forck, and B. Lorentz, "Performance of the closed orbit feedback systems with spatial model mismatch," *Physical Review Accelerators and Beams*, vol. 23, no. 7, p. 072801, 2020.

[14] S. Skogestad and I. Postlethwaite, *Multivariable feedback control: Analysis and Design*. Hoboken, US-NJ: John Wiley, 2005.

[15] N. Bruchon, G. Fenu, G. Gaio, M. Lonza, F. A. Pellegrino, and E. Salvato, "Toward the Application of Reinforcement Learning to the Intensity Control of a Seeded Free-Electron Laser," in *2019 23rd International Conference on Mechatronics Technology (ICMT)*, A. Senatore and T. Q. Dinh, Eds. Salerno: IEEE, oct 2019, pp. 1–6.

[16] N. Bruchon, G. Fenu, G. Gaio, M. Lonza, F. H. O'Shea, F. A. Pellegrino, and E. Salvato, "Basic Reinforcement Learning Techniques to Control the Intensity of a Seeded Free-Electron Laser," *Electronics*, vol. 9, no. 5, 2020. [Online]. Available: https://www.mdpi.com/2079-9292/9/5/781

[17] N. Bruchon, G. Fenu, G. Gaio, S. Hirlander, M. Lonza, F. A. Pellegrino, and E. Salvato, "An Online Iterative Linear Quadratic Approach for a Satisfactory Working Point Attainment at FERMI," *Information*, vol. 12, no. 7, 2021. [Online]. Available: https://www.mdpi.com/2078-2489/12/7/262

[18] A. Ben-Israel and T. N. Greville, *Generalized inverses: theory and applications*. Springer Science & Business Media, 2003, vol. 15.

[19] J. Han, "From PID to Active Disturbance Rejection Control," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 3, pp. 900–906, 2009.

[20] A. Bemporad and A. Garulli, "Output-feedback predictive control of constrained linear systems via set-membership state estimation," *International journal of control*, vol. 73, no. 8, pp. 655–665, 2000.