2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Nonlinear loss-minimizing control of induction motors for EV applications

Apostolos Manasis and George C. Konstantopoulos

Abstract— **In this paper, a novel nonlinear controller for induction motors used in EV applications is proposed to achieve field-oriented operation with minimum losses at the steady state, while inherently guaranteeing the required operating constraints. By taking the expression of the total (iron and copper) losses of the motor, an optimization problem is formulated to minimize the motor losses under the required equality and inequality constraints (torque, voltage and current constraints). Opposed to conventional loss-minimizing control that requires the solution of the entire optimization problem and continuous update of the reference motor currents in order to guarantee all necessary constraints, in this paper, a novel bounded controller is introduced with a specific nonlinear dynamic structure that inherently accomplishes all voltage and current constraints, while achieving field-oriented operation. Hence, a simplified optimization problem is required to be solved once offline to obtain the desired property that the motor currents should satisfy in order to minimize the entire motor losses, thus simplifying the controller implementation. In order to verify the theoretical contribution, the proposed controller is directly compared to the well-known field-oriented control and the conventional loss-minimizing control for the same induction motor control scenario.**

I. Introduction

Squirrel cage induction motors (IM) are widely used in electric vehicle (EV) applications, particularly in dual-motor configurations, due to their increased robustness. Since an IM is driven by a voltage source inverter (VSI), this offers the opportunity for precise torque or speed control. Hence, the VSI-driven induction motor is capable of quickly achieving the required amount of torque, while reaching a high speed operation, as required in EV applications.

In order for the IM to achieve the required torque or speed, an indirect rotor field-oriented control approach (IR-FOC) is often utilized [1]. The aim of the IR-FOC operation is to simplify the electromagnetic torque expression as in a separately excited dc motor [2], by separating the current into the $d-$ and $q-$axes of the synchronously rotating reference frame, using the Park transformation. Thus, the motor variables are decoupled, in order for the motor flux and torque to be controlled from the $d-$ and $q-$axis currents, respectively.

Taking into consideration the limited range of an EV, an optimization process to minimize the motor losses is essential. For this purpose, an optimization problem is formulated to minimize the machine's copper and iron loss under the appropriate operating constraints, which are comprised of the voltage, current and torque constraint expressions. There are numerous articles in the scientific community which are describing the loss-minimizing control (LMC) for induction motors. The main categories of loss-minimizing control are: 1) the search controller (SC) [3], [4] and 2) loss-model-based controller (LMBC) [5]. The first type of LMC utilizes the measurement of the motor input power and adjusts the flux level in consecutive steps [6], with the main goal being the minimization of the input power for given values of torque and speed. The LMBC relies on the machine model and its parameters, in order to calculate the optimal flux that minimizes the total losses. The strong advantage of the first method is the independence of the motor parameters, while the main drawback is the slow convergence and higher torque ripples [7]. The main advantages of the LMBC are the limited torque ripple and the speed of the control operation, but the design directly depends on the motor parameters.

In this paper, the loss-model-based approach is selected. Considering the induction motor dynamic model, the analytical expressions for the iron and copper losses are obtained in order to formulate the optimization problem that aims to minimize the total losses. The LMC approach aims to find the optimum flux level that minimizes the losses while producing a desired torque and simultaneously satisfying several operating constraints (current, voltage constraints). Hence, the analytic solution of this optimization problem can be found using the Kuhn–Tucker theorem [8]. This results in four different solutions for the $d-$ and $q-$axis stator currents, each depending on the operating region of the motor.

The novelty of this paper is the fact that only one of the four solutions is required for the control operation, i.e. leading to a simplified optimization problem that includes only the equality constraint of the motor torque, while the inequality constraints are embedded into the controller structure. In particular, the proposed LMC takes a particular dynamic structure that maintains the controller states within a desired set that represents the inequality constraints. Furthermore, using non-linear ultimate boundedness theory, the proposed non-linear loss-minimizing controller (NL-LMC) is proven to limit the instantaneous values of the $d-$ and $q-$axis stator currents inside a desired set at all times. Hence, compared to the FOC, the NL-LMC achieves the same speed regulation scenario with minimum losses and field-oriented operation at the steady state, while compared to the conventional LMC, the proposed approach can solve a simplified optimization problem once offline, obtain the relationship of the $d-$ and $q-$axis stator currents in order to achieve minimum total

This work was supported under Grant 81359 from the Research Committee of the University of Patras via "C.CARATHEODORY" program. The authors are with the Department of Electrical and Computer Engineering, University of Patras, Rion 26504, Greece. Emails: a.manasis@upatras.gr, g.konstantopoulos@ece.upatras.gr

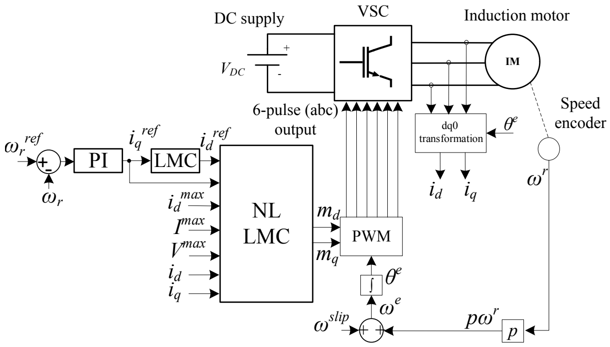

Fig. 1. Proposed non-linear loss-minimizing controller (NL-LMC) structure.

losses, while satisfying the required inequality constraints due to its special dynamic structure.

II. INDUCTION MOTOR MODELING AND FIELD ORIENTED CONTROL

A. Dynamic model of a VSI-fed induction motor

The system under consideration is depicted in Fig. 1, and is consisted of a DC source feeding the three-phase induction motor via a three-phase voltage source inverter (VSI). Using the Park transformation [1], also known as synchronously rotating reference system ($d-q$ frame) and assuming as state variables of the induction motor the following: i_{ds},i_{qs},λ_{dr},λ_{qr}, ω_r, which are the stator currents, the rotor fluxes and the motor speed, the dynamic model of the motor is derived as [2]:

$$\sigma\dot{i}_{ds} = -\left(\frac{R_r L_m^2}{L_r^2} + R_s\right) i_{ds} + \sigma\omega_s i_{qs} + \frac{R_r L_m}{L_r^2}\lambda_{dr} + \frac{L_m}{L_r} p\lambda_{qr}\omega_r + V_{ds} \quad (1)$$

$$\sigma\dot{i}_{qs} = -\left(\frac{R_r L_m^2}{L_r^2} + R_s\right) i_{qs} - \sigma\omega_s i_{ds} + \frac{R_r L_m}{L_r^2}\lambda_{qr} - \frac{L_m}{L_r} p\lambda_{dr}\omega_r + V_{qs} \quad (2)$$

$$\dot{\lambda}_{dr} = \frac{R_r L_m}{L_r} i_{ds} - \frac{R_r}{L_r}\lambda_{dr} + (\omega_s - p\omega_r)\lambda_{qr} \quad (3)$$

$$\dot{\lambda}_{qr} = \frac{R_r L_m}{L_r} i_{qs} - \frac{R_r}{L_r}\lambda_{qr} - (\omega_s - p\omega_r)\lambda_{dr} \quad (4)$$

$$J_m\dot{\omega}_r = -\frac{3L_m}{2L_r} p\lambda_{qr} i_{ds} + \frac{3L_m}{2L_r} p\lambda_{dr} i_{qs} - b\omega_r - T_l \quad (5)$$

where ω_s is the motor synchronous speed, R_s and R_r are the resistances of the stator and rotor, respectively, L_s and L_r are the inductances of the stator and rotor, respectively, L_m is the mutual inductance, p is the number of pole pairs, J_m and b are the total rotor inertia and friction coefficient, respectively, T_l is the load torque and $\sigma = L_s - \frac{L_m^2}{L_r}$ is the leakage coefficient, while the $d-$ and $q-$axis VSI voltage components, i.e. V_{ds} and V_{qs}, represent the control inputs of (1)-(5). The electromagnetic torque is given as:

$$T_e = \frac{3}{2} p \frac{L_m}{L_r}(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}). \quad (6)$$

Using the Park transformation in order to implement the IR-FOC, it gives the opportunity to align the $d-$axis of the synchronous rotating reference frame to the total rotor flux. This alignment results in the following expressions:

$$\lambda_{dr} = \lambda_r, \lambda_{qr} = 0. \quad (7)$$

Thus, the torque equation (6) on the IR-FOC becomes:

$$T_e = \frac{3}{2} p \frac{L_m}{L_r}\lambda_{dr} i_{qs}. \quad (8)$$

Note that the VSI is capable of controlling its output voltage and frequency. Thus, the synchronous frequency ω_s, is produced by the VSI, and it is calculated from the motor speed and the desired slip as $\omega_s = p\omega_r + \omega_{slip}$.

B. Traditional Field Oriented Control

Field Oriented Control (FOC) is a widely used control design for an induction motor [1]. The most common form of FOC is the indirect rotor FOC (IR-FOC), where the rotor flux is aligned to the $d-$axis of the synchronous reference frame. Hence, at steady-state operation, according to (4) the preferred slip is obtained:

$$\omega_{slip} = \omega_s - p\omega_r = \frac{R_r L_m}{L_r \lambda_r} i_{qs}. \quad (9)$$

Therefore, it is essential to know the rotor flux λ_r, which cannot be measured directly. Hence, the amplitude of the rotor flux can be estimated from (3) under FOC operation, i.e. $\lambda_{qr} = 0$, while assuming steady-state operation:

$$\hat{\lambda}_r = \lambda_r^e = L_m i_{ds}^e, \quad (10)$$

where $\hat{\lambda}_r$, λ_r^e are the estimated and steady-state values of the flux, respectively. The above assumption is valid, because in order to achieve a decent performance of the machine in the low/rated speed area, there is no need to chance the flux reference value. Thus, the flux level is maintained constant, and usually the reference values of the controller are the desired speed and the rated flux values. It is noted, that the superscript e in (10) means the steady state condition.

The traditional IR-FOC is implemented using cascaded PI controllers with the appropriate canceling terms [1], [9]. From equation (7) the appropriated $d-$axis current is derived, while the $q-$axis current is obtained through an outer-loop PI controller, which acts as the speed regulator.

C. Minimum losses operation

The loss-minimizing control (LMC) of induction motors has been extensively studied in the literature [5], [7], [10], [11]. The main aim is to ensure that the induction motor achieves the desired operation at the steady-state, e.g. speed regulation, with minimum iron and copper losses. Hence, consider first the existence of an equilibrium point $\left(i_{ds}^e, i_{qs}^e, \lambda_{dr}^e, \lambda_{qr}^e, \omega_r^e\right)$ for system (1)-(5) at the steady state. Consider the total induction motor losses (iron and copper) as $P_{loss} =$

$$R_s\left[(i_{ds}^e)^2 + (i_{qs}^e)^2\right] + R_r\left[(i_{dr}^e)^2 + (i_{qr}^e)^2\right] + \frac{(V_{dm}^e)^2 + (V_{qm}^e)^2}{r_m} \quad (11)$$

where, i_{dr}, i_{qr} are the rotor $d-$ and $q-$axis currents, respectively. r_m is the resistance parallel to the magnetizing inductance, which is used to model the iron loss, and V_{dm}, V_{qm} are the $d-$ and $q-$axis components of the air-gap voltage, at the steady-state; as described in [8]. Note that the total losses in (11) correspond to the model (1)-(5) according to the assumptions presented in [8]. Equation, (11) can be rewritten as:

$$P_{loss} = R_d(\omega_s)\left(i^e_{ds}\right)^2 + R_q(\omega_s)\left(i^e_{qs}\right)^2 \tag{12}$$

with $R_d(\omega_s) = R_s + \frac{\omega_s^2 L_m^2}{r_m}$, $R_q(\omega_s) = R_s + \frac{R_r L_m^2}{L_r^2} + \frac{\omega_s^2 L_m^2 L_{lr}^2}{r_m L_r^2}$ being the equivalent resistors for the $d-$ and $q-$axis.

In order to minimize the losses at the steady-state operation, it is necessary to formulate an optimization problem with the necessary operating constraints. The aim is to minimize the cost function given by (11) while producing a desired torque T_e under current and voltage constraints of the stator. In particular, considering steady-state operation and field-orientation, as explained in the previous subsection, the desired torque T_e can be given from (6) as

$$T_e = \frac{3}{2}p\frac{L_m^2}{L_r}i^e_{ds}i^e_{qs}. \tag{13}$$

. Furthermore, considering the maximum stator current as I_{max}, and the maximum $d-$axis current as $i_{d,max}$, then the following two inequality constraints are introduced

$$\left(i^e_{ds}\right)^2 + \left(i^e_{qs}\right)^2 \leq I^2_{max} \tag{14}$$

$$i^e_{ds} \leq i_{d,max}. \tag{15}$$

Considering the maximum stator voltage as V_{max} [8], i.e. $\left(V^e_{ds}\right)^2 + \left(V^e_{qs}\right)^2 \leq V^2_{max}$, and taking into account that the stator voltage reaches its maximum limit at the start of the field-weakening region [12], where the voltage drop over the stator resistance is negligible, then the voltage constraint can be rewritten as

$$\frac{\left(i^e_{ds}\right)^2}{\left(\frac{V_{max}}{\omega_s L_s}\right)^2} + \frac{\left(i^e_{qs}\right)^2}{\left(\frac{V_{max}}{\omega_s \sigma}\right)^2} \leq 1. \tag{16}$$

Hence, the optimization problems is formulated as:

Minimize $\quad P_{loss}$ from (12) subject to

$$\frac{3}{2}p\frac{L_m^2}{L_r}i^e_{ds}i^e_{qs} = T_e \tag{17}$$

$$\left(i^e_{ds}\right)^2 + \left(i^e_{qs}\right)^2 \leq I^2_{max} \tag{18}$$

$$\frac{\left(i^e_{ds}\right)^2}{\left(\frac{V_{max}}{\omega_s L_s}\right)^2} + \frac{\left(i^e_{qs}\right)^2}{\left(\frac{V_{max}}{\omega_s \sigma}\right)^2} \leq 1 \tag{19}$$

$$i^e_{ds} \leq i_{d,max} \tag{20}$$

Note that (18) represents the area within a circle centered at the origin and (19) represents the area within an ellipse centered at the origin, as well, but with varying radius based on the motor speed. Finally, (20) represents the area on the left-hand side of a line, parallel to i_{qs}-axis line on the $i_{ds}-i_{qs}$ plane. The operating point of the induction motor should lie inside the area formed by the intersection of the above sets.

The optimization problem in order to achieve operation under minimum losses can be solved using Lagrange multipliers and the Kuhn-Tucker theorem. Hence, the desired steady-state values of the currents i^e_{ds} and i^e_{qs} can be calculated under 4 different scenarios, i.e. when the point $\left(i^e_{ds}, i^e_{qs}\right)$ lies at the interior of the intersection of (18), (19) and (20), as well as the cases where it lies on the boundary of each one of (18), (19) or (20). For further details, the reader is referred to [8]. Hence, the design of the conventional loss-minimizing control (LMC) requires a continuous update of the current reference values, which are fed to the field-oriented control structure (i.e IR-FOC), in order to achieve minimum losses with a desired torque level. This complicates the implementation and the desired operation of the conventional LMC, since it is possible to cause chattering effects between two reference values which result from the solution of the optimization problem, when the system operates close to the constraints.

III. Non-Linear Loss-Minimizing Controller (NL-LMC)

A. Reference current values

This paper aims to design a LMC structure that inherently accomplishes the desired inequality constraints, i.e. limits the steady-state currents i^e_{ds} and i^e_{qs} within the desired sets; thus resulting in a simplified optimization problem, consisting of the minimization of the cost function (12) under only the equality constraint (17). This problem can be solved offline once and then fed to the proposed controller.

In particular, using Lagrange multipliers, the optimization problem can be solved and result in the solutions [8]:

$$\left(i^e_{ds}, i^e_{qs}\right) = \left(\left(\frac{R_q(\omega_s)T_e^2}{R_d(\omega_s)(\frac{3}{2}p\frac{L_m^2}{L_r})^2}\right)^{\frac{1}{4}}, \left(\frac{R_d(\omega_s)T_e^2}{R_q(\omega_s)(\frac{3}{2}p\frac{L_m^2}{L_r})^2}\right)^{\frac{1}{4}}\right). \tag{21}$$

One can obtain the stator currents relationship as:

$$i^e_{ds} = i^e_{qs}\sqrt{\frac{R_q(\omega_s)}{R_d(\omega_s)}} \tag{22}$$

where the value of i^e_{qs} can be obtained using a outer-loop PI controller, which acts as a speed regulator (instead from calculating directly from (21) based on the desired torque). The above analysis describes the process for calculating the reference values for the $d-$ and $q-$axis currents in order to accomplish minimum losses operation, which are then fed to the proposed non-linear current controller.

B. Proposed non-linear controller

After the calculation of the desired reference currents that satisfy (22), the aim is to design an inner-loop current controller that regulates the stator currents at the desired reference values while guaranteeing the inequality constraints (18)-(20) at all times. To this end, the proposed non-linear loss-minimizing controller (NL-LMC) is given below:

$$V_{ds} = \bar{v}_d - (\sigma\omega_s i_{qs} + \frac{R_r L_m}{L_r^2}\lambda_{dr} + \frac{L_m}{L_r}p\lambda_{qr}\omega_r) \tag{23}$$

$$V_{qs} = \bar{v}_q - (-\sigma\omega_s i_{ds} + \frac{R_r L_m}{L_r^2}\lambda_{qr} - \frac{L_m}{L_r}p\lambda_{dr}\omega_r), \tag{24}$$

where $\bar{v}_d$ and $\bar{v}_q$ are the new control inputs of the system:

$$\bar{v}_d = -K_p i_{ds} + K_p I_{max} w_d \tag{25}$$

$$\bar{v}_q = -K_p i_{qs} + K_p I_{max} w_q, \tag{26}$$

with w_d and w_q representing the controller states with:

$$\dot{w}_d = K_I(i_{ds}^{ref} - i_{ds})(1 - w_d^2 - w_q^2)(1 - \frac{I_{max}w_d}{i_{d,max}})$$
$$(1 - \frac{(\omega_s L_s I_{max} w_d)^2}{V_{max}^2} - \frac{(\omega_s \sigma I_{max} w_q)^2}{V_{max}^2}) - kw_d \quad (27)$$
$$\dot{w}_q = K_I(i_{qs}^{ref} - i_{qs})(1 - w_d^2 - w_q^2)$$
$$(1 - \frac{(\omega_s L_s I_{max} w_d)^2}{V_{max}^2} - \frac{(\omega_s \sigma I_{max} w_q)^2}{V_{max}^2}) - kw_q. \quad (28)$$

Note that i_{ds}^{ref} and i_{qs}^{ref} represent the reference values of the stator currents that are obtained from the outer-loop controller and satisfy (22), $K_p > 0$ is the controller proportional gain, $K_I > 0$ is the controller integral gain and k is an arbitrarily small positive constant. The entire structure of the proposed NL-LMC is shown in Fig. 1.

By replacing the proposed NL-LMC (23)-(26) into the induction motor dynamics (1)-(2), the closed-loop stator current dynamics become:

$$\sigma \dot{i}_{ds} = -\left(\frac{R_r L_m^2}{L_r^2} + R_s + K_p\right) i_{ds} + K_p I_{max} w_d \quad (29)$$
$$\sigma \dot{i}_{qs} = -\left(\frac{R_r L_m^2}{L_r^2} + R_s + K_p\right) i_{qs} + K_p I_{max} w_q. \quad (30)$$

Now, by selecting the proportional gain K_p such that $K_p \gg \frac{R_r L_m^2}{L_r^2} + R_s$ (as in practice R_s and R_r take relatively small values), and assuming steady-state operation, it yields

$$i_{ds}^e \approx I_{max} w_d^e \quad (31)$$
$$i_{qs}^e \approx I_{max} w_q^e. \quad (32)$$

This means, that the desired inequality constraints (18)-(20) can be rewritten as

$$1 - (w_d^e)^2 - (w_q^e)^2 \geq 0 \quad (33)$$
$$1 - \frac{(\omega_s L_s I_{max} w_d^e)^2}{V_{max}^2} - \frac{(\omega_s \sigma I_{max} w_q^e)^2}{V_{max}^2} \geq 0 \quad (34)$$
$$1 - \frac{I_{max} w_d^e}{I_{d,max}} \geq 0, \quad (35)$$

respectively. This is depicted in Fig. 2. Note that the functions on the left-hand side of inequalities (33)-(35) are utilized in the proposed NL-LMC dynamics (27)-(28). This is particularly important in order to ensure that the controller states do not violate the desired constraints, i.e. for zero initial conditions $w_d(0)$ and $w_q(0)$, the trajectory $(w_d(t), w_q(t))$, $\forall t \geq 0$ remains within the compact and convex set $\mathcal{W} = \mathcal{W}_1 \cap \mathcal{W}_2 \cap \mathcal{W}_3$ with $\mathcal{W}_1 = \left\{w_d, w_q \in \mathbb{R}^2 : 1 - w_d^2 - w_q^2 \geq 0\right\}$, $\mathcal{W}_2 = \left\{w_d, w_q \in \mathbb{R}^2 : 1 - \frac{(\omega_s L_s I_{max} w_d)^2}{V_{max}^2} - \frac{(\omega_s \sigma I_{max} w_q)^2}{V_{max}^2} \geq 0\right\}$ and $\mathcal{W}_3 = \left\{w_d, w_q \in \mathbb{R}^2 : 1 - \frac{I_{max} w_d}{I_{d,max}} \geq 0\right\}$. This is proven using vector field analysis as shown below.

Consider any point (w_d^*, w_q^*) on the boundary of $\mathcal{W}_1$ or $\mathcal{W}_2$. Then, the right-hand sides of (27)-(28) become $f_d(w_d^*, w_q^*) = -kw_d^*$ and $f_q(w_d^*, w_q^*) = -kw_q^*$, respectively, resulting in the vector field

$$f(w_d^*, w_q^*) = -k \begin{bmatrix} w_d^* & w_q^* \end{bmatrix}^T$$

which represents a vector pointing from the point (w_d^*, w_q^*) towards the origin. Since the origin lies at the interior of $\mathcal{W}$, then the vector field points inwards to $\mathcal{W}$. Now consider any point (w_d^*, w_q^*) on the boundary of $\mathcal{W}_3$ which belongs also on the boundary of $\mathcal{W}$. From (27), the $d-$axis vector field becomes $f_d(w_d^*, w_q^*) = -kw_d^*$ while the $q-$ axis vector field could point anywhere on the $d-$axis current limit based on (28). It is easy to see that the total vector field $f(w_d^*, w_q^*)$ will definitely point to the left or on the boundary of $\mathcal{W}_3$ and therefore inwards to $\mathcal{W}$. Hence, according to Nagumo's theorem on invariant sets [13], the set $\mathcal{W}$ is robustly positively invariant with respect to (27)-(28); thus if initially $(w_d(0), w_q(0)) \in \mathcal{W}$, then $(w_d(t), w_q(t)) \in \mathcal{W}$, $\forall t \geq 0$. This property is crucial not only for satisfying the desired inequality constraints, but also for guaranteeing boundedness of the instantaneous values of the stator current.

C. Boundedness of stator currents

Consider the closed-loop current dynamics (29)-(30). By defining the continuously differentiable function:

$$W(t) = \frac{1}{2}\sigma(i_{ds}^2 + i_{qs}^2) \quad (36)$$

its time derivative yields:

$$\dot{W}(t) = \sigma i_{ds} \dot{i}_{ds} + \sigma i_{qs} \dot{i}_{qs}. \quad (37)$$

By substituting (29)-(30) into equation (37):

$$\dot{W}(t) = -\left(\frac{R_r L_m^2}{L_r^2} + R_s\right)(i_{ds}^2 + i_{qs}^2) - K_p(i_{ds}^2 + i_{qs}^2)$$
$$+ K_p I_{max} w_d i_{ds} + K_p I_{max} w_q i_{qs} \quad (38)$$

$$\dot{W}(t) = -\left(\frac{R_r L_m^2}{L_r^2} + R_s\right)\|\boldsymbol{i}\|_2^2 - K_p\|\boldsymbol{i}\|_2^2 + K_p I_{max} \mathbf{w}^T \boldsymbol{i}, \quad (39)$$

with $\boldsymbol{i} = [i_{ds}\, i_{qs}]^T$, $\boldsymbol{w} = [w_d\, w_q]^T$ and $\|\boldsymbol{i}\|_2$ denotes the Euclidean norm of the current vector. Since $\mathbf{w}^T\boldsymbol{i} \leq |\mathbf{w}^T\boldsymbol{i}| \leq \|\mathbf{w}\|_2 \|\boldsymbol{i}\|_2$, then (39) becomes

$$\dot{W}(t) \leq -\left(\frac{R_r L_m^2}{L_r^2} + R_s\right)\|\boldsymbol{i}\|_2^2, \ \forall \|\boldsymbol{i}\|_2 \geq \|\mathbf{w}\|_2 I_{max}. \quad (40)$$

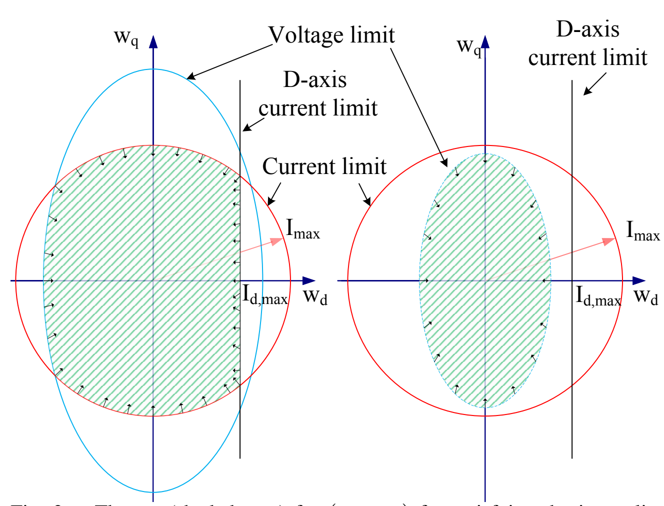

Fig. 2. The set (shaded area) for (w_d, w_q) for satisfying the inequality constraints (33)-(35) under different motor speeds that modify the voltage limit (34), and the vector field at the boundary of the set.

TABLE I
SYSTEM PARAMETERS

Parameters	Value	Parameter	Value
P_n	4 kW	R_r	1.395 Ω
I_s	9 A	r_m	500 Ω
rated speed	1430 rpm	p	2
L_s	178 mH	J_m	0.0131 $kg\cdot m^2$
L_r	178 mH	V_{DC}	1000 V
L_m	172 mH	V_{max}	500 V
R_s	1.405 Ω	$i_{d,max}$	4.68 A

It was shown in the previous subsection that for any $(w_d(0), w_q(0)) \in \mathcal{W}$, then there is $(w_d(t), w_q(t)) \in \mathcal{W}$, $\forall t \geq 0$, which means that $(w_d(t), w_q(t)) \in \mathcal{W}_1$, $\forall t \geq 0$ since $\mathcal{W} \subset \mathcal{W}_1$. Hence, from the structure of $\mathcal{W}_1$, there is $\|\mathbf{w}(t)\|_2 \leq 1$, $\forall t \geq 0$. Therefore, using the ultimate boundedness theory (Theorem 4.18, [14]), expression (40) yields that if initially $\|\boldsymbol{i}(0)\|_2 \leq I_{max}$, then $\|\boldsymbol{i}(t)\|_2 \leq I_{max}$, $\forall t \geq 0$, which proves that the instantaneous values of $i_{ds}(t)$ and $i_{qs}(t)$ are bounded below the maximum current for all $t \geq 0$. It is underlined that opposed to conventional control methods that add a saturation unit at the output of the outer-loop PI speed regulator to ensure that the reference value of i_{qs}^{ref} (and consequently of i_{ds}^{ref}) is bounded, the proposed NL-LMC ensures that the instantaneous values of i_{ds} and i_{qs} are bounded, i.e. even during transients. This is accomplished in addition to the desired inequality constraint satisfaction and is validated in the simulation results that follow.

IV. SIMULATION RESULTS

In order to verify the effective performance of the proposed controller, the VSI-fed IM system is simulated in the MATLAB/Simulink environment. The induction motor parameters are presented in Table I.

The proposed controller (NL-LMC) is compared to the traditional FOC, as described in [12], and the conventional FOC loss-minimizing control, using PI controllers (FOC-LMC) [5]; thus, the same speed regulation scenario is considered. Starting from a motor speed at $\omega_r^{ref} = 1200$ rpm and load torque at 22.1 $N\cdot m$, the speed reference drops to $\omega_r^{ref} = 1000$ rpm at the time instant $t = 1$ sec. At the time instant $t = 2$ sec, the speed reference is set to the rated speed, i.e. $\omega_r^{ref} = 1430$ rpm, and is maintained there. Firstly, the torque load drops to 19.5 $N\cdot m$ at $t = 3$ sec, while, at the time instant $t = 4$ sec it is increased to 26 $N\cdot m$. Then, at the time instant $t = 5$ sec it drops to 24 $N\cdot m$, before it returns to 26 $N\cdot m$ at $t = 6$ sec. It is noted, that in the below figures, the reference or maximum values are depicted with dashed lines.

Fig. 3(a) shows the speed of the IM under the scenario described above. The FOC and the FOC-LMC result in a similar response in terms of overshoot and settling time, while NL-LMC results in a small overshoot under speed reference changes and approximately the same settling time with the aforementioned strategies; however, it appears to have a slightly larger overshoot and settling time under extreme and sudden load torque increases. This is explained

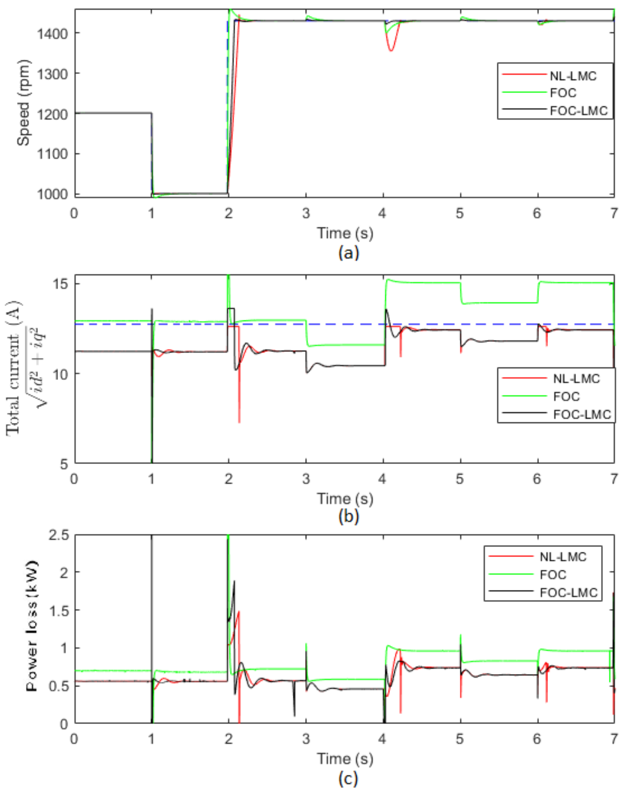

Fig. 3. IM operation under the proposed NL-LMC, traditional IR-FOC (FOC) and conventional FOC with loss-minimizing operation and PI controllers (FOC-LMC): (a) rotor speed, (b) maximum phase current (peak), (c) power losses.

in Fig. 3(b), where the total current limit is depicted, indicating that the NL-LMC has already limited the total current below its maximum opposed to the conventional methods that violate the limit. In Fig. 3(c), the total power losses of the IM are obtained. As it was expected, the IM results in more losses under the FOC control, while the total losses are much lower under the FOC-LMC and the NL-LMC. The response of the $d-$ and $q-$axis currents is illustrated in Fig. 4(a),(b), showing the boundedness of the instantaneous currents, as analytically proven in Section III-C.

Fig. 5 shows the performance of the NL-LMC controlled IM under the high speed operation, as required in EV applications; thus, showing the field-weakening ability of the system. It is underlined that the actual top speed depends on the IM structure and its mechanical resiliency. The torque load is chosen to be disproportional to the IM speed, utilizing the constant power region of the motor [12]. The IM flux depends on the $d-$axis current, thus a reduced $d-$axis current is essential for the field-weakening operation. As depicted in Fig. 5(c), the $d-$axis current is reduced as the IM speed increases, highlighting that the proposed NL-LMC is capable of implementing the flux weakening control. From Fig. 5(b), 5(c) and Fig. 6(a), it is shown that the total phase current is bounded, while the $d-$axis current is always limited below $I_{d,max}$. In Fig. 6(b), it is clear that the inequality constraint (19) holds true for the whole speed range, leading to a successful field-weakening operation.

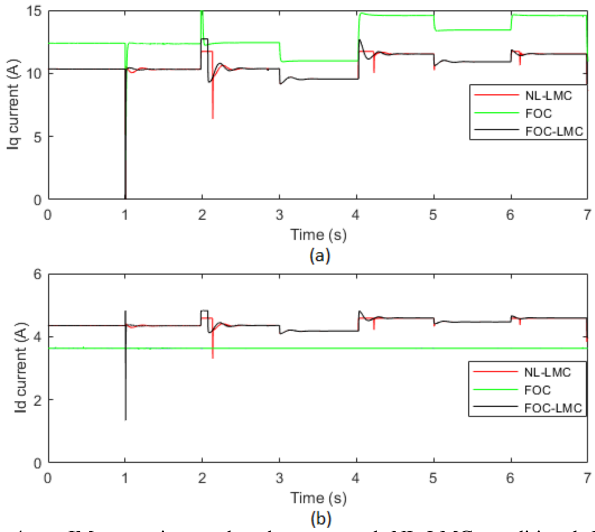

Fig. 4. IM operation under the proposed NL-LMC, traditional IR-FOC (FOC) and conventional FOC with loss-minimizing operation and PI controllers (FOC-LMC): (a) $q-$axis current, (b) $d-$axis current.

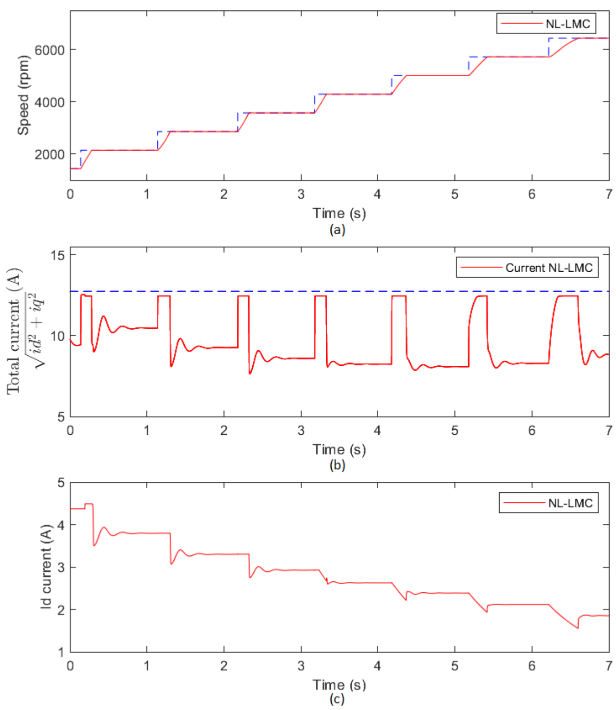

Fig. 5. Field-weakening operation of the IM under the proposed NL-LMC: (a) rotor speed, (b) maximum phase current (peak), (c) $d-$axis current.

V. Conclusion

In this paper a novel NL-LMC was proposed for minimizing the losses of the IM, while achieving accurate speed control with inherent constraint satisfaction. Opposed to conventional LMC strategies that require the solution of an optimization problem and continuous update of the stator currents depending on required constraints, the proposed approach requires the solution of a simplified optimization problem to accomplish minimum total losses, which can be solved once offline, while all inequality constraints are satisfied from the special dynamic structure of the controller. Furthermore, an analysis for the boundedness of the

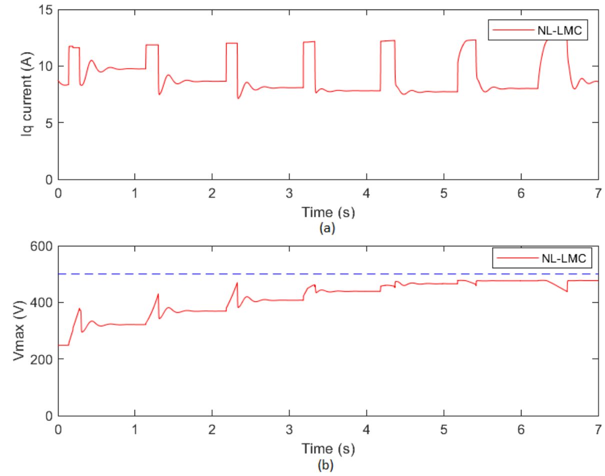

Fig. 6. Field-weakening operation of the IM under the proposed NL-LMC: (a) $q-$axis current, (b) maximum phase voltage (peak).

stator currents was also provided using nonlinear ultimate boundedness theory, proving that the instantaneous stator currents values are always limited. Extended simulation results demonstrate the effectiveness of the proposed NL-LMC compared to the conventional FOC and LMC.

References

[1] B. Bose, *Modern Power Electronics and AC Drives*, N. Prentice-Hall, Ed. Englewood Cliffs, 2001.

[2] A. T. Alexandridis, G. C. Konstantopoulos, and Q. C. Zhong, "Advanced integrated modeling and analysis for adjustable speed drives of induction motors operating with minimum losses," *IEEE Transactions on Energy Conversion*, vol. 30, no. 3, pp. 1237–1246, Sept 2015.

[3] C. Chakraborty, M. Ta, T. Uchida, and Y. Hori, "Fast search controllers for efficiency maximization of induction motor drives based on dc link power measurement," in *Proceedings of the Power Conversion Conference-Osaka 2002*, vol. 2, 2002, pp. 402–408 vol.2.

[4] L. Ramesh, S. P. Chowdhury, S. Chowdhury, A. K. Saha, and Y. H. Song, "Efficiency optimization of induction motor using a fuzzy logic based optimum flux search controller," in *2006 International Conference on Power Electronic, Drives and Energy Systems*, 2006.

[5] M. N. Uddin and S. W. Nam, "New online loss-minimization-based control of an induction motor drive," *IEEE Transactions on Power Electronics*, vol. 23, no. 2, pp. 926–933, 2008.

[6] S. Kaboli, M. R. Zolghadri, and E. Vahdati-Khajeh, "A fast flux search controller for dtc-based induction motor drives," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 5, pp. 2407–2416, 2007.

[7] S. W. Nam and M. N. Uddin, "Model-based loss minimization control of an induction motor drive," in *2006 IEEE International Symposium on Industrial Electronics*, vol. 3, 2006, pp. 2367–2372.

[8] K. Nam, *AC Motor Control and Electrical Vehicle Applications*. CRC Press, 2017.

[9] I. Ferdiansyah, M. R. Rusli, B. Praharsena, H. Toar, Ridwan, and E. Purwanto, "Speed control of three phase induction motor using indirect field oriented control based on real-time control system," in *2018 10th International Conference on Information Technology and Electrical Engineering (ICITEE)*, 2018, pp. 438–442.

[10] Z. Qu, M. Ranta, M. Hinkkanen, and J. Luomi, "Loss-minimizing flux level control of induction motor drives," *IEEE Transactions on Industry Applications*, vol. 48, no. 3, pp. 952–961, 2012.

[11] M. N. Uddin and S. W. Nam, "Development of a nonlinear and model-based online loss minimization control of an im drive," *IEEE Transactions on Energy Conversion*, 2008.

[12] S.-H. Kim, *Electric Motor Control*. Elsevier, 2017.

[13] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[14] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 2001.