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Monotonic tracking of step references for linear switched systems under arbitrary switching signals

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***Abstract*— We consider the analytical controller design for a class of switched linear systems. Under suitable system assumptions, we propose a method using eigenstructure assignment that guarantees the closed-loop switched system is globally asymptotically stable under arbitrary switching signals, and the outputs achieve monotonic step reference tracking from all initial conditions. Additionally, the output signals from both subsystems can be made identical, so the effects of the system switching are not noticeable from the output. A constructive algorithm is provided that yields suitable feedback matrices, and the method is illustrated with a numerical example.**

I. Introduction

The stability analysis of switched systems has received a great deal of attention for many years, [1], [2], [3], [4], and there has also been considerable recent literature dealing with the analytical design of stabilising controllers, see for example [5], [6], [7] for switched linear systems, and more general design methods for stabilising controllers have been synthesised for a large class of systems in [5], [8].

Recent papers on switched systems have investigated the design of feedback control methods that achieve stability under a wide class of switching signals, while also delivering other desirable control performance objectives. Some earlier works in this field are [9] for switched linear descriptor systems, and [10] for switched nonlinear systems, which sought to adapt the composite nonlinear feedback method of [11] to achieve rapid and smooth reference tracking for switched systems.

In [8] a stabilising design for switched linear systems for arbitrary switching is considered. Based on the results in [12] a switched feedback controller is proposed such that the subsystems share the same eigenvectors in closed-loop. This guarantees the existence of a common quadratic Lyapunov function and thus asymptotic stability for arbitrary switching.

The problem of designing state feedback controllers for linear time-invariant systems to achieve the globally monotonic tracking of step references is investigated in [13]. In this problem, state feedback is used to ensure the closed-loop is stabilised and the outputs converge monotonically to their desired reference values, from all initial states. [13] identifies the class of linear systems for which a globally monotonic tracking controller can be obtained, and gives a simple algorithm for computing the feedback law based on the state feedback eigenstructure assignment method of [12].

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In this paper we seek to combine the controller design methods of [8] with those of [13] to design control laws that will stabilise switched systems while also providing monotonic tracking of a step reference. Specifically, we consider switching between two subsystems that share the same output. Thus the two systems represent a plant subject to two differing system dynamics arising from an arbitrary switching signal. We investigate the problem of designing a state feedback control law to ensure closed-loop stability under arbitrary switching, while also ensure the system outputs tracking a constant reference signal with a monotonic transient response. A key component of the design method will be ensuring that both systems share a rectified eigenstructure, with common eigenvectors (but not necessarily common eigenvalues) that have been chosen to yield a step response in which only a single closed-loop mode is visible in each output component, and hence the response converges monotonically to its reference value.

The paper is organised as follows. Section II describes the class of switched systems under consideration, and provides the system assumptions that are needed for the controller design. We state the problems of designing feedback matrices to deliver both global asymptotic stability under arbitrary switching signals, and also the problem of monotonic reference tracking. Section III revisits some earlier results on the monotonic tracking problem for LTI systems, and on the stability of switched systems. Section IV contains the main results of the paper, stating the system assumptions that are required for the solution of our eigenstructure assignment problem, and providing an algorithm for the construction of the feedback matrices that achieve a rectified eigenstructure while assuring monotonic tracking of a step reference for the given switched system. Section V provides a numerical example to demonstrate the implementation of the eigenstucture assignment algorithm on a suitable switched LTI system. Finally Section VI provides a summary of the paper's contribution.

II. Problem Definition

A. Switched linear systems

We consider the switched system composed of the constituent linear time invariant (LTI) systems

$$\Sigma_q : \begin{cases} \dot{x}(t) = A_q\, x(t) + B_q\, u(t), \quad x(0) = x_0 \in \mathbb{R}^n, \\ y(t) = C\, x(t) \end{cases} \tag{1}$$

with $A_q \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $q \in \mathcal{I} = \{1, 2\}$. B_q has full column rank and C has full row rank.

Notice that the matrix C is common to both subsystems. These subsystems constitute the open-loop switched system

$$\Sigma_{\mathrm{OL}}: \left\{ \begin{array}{l} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) \\ y(t) = Cx(t). \end{array} \right. \quad (2)$$

where the switching signal $\sigma : \mathbb{R}^{+} \to \mathcal{I}$ is a piecewise constant function, and $A_{\sigma(t)} = A_q,\ B_{\sigma(t)} = B_q \Leftrightarrow \sigma(t) = q$. The switching signal is assumed to be entirely arbitrary, apart from the mild assumption that it has at most finitely many switches in any finite time interval, to ensure uniqueness of the solutions. While the signal is not assumed to be known for the purpose of controller design, it is assumed that the control input u can adjust to switches instantaneously.

Σ_{OL} is said to be *globally asymptotically stable under arbitrary switching* (GASAS) if the origin of the homogeneous system (with $u \equiv 0$) is asymptotically stable under every admissible switching signal σ, from all initial states x_0. It is well known that Σ_{OL} may be unstable for some switching signals, even when both state matrices A_q are Hurwitz stable matrices [1]. GASAS implies BIBO-stability of Σ_{OL} for bounded inputs u.

The system state is continuous at the switching instances. If the system state is at an equilibrium state of the active system immediately prior to switching, the system output may exhibit transient dynamics due to the switched dynamics. Such transients can be avoided if both systems share the same steady-state point [14].

B. Reference tracking for switched linear systems

In this paper, we consider the combined problems of ensuring GASAS while also ensuring the output y asymptotically tracks a desired step reference $r \in \mathbb{R}^p$ monotonically. For this we require

Assumption 1. *The dimensions of the system state matrices satisfy $n + p < 2m$, and for each $q \in \{1, 2\}$, system Σ_q is right invertible and stabilisable.*

Under these assumptions, we obtain a controller to track an arbitrary step reference signal $r \in \mathbb{R}^p$ as follows. In the spirit of [14] we calculate the joint steady state $x_{\mathrm{ss}} \in \mathbb{R}^n$ and corresponding input $u_{q,\mathrm{ss}}$ satisfying for all $\in \mathbb{R}^m, q \in \mathcal{I}$

$$0 \;=\; A_q\, x_{\mathrm{ss}} + B_q\, u_{q,\mathrm{ss}} \quad (3)$$

$$r \;=\; C\, x_{\mathrm{ss}}\ . \quad (4)$$

Such vectors exist because (3)-(4) can be written as

$$\begin{bmatrix} A_1 & B_1 & 0 \\ A_2 & 0 & B_2 \\ C & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{\mathrm{ss}} \\ u_{1,\mathrm{ss}} \\ u_{2,\mathrm{ss}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} \quad (5)$$

where the zero matrices are of appropriate dimensions. With Assumption 1, the matrix in (5) is wide and hence solutions exist.

Let $F_q \in \mathbb{R}^{n\times m}$ be feedback matrices. For a given switching signal σ, we define $F_{\sigma(t)} = F_q,\ u_{\sigma(t),ss} = u_{q,ss} \Leftrightarrow \sigma(t) = q$. With $G_{\sigma(t)} = -F_{\sigma(t)}x_{ss} + u_{\sigma(t),ss}$, applying the switched control law

$$u(t) = F_{\sigma(t)}x(t) + G_{\sigma(t)} \quad (6)$$

to Σ_{OL} yields the closed-loop control system

$$\Sigma_{\mathrm{CL}}: \left\{ \begin{array}{l} \dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)}F_{\sigma(t)})x(t) + BG_{\sigma(t)} \\ y(t) = Cx(t). \end{array} \right. \quad (7)$$

and employing the change of variable $\xi := x - x_{\mathrm{ss}}$, we obtain the closed loop homogeneous system

$$\Sigma_{\xi}: \left\{ \begin{array}{l} \dot{\xi}(t) = (A_{\sigma}(t) + B_{\sigma}(t)\, F_{\sigma}(t))\, \xi(t), \\ y(t) = C\, \xi(t) + r. \end{array} \right. \quad (8)$$

If the feedback matrices F_q achieve GASAS for Σ_ξ, we conclude that ξ converges to the origin, the state vector x of Σ_{CL} converges to x_{ss}, and the output y converges to r as $t \to \infty$.

We say that the switched system Σ_{CL} achieves *globally monotonic step reference tracking* for arbitrary r under the control law (6), if the tracking error $e(t) = r - y(t) \to 0$ monotonically, in every output component, from any initial condition $x_0 \in \mathbb{R}^n$.

Our aim in this paper is to obtain matrices F_q to achieve GASAS for Σ_{CL}, and also ensure Σ_{CL} achieves globally monotonic step reference tracking for any given r, x_0 and arbitrary switching signals σ. We describe this problem of *globally monotonic step reference tracking under arbitrary switching*. Our approach is based on combining the monotonic controller design methods of [13] with the GASAS design methods of [5] and [8].

III. Preliminary Results

In this section we present some earlier results that will be needed for our controller design methods in Section IV.

A. Globally monotonic tracking controllers for LTI MIMO systems

In the following, we have $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ with

$$\begin{array}{l} \dot{x}(t) = A\, x(t) + B\, u(t), \quad x(0) = x_0 \in \mathbb{R}^n, \\ y(t) = C\, x(t) \end{array} . \quad (9)$$

For the system (9), introduce row matrices $C_{(k)} \in \mathbb{R}^{(p-1)\times n}$ such that, for each $k \in \{0, 1, \ldots, p\}$,

$$C_{(0)} = C, \quad (10)$$

$$C_{(k)} = C \text{ with the } k\text{-th row removed.} \quad (11)$$

We will use $\Sigma_{(k)}$ to denote the LTI system obtained from the triple $(A, B, C_{(k)})$.

Definition 2. *For each $\Sigma_{(k)}$, with $k \in \{0, 1, \ldots, p\}$, and for any $\lambda \in \mathbb{R}$, we define the Rosenbrock system matrices*

$$R_{(k)}(\lambda) := \begin{bmatrix} \lambda I - A & B \\ C_{(k)} & 0 \end{bmatrix} \quad (12)$$

of dimension $(n + p_{(k)}) \times (n + m)$, where $p_{(0)} = p$ and $p_{(k)} = p - 1$. Their kernels may be decomposed as

$$\mathrm{im} \begin{bmatrix} N_{(k)}(\lambda) \\ M_{(k)}(\lambda) \end{bmatrix} := \ker(R_{(k)}(\lambda)), \quad (13)$$

where $N_{(k)}$, $M_{(k)}$ have dimensions $n \times m_{(k)}$ and $m \times m_{(k)}$, respectively, where $m_{(k)} \geq m - p_{(k)}$.

We recall the following lemma from [13].

Lemma 3. *Let $\mathcal{L} = \{\lambda_1, \dots, \lambda_n\} \subset \mathbb{R}$ contain n distinct real numbers, and let the sets $\mathcal{V} = \{v_1, \dots, v_n\} \subset \mathbb{R}^n$ and $\mathcal{W} = \{w_1, \dots, w_n\} \subset \mathbb{R}^m$ be such that*

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} \subset \ker(R_{(0)}(\lambda_i)), \ \textit{for } i \in \{1, \dots, n-p\} \tag{14}$$

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} \subset \ker(R_{(k)}(\lambda_i)), \quad \textit{for } i = n-p+k \textit{ and } k \in \{1, \dots, p\}. \tag{15}$$

Assume that $\mathcal{V}$ is linearly independent, and let

$$F = WV^{-1}, \tag{16}$$

where W and V are matrices whose columns are given by $\mathcal{W}$ and $\mathcal{V}$. Then for all $i \in \{1, \dots, n\}$ and $k \in \{0, 1, \dots, p\}$

$$(A + B\,F)\,v_i \;=\; \lambda_i\,v_i, \tag{17}$$

$$C_{(k)}\,v_i \;=\; 0. \tag{18}$$

Defining $(\alpha_1,\ \alpha_2, \dots, \alpha_n)^\top = V^{-1}x_0$, the output y of (9) *from the input $u = Fx$ is given by*

$$\begin{aligned} y(t) &= Ce^{(A+BF)t}x_0 \\ &= \begin{bmatrix} \beta_1 e^{\mu_1 t} \ \beta_2 e^{\mu_2 t} \ \dots \ \beta_p e^{\mu_p t} \end{bmatrix}^\top \end{aligned} \tag{19}$$

where $\beta_k = \alpha_{n-p+k}$, $\mu_k = \lambda_{n-p+k}$ for $k \in \{1, \dots, p\}$.

Remark 4. *This is a restatement of Lemma 2 in [13]. The assigned closed-loop eigenstructure ensures that $n-p$ of the closed-loop modes are invisible at the outputs, and the remaining p modes are constrained so that they each contribute only to the k-th output component. This ensures that the output term in* (19) *converges to zero monotonically in every component, if the closed-loop eigenvalues in $\mathcal{L}$ have been chosen to be real and negative. Since x_0 is arbitrary, the control law $u = Fx$ ensures that the outputs of the closed-loop system are globally monotonically convergent to zero.*

Viewed from the perspective of linear systems geometry [15], the vectors v_i satisfying (14) *lie in $\mathcal{R}^\star$, the largest output-nulling reachable subspace of system* (9)*. The feedback F obtained in* (16) *is a friend of this subspace. Thus a necessary condition for the existence of a friend that provides globally monotonic convergence is that $dim(\mathcal{R}^\star) = n - p$.*

If we let $\mathcal{R}^\star_{(k)}$ denote the largest output-nulling reachable subspace of system $(A, B, C_{(k)})$, then (15) *requires each $v_i \in \mathcal{R}^\star_{(k)}$, with $k = i - (n-p)$. The v_i must all be chosen so that $\mathcal{V}$ is linearly independent. Theorem 5 of [13] gives necessary and sufficient conditions for such a selection to be possible; the conditions are independent of the choice of desired closed-loop eigenvectors in $\mathcal{L}$.*

B. Global asymptotic stability for arbitrary switching

To obtain GASAS for Σ_{CL}, a key tool will be the method of [8] for obtaining state feedback matrices that assign common eigenvectors for the closed-loop subsystems.

Definition 5. *If there exists a linearly independent set $\mathcal{V} = \{v_i : i \in \{1, \dots, n\}\} \subset \mathbb{R}^n$, and sets $\mathcal{L}_q = \{\lambda_{q,i} : i \in \{1, \dots, n\}\} \subset \mathbb{R}$, with $q \in \mathcal{I}$, and $F_q \in \mathbb{R}^{m \times n}$ such that*

$$(A_q + B_q F_q)v_i = \lambda_{q,i} v_i \tag{20}$$

for each $i \in \{1, \dots, n\}$, then we say that the systems Σ_q are simultaneously eigenvector rectifiable.

Note that for simultaneous eigenvector rectification, we require the feedback matrices F_q to assign identical eigenvectors in $\mathcal{V}$ for both closed-loop subsystems, however the corresponding eigenvalues in $\mathcal{L}_q$ may be different. The following theorem establishes that simultaneous eigenvector rectification with stable eigenvalues implies GASAS.

Theorem 6. *([8], Theorem 3.2) Assume that state feedback matrices $F_q \in \mathbb{R}^{m \times n}$ achieve simultaneous eigenvector rectification for Σ_q, for some $\mathcal{V} \subset \mathbb{R}^n$ and some stable sets of eigenvalues $\mathcal{L}_q \subset \mathbb{R}^-$, $q \in \mathcal{I}$. Then Σ_{CL} is stable under arbitrary switching sequences.*

Remark 7. *An important contribution of [8] is to obtain conditions on Σ_{OL} that would ensure simultaneous eigenvector rectification can be achieved. The system dimension requirement $n < 2m$ is shown to be necessary. For some pairs of subsystems Σ_q the sets of possible closed-loop eigenvalues $\mathcal{L}_q$ may be restricted under feedback rectification, [8].*

IV. Main Results

In this section we present our main results that combine the monotonic tracking results of [13] with the stability results of [8] for switched systems. Our task is to generalise the monotonic tracking results achieved in [13] for a single LTI system to switched systems by simultaneous eigenvector rectification.

A. Eigenvector rectification on output-nulling subspaces

To extend the LTI system results of Section III to the switched system Σ_{OL}, we require some further notation. For each $q \in \mathcal{I}$ and $k \in \{0, 1, \dots, p\}$, we define $\Sigma_{q,(k)} = (A_q, B_q, C_{(k)})$ with $C_{(k)}$ defined as in (11). For convenience we identify $\Sigma_{q,(0)} = \Sigma_q$ in this notation. We let $\mathcal{R}^\star_{q,(k)}$ denote the largest output-nulling reachable subspace of $\Sigma_{q,(k)}$. For any $\lambda \in \mathbb{R}$ we define the Rosenbrock system matrices $R_{q,(k)}(\lambda)$ according to (12) of Definition 2. Their kernels may be decomposed as $N_{q,(k)}(\lambda)$ and $M_{q,(k)}(\lambda)$ in (13).

In terms of this notation we can now formalise the eigenstructure assignment problem to be solved in order to achieve globally monotonic step reference tracking under arbitrary switching for Σ_{OL} as follows:

Problem 8. Eigenstructure assignment. *For the systems Σ_q, obtain feedback matrices F_q that assign closed-loop eigenstructures $\mathcal{V}$, $\mathcal{L}_q \subset \mathbb{R}^-$ such that the outputs arising from each Σ_q take the form* (19)*, and also achieve simultaneous eigenvector rectification* (20).

Requiring that the outputs take the form (19) means that $n-p$ modes in $\mathcal{L}_q$ are each invisible at the outputs, while the remaining p modes are visible at only one of the outputs. Additionlly, we require $\mathcal{L}_q \subset \mathbb{R}^-$ to ensure stability.

To seek conditions on the existence of suitable sets $\mathcal{V}$ and $\mathcal{L}_q$ solving Problem 8, we treat the eigenvalues in $\mathcal{L}_q$ as free parameters and seek n linearly independent closed-loop eigenvectors satisfying

$$v_i \in \operatorname{im} N_{1,(k)}(\lambda_{1,i}) \cap \operatorname{im} N_{2,(k)}(\lambda_{2,i}) \tag{21}$$

for $i = 1, \ldots, n$ and $k = 0, \ldots, p$. The first $n-p$ of these eigenvectors must belong to both $\mathcal{R}^\star_{q,(0)}$, and the remaining p vectors must belong to both $\mathcal{R}^\star_{q,(k)}$, for some k. To ensure the existence of suitable vectors for such a closed-loop eigenstructure, we make the further assumption:

Assumption 9.

$$\begin{aligned}(a)\ \dim\left(\mathcal{R}^\star_{1,(0)} \cap \mathcal{R}^\star_{2,(0)}\right) &= n-p \qquad (22)\\ (b)\ \dim\left(\mathcal{R}^\star_{1,(k)} \cap \mathcal{R}^\star_{2,(k)}\right) &= n-p+1, \qquad (23)\\ &\quad \textit{for } k = \{1, \ldots, p\}\end{aligned}$$

These conditions are necessary for the existence of $n-p$ suitable linearly independent vectors in the intersections of the $\operatorname{im} N_{q,(0)}(\lambda_{q,i})$, and one each from the intersections of the $\operatorname{im} N_{q,(k)}(\lambda_{q,i})$, for some suitable $\lambda_{q,i}$. These vectors construct $\mathcal{V}$, and if it is linearly independent, then the feedback matrices required for the solution of Problem 8 can be obtained using Lemma 3. We solve (14)-(15) for the eigendirections $\mathcal{W}_q = \{w_{q,i} : i \in \{1, \ldots, n\}\}$, and then obtaining F_q from (16).

The subspace intersections in (21) are considered in [8]; it is shown that they can be described as a span of a rational matrix-valued function. For each $k = 0, \ldots, p$, let $N_{q,(k)}(\lambda_q)$ be defined in (13), and let $\lambda_1, \lambda_2 \in \mathbb{R}$. Let $T_{(k)}(\lambda_1, \lambda_2)$ be a transformation matrix that converts $[N_{1,(k)}(\lambda_1)\ \ N_{2,(k)}(\lambda_2)]$ into reduced row-echelon form:

$$T_{(k)}(\lambda_1,\lambda_2)\left[N_{1,(k)}(\lambda_1) \quad N_{2,(k)}(\lambda_2)\right] = \begin{bmatrix} E_{(k),11}(\lambda_1,\lambda_2) & E_{(k),12}(\lambda_1,\lambda_2) \\ 0 & E_{(k),22}(\lambda_1,\lambda_2) \\ 0 & 0 \end{bmatrix} \tag{24}$$

and define the set

$$G_{(k)} := \{(\lambda_1,\lambda_2) : T_{(k)}(\lambda_1,\lambda_2) \text{ is invertible}\} \subset \mathbb{R}^2. \tag{25}$$

Adapting the result in [16] we obtain the following.

Proposition 10. *For any $(\lambda_1,\lambda_2) \in G_{(k)}$, let $Q_{(k)}(\lambda_1,\lambda_2)$ be the rational matrix-valued function given by*

$$Q_{(k)}(\lambda_1,\lambda_2) = T^{-1}_{(k)}(\lambda_1,\lambda_2)\begin{bmatrix} E_{(k),12}(\lambda_1,\lambda_2) \\ 0 \\ 0 \end{bmatrix}, \tag{26}$$

where $E_{(k),12}(\lambda_1,\lambda_2)$ is obtained by the reduced row-echelon form of $[N_{1,(k)}(\lambda_1)\ N_{2,(k)}(\lambda_2)]$ in (24). Then

$$\operatorname{im} N_{1,(k)}(\lambda_1) \cap \operatorname{im} N_{2,(k)}(\lambda_2) = \operatorname{im} Q_{(k)}(\lambda_1,\lambda_2)\ . \tag{27}$$

Remark 11. *The set $G_{(0)}$ is dense in $\mathbb{R}^2$, [8], and denotes the subset of assignable eigenvalues for which the representation $\operatorname{im} Q_{(0)}(\lambda_1,\lambda_2)$ is a valid representation of the intersection $\operatorname{im} N_{1,(0)}(\lambda_1) \cap \operatorname{im} N_{2,(0)}(\lambda_2)$.*

Remark 12. *In case $T_{(0)}(\lambda_1,\lambda_2)$ is singular at points of interest $(\lambda_1^\star,\lambda_2^\star)$, further representations of the intersection can be obtained. At such points $(\lambda_1^\star,\lambda_2^\star)$ the reduced row-echelon form takes a different shape and is obtained by a different transformation $T^\star_{(k)}(\lambda_1^\star,\lambda_2^\star)$. Calculating $Q^\star_{(k)}(\lambda_1^\star,\lambda_2^\star)$ may increase the number of assignable linear independent eigenvectors, see [8] for an example.*

To achieve GASAS for Σ_{CL} with Theorem 6 and monotonicity with Lemma 3 at the given output simultaneously, it must be possible to assign $n-p$ linearly independent eigenvectors $v_i \in \operatorname{im} Q_{(0)}(\lambda_{1,i},\lambda_{2,i})$.

We define

$$d_{(0)} := \dim\operatorname{span}\left(\bigcup_{(\lambda_1,\lambda_2)\in G_{(0)}} \operatorname{im} Q_{(0)}(\lambda_1,\lambda_2)\right) \tag{28}$$

representing the number of linearly independent eigenvectors that can be assigned in $\operatorname{im} Q_{(0)}(\lambda_1,\lambda_2)$ for distinct $\lambda_1 \in \mathcal{L}_1$, $\lambda_2 \in \mathcal{L}_2$, $\mathcal{L}_1 \times \mathcal{L}_2 \subset G_{(0)}$, [8, Prop. 3.3a]. To evaluate (28), we follow [8] and obtain a polynomial representation of the image of $Q_{(0)}(\lambda_{1,i},\lambda_{2,i})$ by multiplying each column with the largest common denominator polynomial $q_\nu(\lambda_1,\lambda_2)$, $\nu = 1, \ldots, z$ with $z = 2m-n-p$ such that

$$P_{(0)}(\lambda_1,\lambda_2) = Q_{(0)}(\lambda_1,\lambda_2)\begin{bmatrix} q_1(\lambda_1,\lambda_2) & & 0 \\ & \ddots & \\ 0 & & q_z(\lambda_1,\lambda_2) \end{bmatrix}. \tag{29}$$

The polynomial matrix $P_{(0)}(\lambda_1,\lambda_2)$ can be written as the matrix polynomial with coefficient matrices $D_{(0),hl} \in \mathbb{R}^{n\times z}$

$$P_{(0)}(\lambda_1,\lambda_2) = \sum_{h=0}^{n}\sum_{l=0}^{n} D_{(0),hl}\lambda_1^h\lambda_2^l\,. \tag{30}$$

With the following lemma the dimension $d_{(0)}$ is readily calculated.

Lemma 13. *Let $D_{(0),hl}$ be the coefficient matrices in (30). Then*

$$d_{(0)} = \operatorname{rk}\left[D_{(0),00} \quad \ldots \quad D_{(0),hl} \quad \ldots \quad D_{(0),nn}\right]. \tag{31}$$

Proof. Let $\tilde{D} = [\tilde{D}_{00} \ldots \tilde{D}_{nn}]$ consist of all linear independent rows of

$$\left[D_{(0),00} \quad \ldots \quad D_{(0),hl} \quad \ldots \quad D_{(0),nn}\right],$$

same partition. Denoting $\tilde{d} = \operatorname{rk}\tilde{D}$, we show that $d_{(0)} = \tilde{d}$. Define

$$\tilde{P}(\lambda_1,\lambda_2) = \sum_{h=0}^{n}\sum_{l=0}^{n} \tilde{D}_{hl}\lambda_1^h\lambda_2^l\,. \tag{32}$$

If there exists $\alpha \in \mathbb{R}^{\tilde{d}} \setminus \{0\}$ satisfying $\alpha^\top \tilde{P}(\lambda_1,\lambda_1) = 0$ for all $(\lambda_1,\lambda_2) \in \mathbb{R}\times\mathbb{R}$. Then with (32) we get

$$\alpha^\top \tilde{P}(\lambda_1,\lambda_2) = \sum_{h=0}^{n}\sum_{l=0}^{n} \alpha^\top \tilde{D}_{hl}\lambda_1^h\lambda_2^l = 0 \in \mathbb{R}^{1\times z}. \tag{33}$$

Further, in the vector space $\mathbb{R}[\lambda_1, \lambda_2]$ over $\mathbb{R}$, the set of polynomials $\{\lambda_1^h \lambda_2^l\}_{h,l=0}^n$ is linearly independent. Hence (33) implies

$$\alpha^\top \tilde{D}_{hl} = 0 \quad \text{for all } h, l = 1, \ldots, n,$$

but this contradicts (31). Hence, there exists no $\alpha \in \mathbb{R}^{\tilde{d}} \setminus \{0\}$ satisfying $\alpha^\top \tilde{P}(\lambda_1, \lambda_2) = 0$ for all $(\lambda_1, \lambda_2) \in \mathbb{R} \times \mathbb{R}$. This implies that

$$\dim \operatorname{span} \left(\bigcup_{(\lambda_1, \lambda_2) \in G_{(0)}} \operatorname{im} \tilde{P}(\lambda_1, \lambda_2) \right) = \tilde{d}$$

for any dense set $G_{(0)} \subset \mathbb{R} \times \mathbb{R}$, [8, Lemma 4.4]. With (29) we have $\tilde{d} = d_{(0)}$. □

Note that $N_{q,(0)}(\lambda_q)$ and thus $Q_{(0)}(\lambda_1, \lambda_2)$ representing the intersection space $\operatorname{im} N_{1,(0)}(\lambda_1) \cap \operatorname{im} N_{2,(0)}(\lambda_2)$ is orthogonal to the output matrix C. Therefore it is possible to assign $d_{(0)} \leq n - p$ linear independent eigenvectors in $\operatorname{im} Q_{(0)}(\lambda_1, \lambda_2)$. The remaining eigenvectors have to be chosen from $\operatorname{im} Q_{(k)}(\lambda_1, \lambda_2),\ k = 1, \ldots, p$.

As $\operatorname{im} N_{q,(0)}(\lambda_q)$ is a subset of every $\operatorname{im} N_{q,(k)}(\lambda_q)$, $k = 1, \ldots, p$, we also have $\operatorname{im} Q_{(0)}(\lambda_1, \lambda_2) \subseteq \operatorname{im} Q_{(k)}(\lambda_1, \lambda_2)$ for every k. Therefore $\operatorname{im} Q_{(k)}(\lambda_1, \lambda_2)$ contains eigenvectors for the closed-loop system. As the intersection spaces $\operatorname{im} Q_{(k)}(\lambda_1, \lambda_2)$ are not orthogonal to the $k-$th output the remaining dimensions of $\mathbb{R}^n$ have to be obtained by eigenvectors drawn from $\operatorname{im} Q_{(k)}(\lambda_1, \lambda_2)$.

B. Globally monotonic tracking subject to arbitrary switching signals

Finally our result on the monotonic tracking of arbitrary step reference signals under arbitrary switching signals is as follows:

Theorem 14. *Let the switched system Σ_{OL} satisfy Assumptions 1 and 9, and assume $d_{(0)} = n - p$ in (28). Let $\mathcal{L}_q = \{\lambda_{q,i} : i \in \{1, \ldots, n\}\} \subset \mathbb{R}^-$ with distinct entries, such that $\mathcal{L}_1 \times \mathcal{L}_2 \subset G_{(0)}$. Then $n - p$ linearly independent eigenvectors $v_i \in \operatorname{im} Q_{(0)}(\lambda_{1,i}, \lambda_{2,i})$ can be chosen.*

Moreover, let there exist p vectors $v_i \in \operatorname{im} Q_{(k)}(\lambda_{1,i}, \lambda_{2,i})$ for $i = d_{(0)} + k$ and $k = 1, \ldots, p$, and the set $\mathcal{V} = \{v_i : i \in \{1, \ldots, n\}\}$ is linearly independent. Then $\mathcal{V}$, $\mathcal{L}_q$ and F_q obtained from (16) solve Problem 8, and the switched control law (6) achieves GASAS for Σ_{CL} with globally monotonic step reference tracking for any $r \in \mathbb{R}^p$.

Proof. With the definition of $d_{(0)}$ in (28) and [8, Prop 3.3a] any choice $(\lambda_{1,i}, \lambda_{2,i}) \in G_{(0)}$ yields $n - p$ linearly independent eigenvectors $v_i \in \operatorname{im} Q_{(0)}(\lambda_{1,i}, \lambda_{2,i})$.

With the additional assumption of p linearly independent vectors obtained from $\operatorname{im} Q_{(k)}(\lambda_{1,i}, \lambda_{2,i})$, V in (16) is invertible and therefore F_q are obtained from (16). The rational matrix $Q_{(k)}(\lambda_{1,i}, \lambda_{2,i})$ represents the intersection (21). Therefore the eigenvectors assigned by (16) rectify the constituent subsystems. Moreover $\operatorname{im} Q_{(k)}(\lambda_1, \lambda_2) \in \mathcal{R}^\star_{q,(k)}$ and therefore the assigned eigenvectors satisfy (17)-(18). Therefore the output of the closed-loop system takes the form (19). This solves Problem 8.

With Assumption 1, x_{ss} and $u_{q,ss}$ satisfying (5) exist for any step reference $r \in \mathbb{R}^p$ yielding, together with F_q, the control law (6). With $\mathcal{L}_q \subset \mathbb{R}^-$ and Theorem 6 we have GASAS for Σ_{CL}.

To show monotonicity of the output consider the switching instant t_j of Σ_{CL}. The feedback matrices F_q satisfy the assumptions of Lemma 3 for each subsystem Σ_q. Therefore the output for $t \in [0, t_{j+1} - t_j)$ is given by

$$y(t_j + t) = \begin{bmatrix} \beta_{q,1} e^{\mu_{q,1} t} & \beta_{q,2} e^{\mu_{q,2} t} & \ldots & \beta_{q,p} e^{\mu_{q,p} t} \end{bmatrix}^\top$$

where $q = \sigma(t_j)$, $\beta_{q,k}$ are the last p rows of $V^{-1} x(t_j)$, and $\mu_{q,k} = \lambda_{q,n-p+k}$, $k = 1, \ldots, p$. Since the eigenstructures of Σ_q are rectified by (6), $V^{-1} x(t_j)$ and thus $\beta_{q,k}$ are shared by all subsystems for any $x(t_j)$. Hence the output of the switched system Σ_{CL} converges monotonically. □

Remark 15. *The rate of convergence of the output at any time $t > 0$ depends upon whether closed-loop poles from $\mathcal{L}_1$ or $\mathcal{L}_2$ are activated. The desired dynamical shape of the output beyond its monotonic characteristics can be assigned by choosing eigenvalues that will be assigned to the eigenvectors in $\mathcal{R}^\star_{q,(k)}$ for $k \neq 0$. In particular, choosing identical eigenvalues for each closed-loop subsystem corresponding to the same output component, results in a smooth output trajectory such that that the effects of system switching is not noticeable from the output.*

The following algorithm describes our procedure for obtaining suitable sets $\mathcal{V}$ and $\mathcal{W}_q = \{w_{q,1}, \ldots, w_{q,n}\} \subset \mathbb{R}^p$, and $\mathcal{L}_q$ that satisfy the conditions of Theorem 14 and hence can be used to obtain the feedback laws F_q that deliver a globally monotonic step response for Σ_{CL} under arbitrary switching, for any desired step reference.

Algorithm 16 (Control law according to Theorem 14)**.**

1) *For each $k \in \{0, \ldots, p\}$ compute the rational matrix functions $Q_{(k)}(\lambda_1, \lambda_2)$ given by (26).*
2) *If $d_{(0)} = n - p$ continue,*
   else: calculate $Q^\star_{(0)}(\lambda_1^\star, \lambda_2^\star)$ as in Remark 11 to obtain additional linear independent eigenvectors in Step 3.
3) *For $i \in \{1, \ldots, n - p\}$ select vectors v_i such that*
   $$v_i \in \operatorname{im} Q_{(0)}(\lambda_{1,i}, \lambda_{2,i})$$
   and v_i are linearly independent.
4) *For $i \in \{n - p + 1, \ldots, n\}$ select vectors v_i such that*
   $$v_i \in \operatorname{im} Q_{(k)}(\lambda_{1,i}, \lambda_{2,i})$$
   where $k = i - (n - p)$.
5) *Form the set $\mathcal{V} = \{v_1, \ldots, v_n\}$ and test that it is linearly independent. If it is not, modify the choice of v_i for some $i \in \{n - p + 1, \ldots, n\}$.*
6) *Select sets of distinct desired closed-loop eigenvalues $\mathcal{L}_q = \{\lambda_{q,i} : i \in \{1, \ldots, n\}\} \subset \mathbb{R}^-$, such that $T_{(k)}(\lambda_{1,i}, \lambda_{2,i})$ is invertible for all i and associated k.*
7) *For each v_i and $\lambda_{q,i}$, solve (14)-(15) for $w_{q,i}$ to obtain $\mathcal{W}_q$, $q \in \mathcal{I}$.*
8) *Obtain F_q from (16).*
9) *For any desired step reference, solve (5) to obtain the control law u in (6).*

V. EXAMPLE

We consider a switched system with two outputs. The dimensions $n = 7$, $m = 5$ and $p = 2$ Assumption 1. Let

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & -4 & 1 & -1 & 0 \\ 2 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -5 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 & 0 \\ 4 & 0 & 0 & -5 & 0 & 3 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 3 & 0 & -3 & 0 & 0 \\ 0 & 1 & -1 & 4 & 0 \\ -1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 5 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & -4 & 3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 & 5 & 0 & -5 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 1 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The output nulling subspaces for each system $\Sigma_{q,(k)}$ may be readily computed with the `rvstar` command in the MATLAB™ GA toolbox [17]. In particular, we obtain

$$\dim(\mathcal{R}^\star_{1,(0)} \cap \mathcal{R}^\star_{2,(0)}) = 5\ . \tag{34}$$

It is readily verified that the second condition in Assumption 9 is also satisfied.

To compute $d_{(0)}$, we first obtain the rational matrix function $Q_{(0)}(\lambda_1, \lambda_2)$ in (26) and the polynomial representation $P_{(0)}(\lambda_1, \lambda_2)$ in (29). Finally the coefficient matrices of $P_{(0)}$ are computed. Note that most coefficient matrices in (29) equal zero such that $P_{(0)}$ can be represented by

$$P_{(0)}(\lambda_1, \lambda_2) = \sum_{h=0}^{2} \sum_{l=0}^{2} D_{(0),hl} \lambda_1^h \lambda_2^l.$$

We obtain

$$\begin{aligned} d_{(0)} = \mathrm{rk} \big[& D_{(0),00}\ D_{(0),01}\ D_{(0),02} \\ & D_{(0),10}\ D_{(0),11}\ D_{(0),12}\ D_{(0),20}\ D_{(0),21}\ D_{(0),22} \big] \end{aligned}$$

$$= \mathrm{rk} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -100 & -120 & 0 & 50 & -20 \\ 0 & 0 & 0 & 180 & -60 & -45 & -90 & 135 & -100 \\ 0 & 0 & 0 & 1380 & 225 & 0 & -495 & 500 & 0 \\ 0 & 0 & 0 & 300 & 360 & 0 & -150 & 60 & 0 \\ -72 & 29 & 24 & -117 & -397 & -319 & 57 & 97 & -60 \end{bmatrix}.$$

It is readily verified that $d_{(0)} = 5$ and thus $n - p$ eigenvectors can be chosen to lie in $\mathcal{R}^\star_{q,(0)}$. The corresponding eigenvalues can be considered as degrees of freedom. Choose the eigenvalue sets $\mathcal{L}_1 = \{-2.5, -3, -4, -5, -6, -1.5, -7\}$ and $\mathcal{L}_2 = \{-0.5, -1.5, -2.5, -3.5, -4.5, -10, -7\}$. Note that the transformation matrix $T_{(0)}(\lambda_{1,i}, \lambda_{2,i})$ in (24) is invertible for the above ordering of eigenvalue sets. The eigenvectors v_i, $i = \{1, \ldots, 5\}$ of the output-nulling subspace are obtained by $v_i \in \operatorname{im} Q_{(0)}(\lambda_{1,i}, \lambda_{2,i})$, such that

$$v_i \in \operatorname{im} N_{1,(0)}(\lambda_{1,i}) \cap \operatorname{im} N_{2,(0)}(\lambda_{2,i}). \tag{35}$$

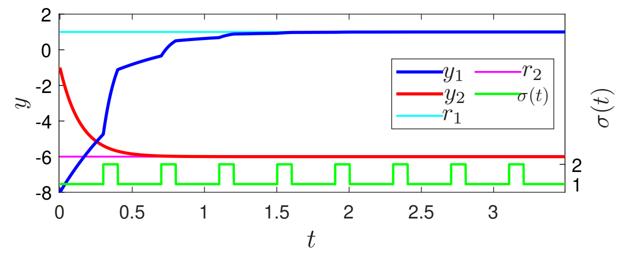

Fig. 1. Rectified eigenstructure with monotonic responses at the outputs

The remaining modes $i = \{6, 7\}$ are assigned to eigenvectors lying in each $\mathcal{R}^\star_{q,(k)}$. We choose $v_6 \in \operatorname{im} Q_{(1)}(\lambda_{1,6}, \lambda_{2,6})$ and $v_7 \in \operatorname{im} Q_{(2)}(\lambda_{1,7}, \lambda_{2,7})$ such that

$$v_6 \in \operatorname{im} N_{1,(1)}(\lambda_{1,6}) \cap \operatorname{im} N_{2,(1)}(\lambda_{2,6}), \tag{36}$$

$$v_7 \in \operatorname{im} N_{1,(2)}(\lambda_{1,7}) \cap \operatorname{im} N_{2,(2)}(\lambda_{2,7}). \tag{37}$$

Therefore the vector v_6 is visible at the first output and the vector v_7 is visible in the second output. We observe that the set $\mathcal{V} = \{v_1, \ldots, v_7\}$ is linearly independent, and use (16) to obtain

$$F_1 = \begin{bmatrix} 1.73 & -0.06 & -9.27 & -0.13 & 0.16 & -0.01 & 3.14 \\ 4.19 & -0.90 & -32.98 & -0.71 & 0.29 & 2.19 & 11.93 \\ 2.23 & -1.56 & -9.27 & -0.13 & 0.16 & -0.01 & 3.14 \\ -0.49 & -1.67 & 5.93 & 0.14 & -0.03 & -0.55 & -2.20 \\ 16.80 & -4.85 & -81.79 & -0.31 & 1.61 & 2.47 & 23.78 \end{bmatrix}$$

and

$$F_2 = \begin{bmatrix} 111.23 & -45.17 & -521.06 & -4.16 & 11.21 & 13.38 & 150.32 \\ 7.39 & -6.58 & -38.11 & -0.19 & 0.71 & 0.17 & 11.47 \\ -7.39 & 3.08 & 38.11 & 0.19 & -0.71 & -1.67 & -11.47 \\ -43.66 & 9.14 & 228.89 & 0.91 & -4.30 & -11.55 & -67.94 \\ -6.39 & 3.68 & 38.51 & 0.19 & -0.71 & -1.67 & -11.47 \end{bmatrix}.$$

For the step reference $r = \begin{bmatrix} 1 & -6 \end{bmatrix}^\top$ we compute x_{ss}, $u_{q,ss}$ satisfying (5) and obtain the control law (6). By Theorem 14, the outputs of Σ_{CL} will track this reference monotonically from any initial state under arbitrary switching.

To demonstrate this behaviour, Figure 1 shows the outputs arising from the initial state $x_0 = \begin{bmatrix} -7 & -1 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}^\top$ under a periodic switching signal that activates Σ_1 for 0.3s followed by Σ_2 for the subsequent 0.1s.

The monotonic convergence to the reference $r = \begin{bmatrix} 1 & -6 \end{bmatrix}^\top$ can be seen. Output y_1 changes its gradient at the switching instances, due to the differing convergence rates of the visible eigenvalues $\lambda_{1,6} = -1.5$ and $\lambda_{2,6} = -10$. For output y_2 we assign the same convergence rate $\lambda_{1,7} = \lambda_{2,7} = -7$ to both subsystems. This renders the output signal smooth and the switching instances are not noticeable at all in y_2.

For comparison we design a switched controller (6) with rectified eigenstructure but without assigning eigenvectors according to the subspaces (14), (15). Using the algorithm in [8] we obtain the feedback matrices

$$\hat{F}_1 = \begin{bmatrix} 1.44 & 0.87 & -5.18 & -0.88 & 0.92 & -1.94 & -1.32 \\ 29.21 & 22.73 & -49.41 & -17.59 & 21.28 & -45.05 & -68.45 \\ 3.14 & 0.98 & -4.73 & -0.64 & 0.60 & -1.18 & -0.46 \\ -18.57 & -13.82 & 41.98 & 10.73 & -13.04 & 25.64 & 36.79 \\ -12.68 & -9.21 & 34.69 & 7.01 & -8.72 & 15.69 & 21.70 \end{bmatrix}$$

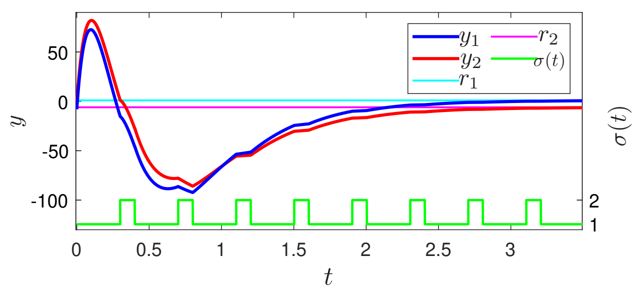

Fig. 2. Rectified eigenstructure without selecting for monotonicity

and

$$\hat{F}_2 = \begin{bmatrix} -437.74 & -363.27 & 738.46 & 248.87 & -308.98 & 618.61 & 987.62 \\ -46.24 & -42.77 & 73.52 & 27.38 & -32.88 & 61.69 & 106.55 \\ -11.15 & -6.16 & 20.58 & 6.43 & -7.97 & 16.79 & 24.82 \\ -11.15 & -6.16 & 20.58 & 6.43 & -7.97 & 16.79 & 24.82 \\ -11.15 & -6.16 & 20.58 & 6.43 & -7.97 & 16.79 & 24.82 \end{bmatrix}$$

assigning the same closed-loop eigenvalues $\mathcal{L}_q$ as before. Note that the resulting closed-loop system has also the same steady state x_{ss}.

The output for the same initial state, the same reference and switching signal is shown in Figure 2. It is easy to see that this controlled switched system has worse characteristics at the considered output. Both outputs exhibit a large over- and undershoot and the switching effects are much more pronounced visible.

Finally we compute the Frobenius norms of the feedback matrices to compare the control effort. For the monotonic design these are given by $\|F_1\| = 95.23$ and $\|F_2\| = 610.73$. For the system only designed with eigenstructure rectification following [8] the norms are $\|\hat{F}_1\| = 135.08$ and $\|\hat{F}_2\| = 1554.19$ are given. This indicates that the superior tracking performance of the monotonic controller design did not require larger control actuation.

VI. Conclusion

We have investigated how to bring together the earlier works on the design of state feedback control laws for the stability of LTI switching systems of [8], with the methods of [13] for the design of globally monotonic tracking controllers. Both of these earlier papers employ eigenstructure assignment methods derived from the classic paper of [12]. We assumed switching between two systems that shared a common state vector and common output. Subject to a suitable assumption on the system dimensions, we were able to design a state feedback switching control law that achieves asymptotic stability for arbitrary switching signals, while also ensuring that the system output track a step reference in a monotonic manner, in all output components, from any initial condition. The principal limitation of the design method is the rather strong assumption on the system dimensions, typically requiring considerably more control inputs than outputs.

Future work will consider the task of adapting the methods of [18] on the design of nonovershooting tracking controllers for switching systems. These methods are applicable to square LTI systems, with equal numbers of inputs and outputs. The controller design objective would be to design state feedback matrices to deliver a nonovershooting step response in all output components for arbitrary switching signals.

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