2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Sampled-data String Stability for a Platoon of heterogeneous vehicles via a mesoscopic approach

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***Abstract*— This paper explores sampled-data techniques to achieve asymptotic string stability in a platoon of autonomous vehicles. This is done making use of both microscopic and macroscopic data that are, however, often available at distinct time instants. The proposed mesoscopic controller is demonstrated to operate effectively, regardless of the involved sampling periods. The theoretical findings are validated through simulations.**

***Index Terms*— Traffic control, autonomous vehicles, sampled-data control, String Stability, platoon control, mesoscopic modeling, macroscopic information.**

I. Introduction

Due to the increasing demand for improved transportation efficiency and enhanced safety, advanced control methods dedicated to Intelligent Transportation Systems (ITS) are needed to address their complex and dynamic nature, where multiple agents interact with each other and the environment. In this framework, current research literature is focusing on how to ensure String Stability (SS) [1], [2]. This property is of foremost importance for developing efficient and safe Cooperative Adaptive Cruise Controllers (CACC) [3].

Historically, distinct cases of information exchange have been considered for each leader-follower vehicles' interaction with the common feature of always sharing some microscopic variables among the whole platoon; e.g., the acceleration of the platoon's leading vehicle (see [1]) or its desired speed profile (see [4]). Recently, few results are considering the possibility to guarantee desired properties via the use of macroscopic information with the aim of avoiding to share the microscopic information of the leading vehicle and, hence, reducing the amount of information that is exchanged [5]–[7]. More in detail, in [5] the authors proved the possibility to obtain SS when using only aggregate (macroscopic) information and measuring local microscopic information, thus obtaining a mesoscopic control law. In particular, the microscopic information available for each vehicle consists of the state of its predecessor only. In [6], those results were extended to the case of disturbances acting on the vehicles and on the information that is shared along the platoon. In [7], a link between the possibility to ensure SS and the way the traffic Fundamental Diagram is impacted was proposed. Other studies aim at mesoscopic traffic modeling for control, e.g., [8]–[11]. However, no formal SS analysis is performed [10], [11]; if any, such analysis is based on the linear tangent model only [8], [9].

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The theoretical results in [5], [6], as well as most of the available literature on this topic, concern continuous-time systems only; i.e., when measurements and the information on the traffic flow are available continuously in time and the control input arbitrarily varies at all times. This framework is far from being realistic as it does not take into account the discrete-time influence of digital devices, which are unavoidable for control implementation and information exchange within the platoon. This limitation highlights the necessity for sampled-data approaches in CACC.

The objective of this paper is to establish a first set of new results to endow a more realistic scenario, in which we let measures be available at sporadic discrete-time instants only and the control inputs be piecewise constant over the sampling period, that cannot be arbitrarily fixed [12]. In this scenario, only few results are available and usually referring to the so-called *emulation-based control*; i.e., the controller is designed by completely neglecting the effect of sampling and then implemented in practice via sample-and-hold devices (see, among many, [13], [14] for a general overview, and [15], [16] for applications to a similar context as ours). As the intuition suggests, those kind of control laws preserve the same performances enforced by the continuous-time (nominal) design only when the involved sampling periods are very small.

Making reference to a discrete-time problem formulation, we design a new simple controller that is capable of enforcing string stability over the traffic flow, despite the effect of sampling (i.e., independently on the length of the involved sampling periods). Furthermore, we contemplate scenarios where the sampling instants for microscopic and macroscopic data are non-simultaneous, as macroscopic information is available intermittently with respect to the microscopic one.

The rest of the paper is organized as follows. In Section II we settle the problem over a suitably defined discrete-time equivalent model of the traffic flow embedding

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sampled-data micro- and macro-scopic information and piecewise constant control action. The main result is proved in Section III with a illustrative simulations reported in Section IV. Section V concludes the paper providing perspectives.

$\mathbb{C}$, $\mathbb{R}$ and $\mathbb{N}$ denote the set of complex, real and natural numbers including 0 respectively. $\mathbb{R}^+$ denotes the set of positive real numbers. I and 0 denote respectively the identity and zero matrices of suitable dimensions. Given a matrix $A \in \mathbb{R}^{n\times n}$, $\sigma\{A\} \subset \mathbb{C}$ is its spectrum. A is said to be Schur if its spectrum is included in the open unit circle of the complex plane (i.e., all its eigenvalues are with norm strictly less than 1 and none is at the origin). $|\cdot| \in \mathbb{R}$ denotes, depending on the argument, either the cardinality of a set $\mathcal{S}$, the absolute value of a complex number $\lambda \in \mathbb{C}$ or the norm of a matrix.

II. Modeling and Problem formulation

A. Microscopic modeling

We consider $N+1$ vehicles implementing CACC that are described by the corresponding longitudinal position, $p_i \in \mathbb{R}^+$, and speed, $0 \le v_i \le v_{\max}$, $v_{\max} \in \mathbb{R}^+$, $\forall\, i \in \mathcal{I}_N^0 := \{0, 1, \dots, N\}$. Then, we define the state of the $i-$th vehicle as

$$x_i = [\, p_i \;\; v_i \,]^\top. \tag{1}$$

Without loss of generality, the low level dynamics describing the power-train can be considered to have been feedback linearised (see [1], [17], [4]), thus allowing for a simplification in considering only the longitudinal dynamics for describing heterogeneous platoons [18], i.e., platoons composed by non identical vehicles. Ignoring both reaction and communication time delays, similarly to [1], [17], [19], [20], the corresponding dynamical system is given by

$$\dot{x}_i = \begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} v_i \\ u_i \end{bmatrix}, \qquad i \in \mathcal{I}_N^0, \tag{2}$$

where $|u_i| \le u_{\max}$, $u_{\max} \in \mathbb{R}^+$, is the control input of the $i-$th vehicle, corresponding to the acceleration. For describing inter-vehicular interactions, we adopt the leader-follower model (see [21]), with respect to which we deduce a global description of the platoon. For including the vehicle $i = 0$, we consider the presence of a virtual leader, indexed by $i = -1$, that precedes the entire platoon, described by

$$\dot{x}_{-1} = \begin{bmatrix} \dot{p}_{-1} \\ \dot{v}_{-1} \end{bmatrix} = \begin{bmatrix} v_{-1} \\ u_{-1} \end{bmatrix}. \tag{3}$$

Then, the state of each car-following situation between vehicle $i-1$ and i is defined by the variable

$$\chi_i = x_i - x_{i-1} = \begin{bmatrix} \Delta p_i \\ \Delta v_i \end{bmatrix} = \begin{bmatrix} p_i - p_{i-1} \\ v_i - v_{i-1} \end{bmatrix}, \; i \in \mathcal{I}_N^0. \tag{4}$$

The resulting microscopic dynamical model of the $i-$th car-following pair of vehicles implementing CACC is

$$\dot{\chi}_i = A\chi_i + B\,(u_i - u_{i-1})\,, \;\; i \in \mathcal{I}_N^0 \tag{5}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{6}$$

At this point, the following assumptions are set.

Assumption 1: The input u_i of each vehicle $i \in \mathcal{I}_N^0$ is a piecewise constant signal over the sampling period of length $T_m \ge 0$, that is $u_i(t) = u_i(t_k)$, $t \in [t_k, t_{k+1})$ and $t_k = kT_m$.

Assumption 2: Each vehicle $i \in \mathcal{I}_N^0$ measures its corresponding microscopic quantities (i.e., the state x_i, the state x_{i-1} and the input u_{i-1} of its predecessor $i-1 \in \mathcal{I}_N^0$) at the sampling instants only, defined as $t_k = kT_m$ with $k \in \mathbb{N}$.

Under the assumptions above, the dynamics of each vehicle (5) at all sampling instants $t_k = kT_m$ and $k \in \mathbb{N}$ can be described by the corresponding sampled-data equivalent model that is given by

$$\chi_i(t_{k+1}) = A_{T_m}\chi_i(t_k) + B_{T_m}\big(u_i(t_k) - u_{i-1}(t_k)\big) \tag{7}$$

with

$$A_{T_m} = e^{AT_m} = \begin{bmatrix} 1 & T_m \\ 0 & 1 \end{bmatrix}, \; B_{T_m} = \int_0^{T_m} e^{A\ell}\mathrm{d}\ell B = \begin{bmatrix} \frac{T_m^2}{2} \\ T_m \end{bmatrix}.$$

In the following, we will refer to (7) as the microscopic sampled-data equivalent model of (5).

To define the equilibrium point of the platoon, we consider that the virtual leader $i = -1$ moves at a constant speed (see [21], [22]) with no disturbance acting over. Thus, the virtual vehicle's speed v_{-1} can be considered as the reference speed of $i = 0$ (see [4]–[6]). Assuming that $\Delta\bar{p} > 0$ is the desired inter-vehicular distance at steady-state and that

$$\Delta p_0(t_k) = -\Delta\bar{p} \quad \forall\, t_k, \tag{8}$$

then the equilibrium point for the $i-$th system of dynamics (5) corresponds to the case where all the vehicles have the same speed and are at the same distance, i.e.,

$$\chi_{e,i} = \bar{\chi} = [-\Delta\bar{p} \;\; 0]^\top, \;\; \forall\, i \in \mathcal{I}_N^0. \tag{9}$$

Since the state vector (4) is defined with respect to the follower vehicle, then the distance Δp_i and the relative speed Δv_i have opposite sign. For this reason, the equilibrium distance in (9) is $-\Delta\bar{p} < 0$. From the platoon point of view, we define the lumped state and the lumped equilibrium for $u_{-1} = 0$ respectively as

$$\chi = [\chi_0^\top \; \chi_1^\top \; \dots \; \chi_N^\top]^\top, \quad \chi_e = [\bar{\chi}^\top \; \bar{\chi}^\top \; \dots \; \bar{\chi}^\top]^\top. \tag{10}$$

B. Macroscopic information modeling

In general, each vehicle receives partial information on the platoon state beyond the ones on the corresponding state and of its own predecessor. For all vehicles $i \in \mathcal{I}_N^0$, such information is referred to as *macroscopic information* and is endowed within a function

$$\psi_{i-1}(\chi_0, \dots, \chi_{i-1}) : \mathbb{R}^2 \times \dots \times \mathbb{R}^2 \to \mathbb{R}^2 \tag{11}$$

with, by definition for $i = 0$, $\psi_{i-1} = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. In the following, we let this quantity be spread over the network only at sporadic time instants that are, in general, distinct than the ones the microscopic information is perceived (Assumption 2).

Assumption 3: Each vehicle $i \in \mathcal{I}_N^0$ measures the macroscopic information function (11) at all $t \in t_{k_M} = k_M T_M$ with $k_M \in \mathbb{N}$ and sampling period verifying $T_M = MT_m$ with $M \in \mathbb{N}\backslash\{0\}$.

In the following, we will refer to $t_k = kT_m$, $t_{k_M} = k_M T_M = k_M MT_m$ as the microscopic and macroscopic sampling instants respectively. Similarly, T_m and T_M are referred to as microscopic and macroscopic sampling periods. Each vehicle defines a sampled-data asynchronous dynamics with the macroscopic and microscopic information available at different sampling instants.

The relationship among macroscopic and microscopic quantities are well-known in the literature. For example, we can consider traffic density as the inverse of inter-vehicular mean distance [23, ch. 2, p. 26], or the speed-density diagram [24, ch. 4] describing the interconnection between local and global quantities. Therefore, several macroscopic quantities can be considered for the purposes of obtaining the desired mesoscopic controller. In the sequel, without loss of generality we leverage the existing relationship between the macroscopic density with the microscopic variance that is described in [24] and modeled in continuous-time in [5], [6]. Then, for each $i \in \mathcal{I}_N^0$, $\mu_{l,i-1}$ and $\sigma^2_{l,i-1}$ denote the inter-vehicular distance ($l = \Delta p$) and speed error ($l = \Delta v$) mean and variance, respectively, computed from vehicle 0 to vehicle $i-1$; i.e., for $l \in \{\Delta p, \Delta v\}$ they are defined as

$$\mu_{l,i-1} = \frac{1}{i}\sum_{j=0}^{i-1} l_j, \;\; \sigma^2_{l,i-1} = \frac{1}{i}\sum_{j=0}^{i-1}(l_j - \mu_{l,i-1})^2. \quad (12)$$

To provide the i-th vehicle with the macroscopic information embedded in $\mu_{l,i-1}$ and $\sigma^2_{l,i-1}$, $i \in \mathcal{I}_N^0$, the macroscopic functions are defined as

$$\psi^l_{i-1} = sign(\mu_{l,i-1} - \bar{\chi}_l)\sqrt{\sigma^2_{l,i-1}}, \; l \in \{\Delta p, \Delta v\}, \quad (13)$$

with $\bar{\chi}_{\Delta p} = -\Delta\bar{p}$ and $\bar{\chi}_{\Delta v} = 0$. Then, (11) specifies as

$$\psi_{i-1} = \begin{bmatrix}\psi^{\Delta p}_{i-1} & \psi^{\Delta v}_{i-1}\end{bmatrix}^\top. \quad (14)$$

Similarly to [5, Theorem 1], we require the conditions than the interconnection terms $\psi_{i-1}(t_{k_M})$ are bounded with respect to the microscopic dynamics, with the bound $|\psi_{i-1}| \le \sum_{j=0}^{i-1} k_{ij}|\tilde{\chi}_j|$, $k_{ij} \in \mathbb{R}^+$, $\forall\, i \in \mathcal{I}_N^0$.

C. Problem statement

Denoting the constant inter-vehicular distance (8) by $\Delta\bar{p}$ the following *spacing policy* is adopted

$$\Delta p^r_i(t_k) = -\Delta\bar{p}, \quad t_k = kT_m, \quad k \in \mathbb{N} \quad (15)$$

Under Assumptions 1, 2 and 3, the objective of the paper is to deduce a piecewise constant dynamic controller asymptotically tracking the desired distance (8) based on asynchronous micro and macroscopic information. More in detail, for all $t_k \in [t_{k_M}, t_{k_M+1})$, we propose a dynamic sampled-data mesoscopic controller of the form

$$u_i(t_k) \;\; = \;\; u_{i-1}(t_k) + F_e\tilde{\chi}_i(t_k) + F_\psi\psi_{i-1}(t_{k_M}) \quad (16)$$

where $t_k = kT_m$, $t_{k_M} = k_M MT_m$ (with $k, k_M \in \mathbb{N}$), $\tilde{\chi}_i = \chi_i - \chi_{e,i}$, for some matrix F_e and F_ψ of suitable dimensions to be defined later on. The problem we address is then formalized as follows.

Problem 1: Consider a platoon of vehicles described by (5) under Assumptions 1, 2 and 3. Let the corresponding sampled-data microscopic model be described by (7). Design a feedback law of the form (16) so that the equilibrium χ_e in (10) of the closed-loop platoon is *Asymptotically String Stable* (ASS); namely, $\chi_{e,i}$ verifies the following properties.

(i) *String Stability* (SS): for all $\varepsilon > 0$, there exists $\delta > 0$ such that, for all $N \in \mathbb{N}$ and $t_k = kT_m$ ($k \in \mathbb{N}$),

$$\max_{i\in\mathcal{I}_N^0} |\chi_i(0) - \chi_{e,i}| < \delta \Rightarrow \max_{i\in\mathcal{I}_N^0} |\chi_i(t_k) - \chi_{e,i}| < \varepsilon; \quad (17)$$

(ii) *Attractiveness*: for all $N \in \mathbb{N}$ and $i \in \mathcal{I}_N^0$

$$\lim_{t_k\to\infty} |\chi_i(t_k) - \chi_{e,i}| = 0. \quad (18)$$

III. Main result

To design the feedback controller, let us consider the coordinate transformation

$$\chi_i \mapsto \tilde{\chi}_i = \chi_i - \chi_{e,i}$$

with the corresponding closed-loop dynamics given by

$$\tilde{\chi}_i(t_{k+1}) = \tilde{A}\tilde{\chi}_i(t_k) + \tilde{B}\psi_{i-1}(t_{k_M}) \quad (19)$$

for $t_k \in [t_{k_M}, t_{k_M+1})$ and

$$\tilde{A} = A_{T_m} + B_{T_m}F_e, \quad \tilde{B} = B_{T_m}F_\psi.$$

At this point, the main result can be enhanced.

Theorem 1: Problem 1 is solved by the controller (16) with $\psi_{i-1} \in \mathbb{R}^2$, F_e and F_ψ such that the conditions hold:

1) the matrix $A_{T_m} + B_{T_m}F_e$ is Schur stable;
2) there exists $c \in \mathbb{R}^+$ such that the macroscopic function (11) verifies, for all $i, j \in \mathcal{I}_N^0$
$$|\psi_{i-1}(\chi_0, \ldots, \chi_{i-1})| \le \sum_{j=0}^{i-1} c|\chi_j - \chi_{e,j}|;$$
3) for $M \in \mathbb{N}\backslash\{0\}$ and $\alpha := -\frac{1}{T_m}\log\{\max_{\lambda\in\sigma\{\tilde{A}\}}\{|\lambda|\}\}$ the parameter
$$\tilde{\gamma} = \frac{c\beta|\tilde{B}|}{1 - e^{-\alpha T_m M}} \quad (20)$$
verifies $\tilde{\gamma} \in (0, 1)$ with $\beta = |\tilde{A}|e^{\alpha T_m M}$.

Proof: We first prove item (i) in Problem 1. To this end, we have to show that, for all $i \in \mathcal{I}_{\mathbb{N}}^0$, the overall closed-loop dynamics (19) is asymptotically stable when 1) holds. This follows from item 1) as $\tilde{A}$ is Schur (that is, with all eigenvalues inside the open unit circle). To see this, we introduce the transformation

$$\boldsymbol{\chi}_i = T\tilde{\chi}_i, \quad T = \begin{bmatrix} I & -CQ \\ 0 & I \end{bmatrix}$$

so that the closed-loop systems gets the form

$$\boldsymbol{\chi}_i(t_{k+1}) = \boldsymbol{A}\boldsymbol{\chi}_i(t_k) + \boldsymbol{B}\boldsymbol{\psi}_{i-1}(t_{k_M})$$

$$\boldsymbol{A} = \begin{bmatrix} A_{T_m} + B_{T_m} F_e & 0 \\ G_e & \Lambda + G_e C Q \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} B_{T_m} F_\psi \\ G_\psi \end{bmatrix}.$$

From the structure of $\boldsymbol{A}$ above, one gets that if 1) holds then $\tilde{A}$ is Schur (that is, with all eigenvalues inside the open unit circle). With this in mind, considering the original coordinates, for all $i \in \mathcal{I}_{\mathbb{N}}^0$ the closed-loop system is in the form of a cascade, that is

$$\tilde{\chi}_0(t_{k+1}) = \tilde{A}\tilde{\chi}_0(t_k) \tag{21a}$$

$$\tilde{\chi}_1(t_{k+1}) = \tilde{A}\tilde{\chi}_1(t_k) + \tilde{B}\psi_0(\tilde{\chi}_0(t_{k_M})) \tag{21b}$$

$$\vdots$$

$$\tilde{\chi}_i(t_{k+1}) = \tilde{A}\tilde{\chi}_i(t_k) + \tilde{B}\psi_{i-1}(\tilde{\chi}_0(t_{k_M}), \dots, \tilde{\chi}_{i-1}(t_{k_M})) \tag{21c}$$

By construction in 1) and because the interconnecting components $\psi_{i-1}(\cdot)$ verify 2) the platoon is exponentially stable [25, Proposition 3.1].

We can now prove string stability (that is (i) in Problem 1). To this end, we first note that, for all $i \in \mathcal{I}_N^0$, $k_M \in \mathbb{N}$ and $j \in \{1, \dots, M\}$ one can rewrite $k = k_M M + j$ and, thus

$$\tilde{\chi}_i(t_k) = \tilde{A}^k \tilde{\chi}_i(t_0) + \sum_{\ell=0}^{k-1} \tilde{A}^{k-\ell-1}\tilde{B}\psi_{i-1}(t_{\lfloor \frac{\ell}{M} \rfloor}). \tag{22}$$

We proceed iteratively exploiting the cascade structure (21). By construction in 1), the leader is exponentially stable and, thus, for all $t_k = kT_m$ and $k \in \mathbb{N}$ it verifies, for some $\beta > 0$

$$|\chi_0(t_k)| \leq \beta e^{-\alpha T_m k}|\chi_0(t_0)|. \tag{23}$$

Consider now the first vehicle (21b) and, in particular, the form (22). Then, the triangular equality and 1) yield

$$|\chi_1(t_k)| \leq \beta e^{-\alpha T_m k}|\chi_1(t_0)| + \Big|\sum_{\ell=0}^{k-1} \tilde{A}^{k-\ell-1}\tilde{B}\psi_0(\tilde{\chi}_0(t_{\lfloor \frac{\ell}{M} \rfloor}))\Big|. \tag{24}$$

At this point, as far as the addend of the right hand side of the inequality above is concerned, one gets

$$\begin{aligned} \Big|\sum_{\ell=0}^{k-1} \tilde{A}^{k-\ell-1}\tilde{B}\psi_0(\tilde{\chi}_0(t_{\lfloor \frac{\ell}{M} \rfloor}))\Big| &\leq \beta\tilde{\gamma} \sup_{\ell \in [t_0, t_k^M]} \big|\tilde{\chi}_0(t_{\ell M})\big| \\ &\leq \beta\tilde{\gamma}\big|\tilde{\chi}_0(t_0)\big| \end{aligned}$$

where the last inequality follows from (23). Defining now (20) and using the bound above into (24), one gets

$$\begin{aligned} |\tilde{\chi}_1(t_k)| &\leq \beta\big|\tilde{\chi}_1(t_0)\big| + \beta\tilde{\gamma}\big|\tilde{\chi}_0(t_0)\big| \\ &\leq \beta\big(1 + \tilde{\gamma}\big)\max\{\big|\tilde{\chi}_0(t_0)\big|, \big|\tilde{\chi}_1(t_0)\big|\} \end{aligned} \tag{25}$$

with, by hypotheses in 3), $\tilde{\gamma} \in (0, 1)$. In addition, $\tilde{\gamma}$ exists as $\tilde{A}$ is Schur by construction in 1).

Similarly for $i = 2$, one gets

$$\begin{aligned} |\tilde{\chi}_2(t_k)| \leq& \beta e^{-\alpha T_m k}|\chi_2(t_0)\big| \\ &+ \Big|\sum_{\ell=0}^{k-1} \tilde{A}^{k-\ell-1}\tilde{B}\psi_1(\tilde{\chi}_0(t_{\lfloor \frac{\ell}{M} \rfloor}), \chi_1(t_{\lfloor \frac{\ell}{M} \rfloor}))\Big|. \\ \leq& \beta\big(1 + \tilde{\gamma} + \tilde{\gamma}^2\big)\max\{\big|\tilde{\chi}_0(t_0)\big|, \big|\tilde{\chi}_1(t_0)\big|\}. \end{aligned} \tag{26}$$

Iterating for the i^{th} vehicle, one gets

$$|\tilde{\chi}_i(t_k)| \leq \beta\big(1 + \tilde{\gamma} + \dots + \tilde{\gamma}^i\big)\max_{i \in \mathcal{I}_N^0}\{\big|\tilde{\chi}_i(t_0)\big|\} \tag{27}$$

that yields, since $\tilde{\gamma} \in (0, 1)$,

$$|\tilde{\chi}_i(t_k)| \leq \frac{\beta}{1 - \tilde{\gamma}}\max_{i \in \mathcal{I}_N^0}\{\big|\tilde{\chi}_i(t_0)\big|\}$$

and thus (i) follows. Finally, (ii) in Problem 1 follows from asymptotic stability and string stability. ■

IV. Simulations

The control proposed in (16) is simulated in Matlab2022b©. Based on the modeling in (7), we consider a platoon of 10 vehicles in the two situations below.

a) Both the microscopic and the macroscopic information are synchronous with $T_m = T_M = 0.2$s (hence, $M = 1$). This choice of the sampling periods allows for a possible comparison with respect to the continuous-time (CT) controller in [5], [6]. The considered value for $\tilde{\gamma}$ in (20) is 0.7841.

b) The sampling period of the microscopic information is $T_m = 0.2$s, whereas the macroscopic one is $T_M = 2s$ (hence, $M = 10$). This choice produces $\tilde{\gamma} = 0.1644$ in (20).

In both situations the desired distance along the platoon is $\Delta p_i^r(t_k) = 20\ m$, and the leader starts with a speed of $10\ m/s$. The vehicles' speeds are $0 \leq v_i \leq 36\ m/s$, while the accelerations are $-7 \leq u_i \leq 7\ m/s^2$.

For providing a fair comparison with the continuous-time approach in [5], [6], each simulation consists of the following different phases.

- From $t = 0$ s to $t = 25$ s, for all vehicles $i \in \mathcal{I}_N^0 \backslash \{9\}$ start with $\Delta p_i(0) = 20$ m and $\Delta v_i(0) = 10$ m/s; for $i = 9$ we fix $\Delta p_9(0) = 20$ m and $\Delta v_9(0) = 12$ m/s. The leader tracks a piece-wise constant speed reference, with no disturbance acting over.
- From $t = 25$ s to $t = 30$ s, a constant disturbance of $1\ m/s^2$ is introduced on the leader's control input (acceleration). Since the disturbance is an external input, it is not communicated to the follower and can possibly propagate along the platoon.
- From $t = 30$ s to $t = 60$ s, a sinusoidal disturbance affects the control input (acceleration) of the leader, while its speed reference is piece-wise constant.

Still for the sake of comparison, eigenvalues of the closed-loop discrete-time system in (7) (i.e., of the matrix $A_{T_m} + B_{T_m}F_e$) coincide with the exact discretization of the continuous-time (CT) ones in [5]. However, we remark that the CT controller in [5] works in favored circumstances itself, as the information flow is continuously and synchronously exchanged over time; the input can be arbitrarily changed at all time instants. In addition, we highlight that the control in [5] consists of a filter aimed at smoothing the collected macroscopic function and that, hence, guarantees an instrinc filtering action over disturbances. In the current discrete time setting, the smoothing filter is not needed so that a static controller is enough. In the following, a color scale from dark to light blue is used to represent the pairs of vehicles of the platoon from the head pair $(0,1)$ (dark blue) to the tail one $(N-1,N)$ (light blue). In the Figures, we denote by SD the sampled-data case.

Figs. 1, 2 and 3 show the distances, differences of speeds and macroscopic variables with respect to case a). Due to the disturbance acting over, after 25 s, the leader can no longer keep the desired trajectory. Nonetheless, it succeeds to properly counteract to the speed disturbance even if a steady-state error is generated. Still, each follower in the platoon can track the desired distance and speed, when the disturbance is acting on the leader (after 25 s) and it is not (from 0 to 25 s). As a matter of fact and as one might expect, we observe that only vehicles 8 and 9 do not track the desired distance and speed differences in the first phase; i.e., when disturbances are off and there is an initial displacement of the vehicle 8 from the equilibrium. In the remaining of the simulation, the error with respect to the desired values has a magnitude smaller than 10^{-5}. This is a clear improvement with respect to the CT case, as Figs. 4 and 5 testify. Indeed, in the latter case, the effect of disturbances over the leader affect the behavior of the whole platoon (4) contrarily to the sampled-data case (see [5] for more details).

Figs. 6, 7 and 8 show the results of simulations in case b), where the sampling instants for microscopic and macroscopic information are distinct. Here, the tracking performances are similar to case a); clearly, transients are less smooth with respect to the latter. This result was readily apparent, since the updates on the macroscopic information take place less often. Nevertheless, the controller produces good results also with respect to the transients. The dynamics of the macroscopic information of case a) in Fig. 3 and of the case b) in Fig. 8 are similar. Simulations support the theoretical investigations. The mesoscopic controller in (16) solves Problem 1 with performances that are comparable with the ones of the continuous-time controller proposed in [5], despite the effect of sampling and holding devices.

V. Conclusions

In this work, we proposed a new sampled-data control law capable of enforcing string stability over a platoon of heterogeneous vehicles, independently on the length of the involved sampling periods. We highlight that the proposed approach extends, along the same lines, to the case in which Assumption 3 is weakened to demand $T_M = MT_m + r$ with $r \in [0, T_m)$. Current work is addressing the case of asynchronous vehicles (i.e., each with its own sampling period).

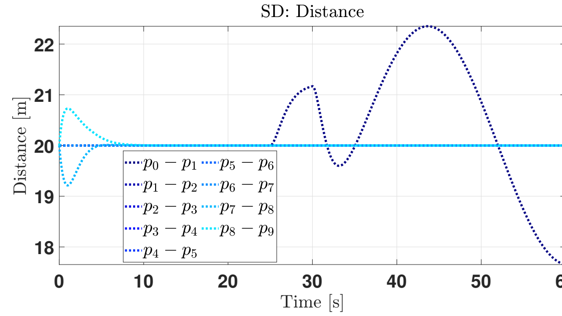

Fig. 1. SD, Case a): Distances. The color scale from dark to light blue represents the inter-vehicular distances between the vehicles of the platoon from the head pair $(0,1)$ (dark blue) to the tail one $(N-1,N)$ (light blue).

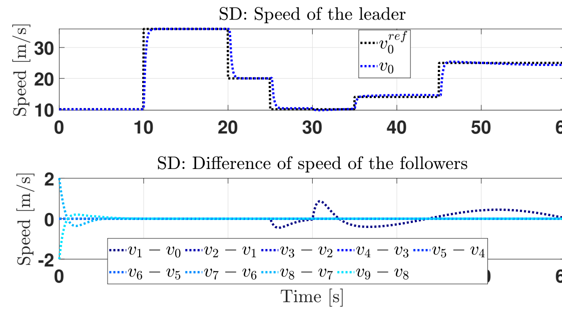

Fig. 2. SD, Case a): the leader's Speed and the Speed differences. In the upper part, the blue line represents the speed of the leader with respect to its desired reference (black line). In the lower part, the color scale from dark to light blue represents the inter-vehicular distances between the vehicles of the platoon from the head pair $(0,1)$ (dark blue) to the tail one $(N-1,N)$ (light blue).

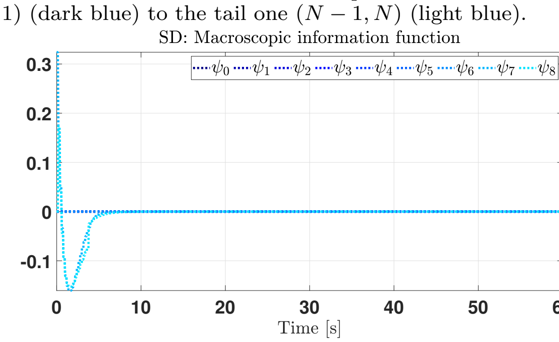

Fig. 3. SD, Case a): The macroscopic information. The color scale from dark to light blue represents the macroscopic variable ψ_i associated to the i-th vehicles of the platoon from $i=0$ to $i=N$.

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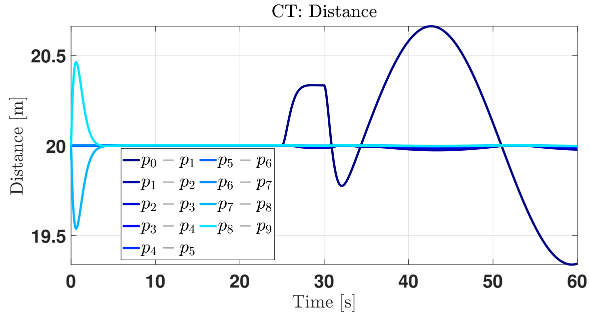

Fig. 4. Continuous time: Distances. Same settings as in Fig. 1.

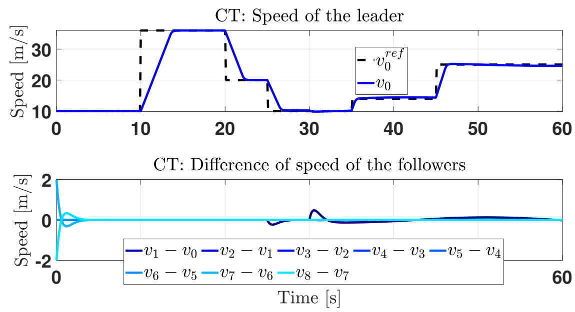

Fig. 5. Continuous time: the Speed of the leader and the Speed differences. The settings are the same as in Fig. 2.

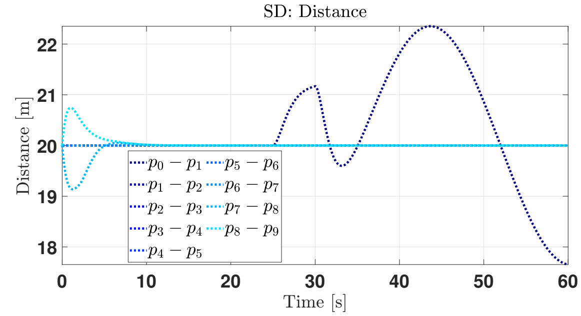

Fig. 6. SD, Case b): Distances. Same settings as in Fig. 1.

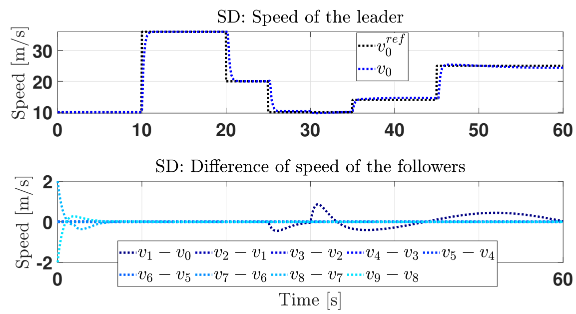

Fig. 7. SD, Case b): the Speed of the leader and the Speed differences. Same settings as in Fig. 2.

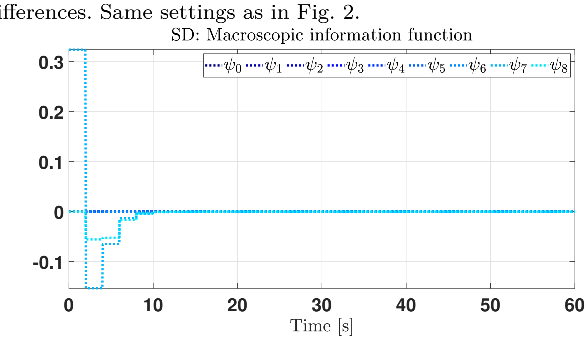

Fig. 8. SD, Case b): The macroscopic information. Same settings as in Fig. 3.

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