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Periodic Event-triggered Control for Stabilization of Lur'e Systems under Saturating Actuators*

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Abstract— **This paper deals with the stability analysis of continuous-time Lur'e systems under dynamic periodic event-triggered saturating control. A looped-functional approach is applied to handle the continuous-time dynamics of the plant and the aperiodic control signal updates. Considering the emulation design, sector-based inequalities, and Lyapunov Theory arguments, sufficient conditions are derived to ensure local asymptotic stability of the origin of the closed-loop system. Then, a convex optimization problem is proposed to synthesize the event generator parameters aiming to reduce the number of events regarding the time-based implementation. The proposed approach is illustrated by a numerical example.**

I. Introduction

In the last years, the advancement of digital technology has significantly impacted the implementation of modern control systems. Although closed-loop stability and performance remain the two primary goals, the industry demands new requirements, e.g., modularity, and reduced cost [1]. The control community has introduced the paradigm of networked control systems (NCS) to cope with some of these challenges. In this case, the distributed control loops are implemented over a shared digital communication network, which makes the analysis and design of control systems more complex when compared to the traditional point-to-point architecture. This is mainly due to network-induced effects such as bandwidth limitation and packet dropouts [2].

The event-triggered control (ETC) strategy has emerged as a promising solution to tackle these issues. The basic idea requires monitoring system variables in a periodic or continuous-time fashion, and updating the control signal only when a triggering criterion is satisfied [3]. If the triggering condition is evaluated at periodic monitoring instants, then the control updates always occur at integer multiples of the monitoring period, which gives rise to the term periodic ETC (PETC). Moreover, the ETC literature includes two design approaches: emulation and co-design. In the emulation, the aim is to compute only the event-triggering mechanism (ETM), with a stabilizing controller given *a priori*, such that the closed-loop stability is guaranteed for the periodic or continuous-time implementation. In the co-design, the goal is to determine the control law and the ETM simultaneously.

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In both cases, the aim is to compute a suitable ETM capable of striking a balance between performance and reduction of events. The most simple approach involves a static ETM to decide when the control signal needs to be updated. Thus, an event occurs only when a scalar function of the states at the current instant and last event instant exceeds a certain constant threshold. Reference [4] proposed a new triggering criterion, based on a dynamic threshold that evolves with time, to further reduce the number of events.

PETC is an alternative to the continuous-time monitoring of the triggering rule (see, e.g., [1], [2]) best suited for digital implementation, with the advantage of implicitly ruling out the occurrence of Zeno behavior [5]. However, the dynamic PETC brings new challenges from an analysis and design perspective. For instance, following an emulation approach and the hybrid systems formalism, [6] proposes a dynamic PETC strategy under an output feedback controller to stabilize a linear time-invariant system. Reference [7] presents a co-design method to determine a gain-scheduling controller and a dynamic PETC policy for quasi-linear parameter-varying systems subject to $\mathscr{L}_2$ disturbances and delays.

A challenging task in NCS is the non-uniform (aperiodic) sampling that may occur in some implementations. Reference [8] introduced a novel method to derive stability conditions for linear aperiodic sampled-data systems through the use of looped-functionals. This approach has also been applied for nonlinear systems. For instance, [9] investigates the effect of saturating inputs in the plant model, which is an ubiquitous problem in engineering applications [10]. In the ETC framework, [11] proposes an observer-based ETC scheme, under a static ETM with continuous-time monitoring and dwell-time, for systems with slope-restricted nonlinearities. Note that none of these papers considers a dynamic PETC policy.

In this work, we present a dynamic PETC strategy for a continuous-time saturated Lur'e system. To handle the continuous-time evolution of the plant states and the discrete-time updates of the controller, we apply a looped-functional approach. Following an emulation design, sector-based inequalities, and Lyapunov Theory, conditions based on linear matrix inequalities (LMIs) are derived to ensure local asymptotic stability of the origin of the closed-loop system. Then, we propose an optimization problem to compute the ETM parameters aiming to reduce the number of events when compared to a time-triggered policy. Besides, we guarantee that the trajectories converge to the origin provided that the initial condition belongs to a given admissible set. Finally, we illustrate the proposed method with a numerical example.

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Notation: $\mathbb{N}_k^*$, $\mathbb{R}^+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{S}^n$ denote the sets of natural numbers belonging to the interval from 1 to k, positive real numbers, n-dimensional vectors, $n\times m$ real matrices, and symmetric matrices of order n, respectively. Given a matrix $A\in\mathbb{R}^{n\times m}$, $A_{(i,\,j)}$ corresponds to its $(i,\,j)$-th element. For a square matrix A, $\mathrm{tr}(A)$ indicates its trace, $A>0$ means that A is symmetric positive definite, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ represent its largest and smallest eigenvalue, respectively. $He\{A\}>0$ refers to $A^\top+A>0$ and x_i is the i-th component of vector x. Given a positive scalar T, the notation $\mathbb{K}^n_{[0,\,T]}$ represents the set of continuous functions from $[0,\,T]$ into $\mathbb{R}^n$. I and 0 are identity and null matrices of appropriate dimensions. The notation $\star$ indicates symmetric blocks within a matrix. $diag\{X,Y\}$ denotes the block-diagonal matrix composed by matrices X and Y.

II. Preliminaries

A. Lur'e system

Consider the continuous-time Lur'e system represented by

$$\begin{cases}\dot{x}(t)=Ax(t)+B\sigma(y(t))+Esat(v(t)), \quad x(0)=x_0\\ y(t)=Cx(t),\end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^m$, $v(t)\in\mathbb{R}^p$ are the state, nonlinearity input and control signal, respectively. All matrices present compatible dimensions and have constant real-valued elements. $\sigma(\cdot)\in\mathbb{R}^m$ is a time-invariant, memory-less, vector-valued nonlinear function with components $\sigma_j:\mathbb{R}\to\mathbb{R}$, for all $j\in\mathbb{N}_m^*$, verifying $\forall y\in\mathbb{R}^m$

(P1) $\sigma_j(y)=\sigma_j(y_j)$, $\sigma_j(0)=0$;

(P2) $\sigma_j(y_j)y_j\geq 0$;

(P3) $0\leq\frac{d}{dy_j}\{\sigma_j(y_j)\}\leq\Lambda_j$, with $\Lambda_j>0$.

Properties (P1)-(P3) imply that $\sigma(\cdot)$ is a decentralized, slope-restricted nonlinearity, where each component belongs to the first and third quadrants. Besides, the property (P3) guarantees that there exists a non-negative scalar Ω_j, for each Λ_j, satisfying $0\leq\Omega_j\leq\Lambda_j$ such that $\sigma_j(y_j)$ belongs to the sector $[0,\Omega_j]$. Then, considering $\Lambda:=diag\{\Lambda_1,\ldots,\Lambda_m\}$ and $\Omega:=diag\{\Omega_1,\ldots,\Omega_m\}$, we conclude that $\sigma(\cdot)$ globally verifies the classical sector condition $\sigma^\top(y)J_1[\sigma(y)-\Omega y]\leq 0$, $\forall y\in\mathbb{R}^m$, for any diagonal matrix $J_1\in\mathbb{S}^m$, $J_1>0$ [11].

The term $sat(\cdot):\mathbb{R}^p\to\mathbb{R}^p$ is a decentralized symmetric vector-valued static saturation function [10]. Each component is defined as $sat_r(v)=sign(v_r)min\{|v_r|,\,u_{0r}\}$ for $r\in\mathbb{N}_p^*$, and $\pm u_{0r}$ are the control amplitude bounds of the r-th input signal. Then, from the deadzone function $\psi(v(t)):=v(t)-sat(v(t))$, it follows that (1) can be rewritten as

$$\dot{x}(t)=Ax(t)+B\sigma(Cx(t))+Ev(t)-E\psi(v(t)). \tag{2}$$

To avoid unnecessary control updates that often occur in periodic sampled-data control, we follow a dynamic PETC approach. The idea is to sample the plant state with a period $T>0$, compute the value of a triggering function, and check whether it exceeds a certain dynamic threshold that evolves with time. This enables the ETM to update the controller only when needed to ensure closed-loop stability or performance requirements. Thus, when the triggering criterion is verified, one event occurs, and the control action is updated and kept constant by a zero-order hold until the next event.

Similarly to [7], we consider the following rule to determine the event instants:

$$k_{i+1}T=\min_{k,i\in\mathbb{N}}\{kT>k_iT:f(\delta(kT),x(kT))>\frac{1}{\theta}\eta(kT)\}, \tag{3}$$

where $k_0=0$, and k_iT denotes the i-th event instant. The variable $\eta:\mathbb{R}\to\mathbb{R}$ represents the dynamic threshold, θ is a positive real scalar and $f(\cdot,\cdot)$ is the triggering function. All these elements are design parameters that will be determined later. The variable $\delta(\cdot)$ stands for the state deviation between the last event and the current sample, that is

$$\delta(kT)=x(k_iT)-x(kT), \text{ for } kT\in[k_iT,k_{i+1}T). \tag{4}$$

The sampling instants form a sequence of increasing positive real scalars $\{kT\}_{k\in\mathbb{N}}\subset\mathbb{R}$ with no accumulation points such that $\bigcup_{k\in\mathbb{N}}[kT,(k+1)T)=[0,+\infty)$, thereby eliminating the possibility of Zeno behavior [5]. In this article, we consider a quadratic triggering function as in [11]:

$$f(\delta(kT),x(kT))=\delta^\top(kT)Q_\delta\delta(kT)-x^\top(kT)Q_xx(kT), \tag{5}$$

with Q_δ and Q_x being symmetric positive definite matrices. It represents a measure of the relative error between the states at the last event and at the current sampling instant.

In addition, the continuous-time variable $\eta(t)$ satisfies the first-order differential equation

$$\dot{\eta}(t)=-\lambda\eta(t)-f(\delta(kT),x(kT)),\forall t\in[kT,(k+1)T), \tag{6}$$

with a given initial condition $\eta(0)=\eta_0$ and a design parameter $\lambda>0$ that affects its decay rate. To derive a suitable dynamic threshold, it is necessary to avoid negative values of $\eta(t)$, as they can cause triggers to occur earlier than in the static ETM case. This is ensured by imposing $\eta_0\geq 0$ and tuning parameters λ and θ such that $\theta\geq\frac{1}{\lambda}(e^{\lambda T}-1)$. Lemma 3 from [7] provides a formal proof of this result.

In order to control (2), we propose the following event-based state-feedback control law

$$v(t)=v(k_iT)=Kx(k_iT),\forall t\in[k_iT,k_{i+1}T),\forall i\in\mathbb{N}, \tag{7}$$

where $K\in\mathbb{R}^{p\times n}$ is the controller gain matrix. Considering $\delta(kT)$ defined in (4), (7) can be rewritten as $v(t)=Kx(kT)+K\delta(kT)$. Applying this in (2) leads to the following closed-loop dynamics for all $t\in[k_iT,k_{i+1}T)$

$$\begin{aligned}\dot{x}(t)=&Ax(t)+B\sigma(Cx(t))+EKx(kT)+EK\delta(kT)\\&-E\psi(Kx(kT)+K\delta(kT)).\end{aligned} \tag{8}$$

B. Problem formulation

Due to the input saturation, global stability of the origin may not be achievable when the open-loop system is not asymptotically stable [10]. Thus, we will focus on providing local stability conditions of the origin of the nonlinear system described by (8) with the dynamic ETM (3). With this aim, we consider a compact domain of admissible initial conditions $\mathscr{D}\subseteq\mathscr{R}_a\subseteq\mathbb{R}^n$, where $\mathscr{R}_a$ denotes the region of attraction of the origin. Based on this setup, the problem under consideration is defined as follows:

Problem. Given a feedback controller gain K, positive scalars T, λ, θ, and a suitable domain $\mathscr{D} \subseteq \mathbb{R}^n$, compute the parameters Q_δ and Q_x of the triggering function (5) such that all trajectories of the closed-loop system (8) starting at $\mathscr{D}$ converge asymptotically to the origin, while reducing the number of events regarding the time-triggered control policy.

III. Main Result

A. Looped-functional approach

To derive the stability conditions, the idea is to represent the state trajectories in a lifted domain. Define $\chi_k(\tau) := x(\tau + kT)$, $\sigma_k(\tau) := \sigma(Cx(\tau + kT))$, $\delta_k(\tau) := \delta(\tau + kT)$, $\eta_k(\tau) := \eta(\tau + kT)$, $\dot{\chi}_k(\tau) := \frac{d}{d\tau}\chi_k(\tau)$, with $\tau = t - kT$, $\tau \in [0, T)$, $k \in \mathbb{N}$. Thus, the closed-loop system (8) in the inter-sampling interval can be described as

$$\dot{\chi}_k(\tau) = A\chi_k(\tau) + B\sigma_k(\tau) - E\psi_k(0) + EK\chi_k(0) + EK\delta_k(0), \quad (9)$$

with $\psi_k(0) := \psi(K\chi_k(0) + K\delta_k(0))$. The dynamic threshold (6) evolves on the lifted domain according to

$$\dot{\eta}_k(\tau) = -\lambda\eta_k(\tau) - \delta_k^\top(0)Q_\delta\delta_k(0) + x_k^\top(0)Q_x x_k(0). \quad (10)$$

Consider now the function $V(x(t), \eta(t)) = H(x(t)) + \eta(t)$, where $H : \mathbb{R}^n \to \mathbb{R}^+$ is a scalar function of the plant states for which there exist positive real scalars $\mu_2 > \mu_1$ such that

$$\forall x \in \mathbb{R}^n, \quad \mu_1 \|x(t)\|^2 \le H(x(t)) \le \mu_2 \|x(t)\|^2. \quad (11)$$

From (11) and the fact that the threshold $\eta(t)$ is bounded and positive for all $t \ge 0$, we have

$$\mu_1 \|x(t)\|^2 + |\eta(t)| \le V(x(t), \eta(t)) \le \mu_2 \|x(t)\|^2 + |\eta(t)|,$$

which means that $V(\cdot,\cdot)$ is a radially unbounded function.

Let $\mathscr{V}_0 : [0, T] \times \mathbb{K}^n_{[0,T]} \to \mathbb{R}$ be a continuous-time functional and differentiable over $\tau \in [0, T)$, which satisfies for all $z \in \mathbb{K}^n_{[0,T]}$

$$\mathscr{V}_0(T, z) = \mathscr{V}_0(0, z). \quad (12)$$

Based on [9], we propose the following theorem that establishes the basis to ensure the local asymptotic stability of the origin of the closed-loop system (8).

Theorem 1: Consider positive scalars T, θ, and λ satisfying $\theta \ge \frac{1}{\lambda}(e^{\lambda T} - 1)$ and $0 \le \eta_0 < 1$. Let η and H be functions as defined before, and the looped-functional $\mathscr{V}_0(\tau, \chi_k)$ verifying (12). If there exist matrices $G_1, G_2 \in \mathbb{R}^{p \times n}$ and a positive definite diagonal matrix $J_2 \in \mathbb{S}^p$ such that

$$\begin{aligned} \left\| \begin{bmatrix} (K - G_1)_r & (K - G_2)_r \end{bmatrix} \begin{bmatrix} \chi_k(0) \\ \delta_k(0) \end{bmatrix} \right\|^2 &\le f(\delta_k(0), \chi_k(0)) \\ -\frac{1}{\theta}\eta_k(0) + u_{0r}^2(H(\chi_k(0)) + \eta_k(0)) &\quad \forall r \in \mathbb{N}_p^* \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d}{d\tau}H(\chi_k(\tau)) + \frac{d}{d\tau}\mathscr{V}_0(\tau, \chi_k) - \lambda\eta_k(\tau) - \delta_k^\top(0)Q_\delta\delta_k(0) \\ + \chi_k^\top(0)Q_x\chi_k(0) - 2\Upsilon < 0 \quad \forall k \in \mathbb{N}, \tau \in [0, T) \end{aligned} \quad (14)$$

where $\Upsilon = \psi_k^\top(0)J_2[\psi_k(0) - G_1\chi_k(0) - G_2\delta_k(0)]$, are satisfied along the trajectories of the closed-loop system (9), under the triggering rule (3) with the dynamic threshold given by (10), it follows that:

(i) For all $k \in \mathbb{N}$ and $\tau \in [0, T]$,

$$\Delta V(k) = V(\chi_k(T), \eta_k(T)) - V(\chi_k(0), \eta_k(0)) < 0; \quad (15)$$

(ii) The trajectories of the closed-loop system (8) starting in the set $\mathscr{X}_0 = \{x \in \mathbb{R}^n : H(x) \le 1 - \eta_0\}$, under the PETC strategy (3), are bounded and converge asymptotically to the origin.

Proof: Rewriting (14) as $\frac{d}{d\tau}V(\chi_k(\tau), \eta_k(\tau)) + \frac{d}{d\tau}\mathscr{V}_0(\tau, \chi_k) - 2\Upsilon < 0$ from the dynamics of $\eta_k(\tau)$ in (10), and integrating this inequality over the interval $[0, T)$, considering that the looping condition (12) holds, relation (15) follows directly provided that $\chi_{ak}(0) = \begin{bmatrix} \chi_k^\top(0) & \delta_k^\top(0) \end{bmatrix}^\top \in \mathscr{S}_0 = \{\chi_{ak}(0) \in \mathbb{R}^{2n} : |(K_a - G_a)_r\chi_{ak}(0)| \le u_{0r}, r \in \mathbb{N}_p^*\}$, with $K_a = \begin{bmatrix} K & K \end{bmatrix}$ and $G_a = \begin{bmatrix} G_1 & G_2 \end{bmatrix}$. In this case, the symmetric polyhedral set $\mathscr{S}_0$ defines the region of validity of the generalized sector condition $\Upsilon \le 0$ [10]. Before determining if $\chi_{ak}(0) \in \mathscr{S}_0$ from (13), we need to show the ETM inequality. Note that the ETM (3) ensures $f(\delta_k(0), \chi_k(0)) - \frac{1}{\theta}\eta_k(0) \le 0$ for all $k \in (k_i, k_{i+1})$. On the other hand, $\delta_k(0)$ is reset to zero when an event occurs, and thus $f(\delta_k(0), \chi_k(0)) = -\chi_k^\top(0)Q_x\chi_k(0) < 0$. From this reasoning, since $\eta_k(0) \ge 0$ for all k, we have $f(\delta_k(0), \chi_k(0)) - \frac{1}{\theta}\eta_k(0) \le 0$ for all $k \in \mathbb{N}$. In turn, condition (13) leads to

$$\|(K_a - G_a)_r\chi_{ak}(0)\|^2 \le u_{0r}^2\big(H(\chi_k(0)) + \eta_k(0)\big), \quad (16)$$

from which we conclude that $\chi_{ak}(0) \in \mathscr{S}_0$ as long as $V(\chi_k(0), \eta_k(0)) = H(\chi_k(0)) + \eta_k(0) \le 1$.

Consider the set $\mathscr{X} = \{x \in \mathbb{R}^n : H(x) \le 1\}$, such that, by construction, $\mathscr{X}_0 \subseteq \mathscr{X}$. Using induction arguments, we will prove that the generalized sector condition is verified for all $k \in \mathbb{N}$. If $\chi_0(0) \in \mathscr{X}_0$ (i.e. $H(\chi_0(0)) + \eta_0(0) \le 1$), (16) implies that $\chi_{a0}(0) \in \mathscr{S}_0$, and according to Lemma 1.6 from [10], $\Upsilon \le 0$. Hence, the total variation of the Lyapunov function is negative, i.e., $V(\chi_0(T), \eta_0(T)) < V(\chi_0(0), \eta_0(0))$, which in turn implies that

$$H(\chi_0(T)) \le V(\chi_0(T), \eta_0(T)) < V(\chi_0(0), \eta_0(0)) \le 1,$$

and thus $\chi_1(0) = \chi_0(T)$ also belongs to $\mathscr{X}$ by continuity of the system trajectories (8). Repeating these arguments for $k \ge 1$, we conclude that $\chi_k(0) \in \mathscr{X}$ for all $k \in \mathbb{N}$, the sector condition holds, and $V(\chi_k(0), \eta_k(0))$ is decreasing at the sampling instants, which implies that both sequences $\eta_k(0) \to 0$ and $\chi_k(0) \to 0$ as $k \to \infty$.

Now, the idea is to prove that the continuous-time trajectories are bounded and converge asymptotically to the origin. For this, we define the auxiliary set

$$\begin{aligned} \mathscr{S}_\alpha = \{\alpha = [\tilde{\alpha}^\top \ \hat{\alpha}^\top]^\top \in \mathbb{R}^{m+p} : 0 \le \tilde{\alpha}_j \le \Omega_j \text{ and} \\ 0 \le \hat{\alpha}_r \le 1, j \in \mathbb{N}_m^*, r \in \mathbb{N}_p^*\}. \end{aligned}$$

Recall that the nonlinear functions $\sigma(\cdot)$ and $sat(\cdot)$ globally belong to sectors $[0, \Omega]$ and $[0, I]$, respectively. Hence, for each $t \ge 0$, there exist $\alpha(t) \in \mathscr{S}_\alpha$ such that $\sigma_j(y(t)) = \tilde{\alpha}_j(t)y_j(t)$ and $sat_r(v(t)) = \hat{\alpha}_r(t)v_r(t)$. Then, defining $\tilde{\Xi}(t) := diag\{\tilde{\alpha}_1(t), \ldots, \tilde{\alpha}_m(t)\}$ and $\hat{\Xi}(t) := diag\{\hat{\alpha}_1(t), \ldots, \hat{\alpha}_p(t)\}$, the closed-loop system (8) can be represented by the following linear time-varying system

$$\dot{x}(t) = (A + B\Xi(t)C)x(t) + E\hat{\Xi}(kT)Kx(kT) + E\hat{\Xi}(kT)K\delta(kT), \quad \forall t \in [k_iT, k_{i+1}T), i \in \mathbb{N}. \tag{17}$$

Defining $\Xi_k(\tau) := \Xi(\tau + kT)$ and $\hat{\Xi}_k(0) := \hat{\Xi}(kT)$, the system (17) in the inter-sampling interval can be described $\forall k \in \mathbb{N}$ and $\tau \in [0, T)$ as follows

$$\dot{\chi}_k(\tau) = (A + B\Xi_k(\tau)C)\chi_k(\tau) + \bar{\Xi}_k(0) \tag{18}$$

with $\bar{\Xi}_k(0) := E\hat{\Xi}_k(0)K(\chi_k(0) + \delta_k(0))$.

Note that the solutions $x(t)$ of the linear time-varying system in (18), for each admissible function $\alpha(t) \in \mathscr{S}_\alpha$, can be expressed in terms of the state transition matrix $\Phi_\alpha(t, t_0)$. Considering now $\alpha_k(\tau) := \alpha(\tau + kT)$, with $\alpha_k \in \mathbb{K}^{m+p}_{[0,T]}$, we define $\Phi_{\alpha_k}(\tau, s) := \Phi_\alpha(\tau + kT, s + kT)$ as the restriction of the matrix-valued function $\Phi_\alpha(t, t_0)$ in the closed inter-sampling intervals $[kT, (k+1)T]$, for all $k \in \mathbb{N}$. Using this definition, we can compute the trajectories as

$$\chi_k(\tau) = \Phi_{\alpha_k}(\tau, 0)\chi_k(0) + \int_0^\tau \Phi_{\alpha_k}(\tau, s)\bar{\Xi}_k(0)ds, \tag{19}$$

which are bounded according to the relation

$$\|\chi_k(\tau)\| \leq \|\Phi_{\alpha_k}(\tau, 0)\| \, \|\chi_k(0)\| + \gamma \tag{20}$$

with $\gamma := \left\|\int_0^\tau \Phi_{\alpha_k}(\tau, s)E\hat{\Xi}_k(0)Kds\right\| (\|\chi_k(0)\| + \|\delta_k(0)\|)$.

Based on the triggering criterion (3), it follows that

$$\|\delta_k(0)\| \leq \sqrt{\frac{1}{\theta\lambda_{min}(Q_\delta)}}\sqrt{\eta_k(0)} + \sqrt{\frac{\lambda_{max}(Q_x)}{\lambda_{min}(Q_\delta)}}\|\chi_k(0)\| \tag{21}$$

holds for all $k \in \mathbb{N}$. Replacing (21) in (20), and considering all the admissible functions $\alpha_k(\tau) \in \mathscr{S}_\alpha$ for all $\tau \in [0, T]$, there exist two positive real scalars

$$\beta_1 = \sup_{\alpha_k \in \mathbb{K}^{m+p}_{[0,T]}, \alpha_k(\tau) \in \mathscr{S}_\alpha, \tau \in [0,T]} \|\Phi_{\alpha_k}(\tau, 0)\|,$$

$$\beta_2 = \sup_{\alpha_k \in \mathbb{K}^{m+p}_{[0,T]}, \alpha_k(\tau) \in \mathscr{S}_\alpha, \tau \in [0,T]} \left\|\int_0^\tau \Phi_{\alpha_k}(\tau, s)E\hat{\Xi}_k(0)Kds\right\|,$$

such that $\|\chi_k(\tau)\| \leq \bar{\beta}_1\|\chi_k(0)\| + \bar{\beta}_2\sqrt{\eta_k(0)}$, with positive real scalars $\bar{\beta}_1 := (\beta_1 + \beta_2(1 + \sqrt{\lambda_{max}(Q_x)/\lambda_{min}(Q_\delta)}))$ and $\bar{\beta}_2 := \beta_2\sqrt{1/(\theta\lambda_{min}(Q_\delta))}$.

From this reasoning, we see that $\chi_k(\tau)$ is bounded for all $k \in \mathbb{N}$ and all τ. Therefore, we conclude that the continuous-time trajectories $x(t)$ do not diverge in the inter-sampling intervals. Furthermore, since (15) ensures that $\eta_k(0) \to 0$ and $\chi_k(0) \to 0$ as $k \to \infty$, it follows that the solutions $x(t) \to 0$ as $t \to \infty$, which concludes the proof. ■

B. Local stability conditions

Before stating the main result, which allow us to determine the dynamic ETM (3) while guaranteeing closed-loop stability, we define the following auxiliary matrices:

$M_0 = [A \;\; EK \;\; 0 \;\; -I \;\; B \;\; -E \;\; 0 \;\; EK]$,
$M_1 = [I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$, $M_2 = [0 \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$,
$M_3 = [0 \;\; 0 \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$, $M_4 = [0 \;\; 0 \;\; 0 \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0]$,
$M_5 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; I \;\; 0 \;\; 0 \;\; 0]$, $M_6 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I \;\; 0 \;\; 0]$,
$M_7 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I \;\; 0]$, $M_8 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I]$,

$$M_9 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & -2I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_{12} = M_1 - M_2, \quad \tilde{M}_2 = [M_2^\top \;\; M_6^\top \;\; M_8^\top]^\top.$$

Theorem 2: Given positive scalars T, λ, $\theta \geq \frac{1}{\lambda}(e^{\lambda T} - 1)$ and $0 \leq \eta_0 < 1$, if there exist positive definite matrices P, R, X_6, Q_δ, $\bar{Q}_x \in \mathbb{S}^n$, positive definite diagonal matrices $J_1 \in \mathbb{S}^m$, $J_2 \in \mathbb{S}^p$, matrices X_1, S_1, $Q_1 \in \mathbb{S}^n$, $X_4 \in \mathbb{S}^p$, X_3, S_2, S_4, Q_2, $Q_4 \in \mathbb{R}^{n\times n}$, X_5, $\bar{G}_1$, $\bar{G}_2 \in \mathbb{R}^{p\times n}$, X_2, S_3, $Q_3 \in \mathbb{R}^{n\times p}$, $Y \in \mathbb{R}^{n\times(4n+m+p)}$, $Z \in \mathbb{R}^{(4n+m+p)\times 2n}$ and a scalar $\varepsilon > 0$ such that the following matrix inequalities are satisfied

$$\begin{bmatrix} F_1 + T(-F_2 + F_3 + F_4) & M_2^\top \\ \star & -\bar{Q}_x \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} F_1 - T(F_2 + F_4 + F_5) & \begin{bmatrix} Z \\ 0 \end{bmatrix} & M_2^\top \\ \star & -\frac{1}{T}\begin{bmatrix} R & 0 \\ \star & 3R \end{bmatrix} & 0 \\ \star & \star & -\bar{Q}_x \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} P & 0 & 0 & (J_{2_{(r,r)}}K_r - \bar{G}_{1r})^\top & I \\ \star & Q_\delta & 0 & (J_{2_{(r,r)}}K_r - \bar{G}_{2r})^\top & 0 \\ \star & \star & 1 - \frac{1}{\theta} & 0 & 0 \\ \star & \star & \star & 2\varepsilon J_{2_{(r,r)}} - \frac{\varepsilon^2}{u_{0r}^2} & 0 \\ \star & \star & \star & \star & \bar{Q}_x \end{bmatrix} > 0 \quad \forall r \in \mathbb{N}_p^* \tag{24}$$

where

$$F_1 = He\Big\{M_1^\top PM_4 - M_{12}^\top(S_2M_2 + S_3M_6 + S_4M_8) - \begin{bmatrix} Z \\ 0 \end{bmatrix} M_9 - M_5^\top J_1(M_5 - \Omega CM_1) - M_6^\top(J_2M_6 - \bar{G}_1M_2 - \bar{G}_2M_8) + \begin{bmatrix} Y & 0 \end{bmatrix}^\top M_0\Big\} - M_{12}^\top S_1M_{12} - M_7^\top \lambda M_7 - M_8^\top Q_\delta M_8,$$

$$F_3 = He\Big\{M_4^\top(S_1M_{12} + S_2M_2 + S_3M_6 + S_4M_8) + M_1^\top(Q_1M_3 + Q_2M_2 + Q_3M_6 + Q_4M_8)\Big\} + M_4^\top RM_4,$$

$$F_2 = M_3^\top Q_1M_3, \; F_4 = \tilde{M}_2^\top X\tilde{M}_2, \; X = \begin{bmatrix} X_1 & X_2 & X_3 \\ \star & X_4 & X_5 \\ \star & \star & X_6 \end{bmatrix},$$

$$F_5 = He\Big\{M_3^\top(Q_2M_2 + Q_3M_6 + Q_4M_8)\Big\},$$

then all trajectories of the closed-loop system (8) starting in the set $\mathscr{X}_0 = \{x \in \mathbb{R}^n : x^\top Px \leq 1 - \eta_0\}$, under the triggering rule (3), triggering function (5) with Q_δ, $Q_x = \bar{Q}_x^{-1}$, and dynamic strategy described in (6), are bounded and converge asymptotically to the origin.

Proof: Consider the candidate function $V(x(t), \eta(t)) = H(x(t)) + \eta(t)$, where $H(x(t)) = x^\top(t)Px(t)$ with a positive definite matrix $P \in \mathbb{S}^n$. Note that the function $H(x(t))$ satisfies (11), with scalars $\mu_1 = \lambda_{min}(P)$ and $\mu_2 = \lambda_{max}(P)$. Besides, we consider the following candidate functional $\mathscr{V}_0(\tau, \chi_k)$:

$$\mathscr{V}_0(\tau, \chi_k) = (T - \tau)\Big\{\zeta_k^\top(\tau)[S_1\zeta_k(\tau) + 2S_2\chi_k(0) + 2S_3\psi_k(0) + 2S_4\delta_k(0)] + \tau\vartheta_k^\top(\tau)[Q_1\vartheta_k(\tau) + 2Q_2\chi_k(0) + 2Q_3\psi_k(0) + 2Q_4\delta_k(0)] + \tau\bar{\chi}_k^\top(0)X\bar{\chi}_k(0) + \int_0^\tau \dot{\chi}_k^\top(\omega)R\dot{\chi}_k(\omega)d\omega\Big\},$$

with $\zeta_k(\tau) := \chi_k(\tau) - \chi_k(0)$, $\vartheta_k(\tau) := \frac{1}{\tau}\int_0^\tau \chi_k(\omega)d\omega$, and $\bar{\chi}_k(0) := \begin{bmatrix} \chi_k^\top(0) & \psi_k^\top(0) & \delta_k^\top(0) \end{bmatrix}^\top$.

Note that $\mathscr{V}_0(\tau,\chi_k)$ satisfies the looping condition (12), with $\mathscr{V}_0(0,\chi_k) = \mathscr{V}_0(T,\chi_k) = 0$. Thus, if relation $\Gamma(\tau,\chi_k,\eta_k) = \frac{d}{d\tau}\mathscr{W}(\tau,\chi_k,\eta_k) - 2\sigma_k^\top(\tau)J_1[\sigma_k(\tau) - \Omega C\chi_k(\tau)] - 2\Upsilon < 0$ is verified along trajectories of the closed-loop system (9) with $\mathscr{W}(\tau,\chi_k,\eta_k) := V(\chi_k(\tau),\eta_k(\tau)) + \mathscr{V}_0(\tau,\chi_k)$ and $\chi_{ak}(0) \in \mathscr{S}_0$, it follows that condition (14) holds. Besides, from the satisfaction of properties (P1)-(P3), it follows that $2\sigma_k^\top(\tau)J_1[\sigma_k(\tau) - \Omega C\chi_k(\tau)] \le 0$. Computing $\Gamma(\tau,\chi_k,\eta_k)$ leads to

$$\begin{aligned}
\Gamma(\tau,\chi_k,\eta_k) &= 2\chi_k^\top(\tau)P\dot{\chi}_k(\tau) - \lambda\eta_k(\tau)\\
&- \delta_k^\top(0)Q_\delta\delta_k(0) + \chi_k^\top(0)Q_x\chi_k(0)\\
&- \zeta_k^\top(\tau)[S_1\zeta_k(\tau) + 2S_2\chi_k(0) + 2S_3\psi_k(0) + 2S_4\delta_k(0)]\\
&+ (T-\tau)\dot{\chi}_k^\top(\tau)[2S_1\zeta_k(\tau) + 2S_2\chi_k(0) + 2S_3\psi_k(0)\\
&+ 2S_4\delta_k(0)] - \tau\vartheta_k^\top(\tau)[2Q_2\chi_k(0) + 2Q_3\psi_k(0) + 2Q_4\delta_k(0)]\\
&- T\vartheta_k^\top(\tau)Q_1\vartheta_k(\tau) + (T-\tau)\chi_k^\top(\tau)[2Q_1\vartheta_k(\tau) + 2Q_2\chi_k(0)\\
&+ 2Q_3\psi_k(0) + 2Q_4\delta_k(0)] + (T-2\tau)\bar{\chi}_k^\top(0)X\bar{\chi}_k(0)\\
&- \int_0^\tau \dot{\chi}_k^\top(\omega)R\dot{\chi}_k(\omega)d\omega + (T-\tau)\dot{\chi}_k^\top(\tau)R\dot{\chi}_k(\tau)\\
&- 2\sigma_k^\top(\tau)J_1[\sigma_k(\tau) - \Omega C\chi_k(\tau)] - 2\Upsilon.
\end{aligned} \tag{25}$$

Introducing the auxiliary vector

$$\begin{aligned}
\xi_k(\tau) = [&\chi_k^\top(\tau)\ \chi_k^\top(0)\ \vartheta_k^\top(\tau)\ \dot{\chi}_k^\top(\tau)\\
&\sigma_k^\top(\tau)\ \psi_k^\top(0)\ \sqrt{\eta_k(\tau)}\ \delta_k^\top(0)]^\top \in \mathbb{R}^{5n+m+p+1},
\end{aligned}$$

the Wirtinger inequality [12] can be used to derive an upper bound to the integral term in (25) as follows

$$\begin{aligned}
&-\int_0^\tau \dot{\chi}_k^\top(\omega)R\dot{\chi}_k(\omega)d\omega \le\\
&\xi_k^\top(\tau)\left\{-He\left\{\begin{bmatrix}Z\\0\end{bmatrix}M_9\right\} + \tau\begin{bmatrix}Z\\0\end{bmatrix}\begin{bmatrix}R & 0\\ \star & 3R\end{bmatrix}^{-1}\begin{bmatrix}Z\\0\end{bmatrix}^\top\right\}\xi_k(\tau),
\end{aligned} \tag{26}$$

with $Z \in \mathbb{R}^{(4n+m+p)\times 2n}$. In addition, since $M_0\xi_k(\tau) = 0$ along the trajectories of the system (9), we have that

$$2\xi_k^\top(\tau)\begin{bmatrix}Y & 0\end{bmatrix}^\top M_0\xi_k(\tau) = 0 \tag{27}$$

for any matrix $Y \in \mathbb{R}^{n\times(4n+m+p)}$.

Then, from (25)-(27), condition $\Gamma(\tau,\chi_k,\eta_k) < 0$ is verified if

$$\begin{aligned}
\xi_k^\top(\tau)\Bigg\{&F_1 + M_2^\top Q_x M_2 - TF_2 + (T-\tau)F_3 + (T-2\tau)F_4\\
&- \tau F_5 + \tau\begin{bmatrix}Z\\0\end{bmatrix}\begin{bmatrix}R & 0\\ \star & 3R\end{bmatrix}^{-1}\begin{bmatrix}Z\\0\end{bmatrix}^\top\Bigg\}\xi_k(\tau) < 0
\end{aligned} \tag{28}$$

holds for all $\tau \in [0,T]$ and $\chi_{ak}(0) \in \mathscr{S}_0$. Now, observe that (28) can also be expressed in a quadratic form $\xi_k^\top(\tau)\mathscr{M}(\tau)\xi_k(\tau) < 0$, where $\mathscr{M}(\tau)$ is affine with respect to τ. Therefore, by applying convexity arguments, it suffices to ensure $\mathscr{M}(\tau) < 0$ for $\tau = 0$ and $\tau = T$ to guarantee $\Gamma(\tau,\chi_k,\eta_k) < 0$ for all $\tau \in [0,T]$.

Applying $\tau = 0$ in $\mathscr{M}(\tau) < 0$ leads directly to (22), after a Schur's complement in the quadratic term $M_2^\top Q_x M_2$. Furthermore, one retrieves condition (23) considering $\tau = T$ in $\mathscr{M}(\tau) < 0$, and applying a Schur's complement twice. Hence, (22) and (23) imply that condition (14) of Theorem 1 is satisfied.

Now, we will show that (24) ensures $\chi_{ak}(0) \in \mathscr{S}_0$ for all $k \in \mathbb{N}$, provided that $\chi_0(0) \in \mathscr{X}_0$. First, note that relation

$$\left(\varepsilon/u_{0r}^2 - J_{2(r,r)}\right)u_{0r}^2/\varepsilon\left(\varepsilon/u_{0r}^2 - J_{2(r,r)}\right)^\top \ge 0 \tag{29}$$

holds for any scalar $\varepsilon > 0$ and $r \in \mathbb{N}_p^*$. By simple manipulations, we get

$$2\varepsilon J_{2(r,r)} - \varepsilon^2/u_{0r}^2 \le J_{2(r,r)}^2 u_{0r}^2. \tag{30}$$

Then, from (24) and (30), condition

$$\begin{bmatrix}
P & 0 & 0 & (J_{2(r,r)}K_r - \bar{G}_{1r})^\top & I\\
\star & Q_\delta & 0 & (J_{2(r,r)}K_r - \bar{G}_{2r})^\top & 0\\
\star & \star & 1-\frac{1}{\theta} & 0 & 0\\
\star & \star & \star & J_{2(r,r)}^2 u_{0r}^2 & 0\\
\star & \star & \star & \star & \bar{Q}_x
\end{bmatrix} > 0 \quad \forall r \in \mathbb{N}_p^* \tag{31}$$

also holds. Now, pre- and post-multiplying (31) by $diag\{I, I, I, J_2^{-1}, I\}$, considering $G_1 = J_2^{-1}\bar{G}_1$, $G_2 = J_2^{-1}\bar{G}_2$, applying Schur's complement twice, pre- and post-multiplying the resulting inequality by the vector $[\chi_{ak}^\top(0)\ \sqrt{\eta_k(0)}]^\top$ and its transpose, respectively, leads to (13) of Theorem 1.

At this point, it follows that the satisfaction of (22)-(24) implies that all conditions of Theorem 1 are verified, which concludes the proof. ■

Remark 1. Similarly to Theorem 2, we can also derive local stability conditions for the closed-loop system under a static ETM. It suffices to disregard the dynamic threshold in the matrix inequalities presented in Theorem 2 by removing the row and the column corresponding to $\sqrt{\eta}$ in (22)-(24).

IV. Optimization Problem

Here we present a convex optimization problem to systematically compute the ETM parameters while ensuring local asymptotic stability of the origin of closed-loop system. We assume $\mathscr{D}$ as an ellipsoidal set $\mathscr{D} = \{x \in \mathbb{R}^n : x^\top P_0 x \le 1 - \eta_0\}$, with a given positive definite matrix $P_0 \in \mathbb{R}^{n\times n}$. Then, if (22)-(24) and

$$P_0 > P \tag{32}$$

are satisfied, we conclude that $\mathscr{D} \subset \mathscr{X}_0$ and all trajectories of the closed-loop system (8) starting in $\mathscr{D}$ converge asymptotically to the origin.

Now, the idea is to design Q_δ and Q_x aiming to reduce the number of events when compared to the time-based implementation. Note that an event does not occur while

$$\delta^\top(kT)Q_\delta\delta(kT) - x^\top(kT)Q_x x(kT) \le (1/\theta)\eta(kT) \tag{33}$$

is verified. Thus, to reduce the triggering activity, we can indirectly minimize the eigenvalues of Q_δ by minimizing its trace, and indirectly maximize the eigenvalues of Q_x by minimizing the trace of its inverse $\bar{Q}_x$. Based on this reasoning, we propose the following optimization problem:

$$\begin{aligned}
&\min\ \operatorname{tr}(Q_\delta) + \operatorname{tr}(\bar{Q}_x)\\
&\text{subject to: } (22), (23), (24), (32).
\end{aligned} \tag{34}$$

It is important to emphasize that the constraints presented in (34) are LMIs as long as the scalar ε is given *a priori*. Thus, this offline optimization problem can be solved by applying a grid search algorithm on the ε.

V. Numerical Example

Consider the following Lur'e system

$$\dot{x} = \begin{bmatrix} -\frac{1}{4} & 1 \\ 1 & -\frac{1}{4} \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \sigma(x_1) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u,$$

$$\sigma(x_1) = \begin{cases} 0.05x_1 + 0.05 & \text{if } x_1 > 1 \\ 0.1x_1, & \text{if } -1 \le x_1 \le 1, \\ 0.05x_1 - 0.05 & \text{if } x_1 < -1 \end{cases}$$

controlled by (7), with $K = \begin{bmatrix} -0.4639 & -0.6668 \end{bmatrix}$, saturation level $u_0 = 2.5$, and sampling period $T = 0.8$. Note that the $\sigma(\cdot)$ globally belongs to the sector $[0, 0.1]$. We choose $\eta_0 = 0$ and the set $\mathscr{D}$ is specified by defining $P_0 = 0.1I$. In addition, we assume the ETM parameters $\lambda = 0.1$ and $\theta = 2$.

Thus, solving the optimization problem (34)[1], we get $\varepsilon = 4.11$ and the matrices

$$Q_\delta = \begin{bmatrix} 2.070 & 2.974 \\ 2.974 & 4.275 \end{bmatrix}, Q_x = \begin{bmatrix} 0.014 & -0.010 \\ -0.010 & 0.036 \end{bmatrix}, P = \begin{bmatrix} 0.082 & 0.018 \\ 0.018 & 0.083 \end{bmatrix}.$$

Using these parameters, we simulated the closed-loop system under the dynamic PETC with initial condition $x_0 = \begin{bmatrix} 1.76 & 2.65 \end{bmatrix}^\top$ and plotted the state norm, the control input and event instants for $t \in [0, 25]$ in Fig. 1. The dashed-line represents the saturation level in the middle plot, and minimum inter-event time in the bottom plot. Observe that $\|x(t)\|$ goes to zero as time increases. Note that a saturation occurs in the beginning of the simulation. Finally, the dynamic ETM has triggered 12 events, as illustrated in the last plot, versus the 32 control updates obtained with a time-triggered control scheme (i.e. the controller is updated at each sampling-time).

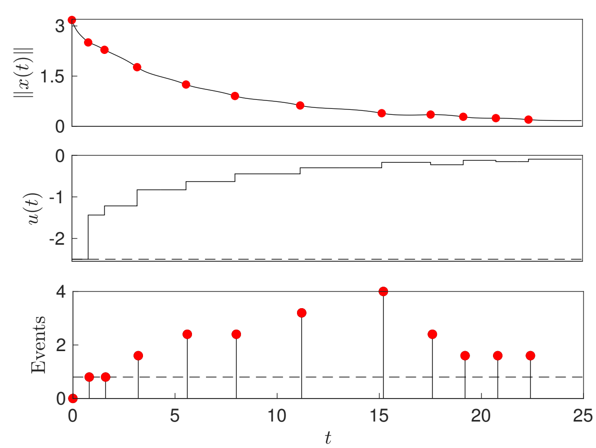

Fig. 1. State norm, control input and event instants.

To show the effect of $\mathscr{D}$ on the number of events, we define $P_0 = \mu I$, solve optimization problem (34) for the values of μ shown in the first column of Table I, and simulate the closed-loop system considering static and dynamic ETMs. Then, we compute the average number of events, rounded up to the nearest integer, for 100 initial conditions uniformly distributed on the boundary of $\mathscr{D}$ and $t \in [0, 200]$. Note that lower values of μ, which correspond to larger $\mathscr{D}$ sets, result in higher number of events. Moreover, the dynamic ETM has efficiently reduced the triggering activity with respect to the time-based implementation that would have 251 events in this interval. We can also observe that the number of events with the dynamic ETM is smaller than what would be with a static ETM (see Remark 1).

[1]We also consider additional constraints $\lambda_{max}(Q_\delta) < 10^4 \lambda_{min}(Q_\delta)$ and $\lambda_{max}(\bar{Q}_x) < 10^4 \lambda_{min}(\bar{Q}_x)$ in (34) to avoid ill-conditioned matrices.

TABLE I

Average number of events with $P_0 = \mu I$.

μ	**Static ETM**	**Dynamic ETM**
0.60	126	93
0.30	127	96
0.09	242	106
0.08	Infeasible	Infeasible

VI. Conclusion

In this paper, we addressed the stability analysis of continuous-time Lur'e systems with dynamic PETC and saturating control. Following an emulation-based design, along with properties of nonlinear functions, Lyapunov Theory and a looped-functional approach, conditions in the form of LMIs were derived to ensure regional asymptotic stability of the origin of the closed-loop system. These conditions were then cast into a convex optimization problem to synthesize the triggering function parameters aiming to reduce the number of control updates when compared to the time-based updating policy, while guaranteeing regional asymptotic stability with respect to a given set of initial conditions. Finally, an numerical example was presented to show the proposed method.

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