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Efficient Demand Management For The On-Time Arrival Problem: A Convexified Multi-Objective Approach Assuming Macroscopic Traffic Dynamics

C. Menelaou, S. Timotheou, P. Kolios and C.G. Panayiotou

Abstract— **This work proposes a novel solution approach to address the On-Time Arrival (OTA) problem, considering macroscopic traffic dynamics. The OTA problem is formulated as a nonconvex, nonlinear, multi-objective optimization problem considering two objective criteria. The first criterion aims at minimizing the travel time of all drivers in the network to prevent congestion, while the second criterion seeks to minimize the discrepancy between the desired and actual arrival time. The proposed formulation is solved efficiently through an approximated convex solution that leverages the Normal Boundary Intersection (NBI) method to efficiently generate a representative sample of the Pareto Front. Additionally, a solution methodology based on the Nash Bargaining Game is proposed to select a unique solution across all the Pareto points. Finally, simulation results demonstrate that the proposed solution can eliminate congestion while ensuring that most drivers will arrive at their destination on their desired time.**

I. Introduction

Despite the remarkable technological advancements and the plethora of traffic management schemes available today, traffic congestion remains a persistent problem [1] that is expected to worsen. The main reason for this is that the majority of these schemes are focused on improving the needs of individuals rather than the system optimum [2]. However, recent research has demonstrated that an effective way to address traffic congestion is through the integration of traffic and demand management strategies [3]. By combining these approaches, traffic flows can be redistributed in both time and space domains to optimize the overall system performance [4].

Traffic demand management strategies primarily aim to manage the inflow rate of vehicles into the network by influencing drivers to choose earlier/later trip departure times or alternative modes of transport [5]. Similarly, several demand management approaches have been introduced for the On-Time Arrival (OTA) problem, many of which are combined with route guidance [6]. The On-Time Arrival (OTA) problem is concerned with determining optimal departure times and routes for drivers, such that the likelihood of their on-time arrival is maximized while minimizing the risk of arriving too early or too late at their destination [7]. Another OTA approach is to consider macroscopic traffic dynamics [8]. Macroscopic dynamics are expressed using the three traffic parameters of speed, flow, and density, which are used to construct the Macroscopic Fundamental Diagram (MFD) [9]. Recently, many efforts have focused on using macroscopic traffic dynamics for demand management purposes, as in the work presented in [10], which developed a multi-regional demand management method to shift drivers' departure times.

This work proposes a multi-objective formulation for the OTA problem that considers macroscopic traffic dynamics. The objective is to minimize the difference between the actual and desired arrival times for each OD pair and at the same time to avoid the emergence of congestion. To generate a representative sample of the Pareto front, the Normal Boundary Intersection (NBI) method is used, which employs a scalarization technique that leads to generation of a near-uniform spread of the Pareto points. To ensure computational efficiency, a convex formulation of the NBI that approximates all nonlinear nonconvex constraints with convex ones is proposed. Additionally, this work introduces a methodology for identifying a single Pareto optimal solution based on the Nash bargain theory [11]. The Nash bargain solution identifies the Pareto point that maximizes the product of the net benefit of the two conflicting objective functions, enabling the identification of the "best" Pareto solution by solving a single optimization problem over all the points of the Pareto Front. Overall, the contribution of this work to the MFD-based OTA problem, are:

- Reformulates the OTA problem to generate a representative Pareto frontier using the NBI method. To address the nonconvexity of this problem, this work introduces an approximate linear solution that provides a lower-bound solution to the original nonconvex problem.
- Introduces a rolling-horizon algorithmic approach [12] that projects the lower-bound solution into the feasible domain of the original non-convex problem. This projection results in an upper-bound solution.
- Provides an optimization procedure that leverages the Nash bargaining game theory [11] to pinpoint the "optimal" trade-off solution among all the Pareto points.

The rest of this paper is structured as follows. Section II introduces the demand and traffic flow models, which

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are presented in subsections II-A and II-B, respectively. Section III describes the Pareto generation procedure for the OTA problem, while, Section IV, mathematically formulates the Nash solution that selects the "best" trade-off over all generated Pareto points. Section V evaluates the proposed methodologies, and finally Section VI concludes this work and discusses potential areas for future research.

II. Methodology

A. Demand Model

Let an urban area partitioned into a set of regions, i.e., $\mathcal{R} = \{1, \dots, R\}$ with the sets $\mathcal{O}$ and $\mathcal{D}$ denote the set of regions considered as origins, $\mathcal{O} \subseteq \mathcal{R}$ and destinations $\mathcal{D} \subseteq \mathcal{R}$, respectively. This work assumes that vehicular flows that intent to use the road infrastructure communicate beforehand the time that they desired to arrive at their destination. Let, $d^D_{od}(k)$ denote the number of vehicles traveling from $o \in \mathcal{O}$ to $d \in \mathcal{D}$ and they desire to arrive at d during time-step $k \in \mathcal{K}$, where the set $\mathcal{K} = \{1, 2, \dots, K\}$ denotes the considered time horizon.

To model demand dynamics we introduce the variables $\tilde{d}_{od}(k)$ and $D_{od}(k)$, representing the *admitted external demand* and *remaining external demand*, from $o \in \mathcal{O}$ to $d \in \mathcal{D}$, during time-step k, respectively. Hence, the admitted external demand denotes the number of vehicles that actually enter o towards d at time-step k, that is limited by the factors of:

1) The capacity of the region to serve more vehicles.
2) The maximum possible demand that can physically enter region $o \in \mathcal{O}$, defined by the parameter D^{MAX}_o.
3) The traffic management scheme that manages the entry rate of vehicles.

Furthermore, the remaining external demand denotes the number of vehicles that are remaining as they are not yet admitted to enter the network. To keep track of the demand that is going to be served during the following time-steps, the dynamics of the remaining external demand at time-step k are mathematically expressed as:

$$D_{od}(k+1) = D_{od}(k) - \tilde{d}_{od}(k), \tag{1}$$

and we make the following assumptions:

- The demand is known beforehand (i.e., $D_{od}(0) = \sum_k d^D_{od}(k)$),
- At the end of the time horizon all the vehicles will be served ($D_{od}(K) = 0$).

B. Traffic Flow Model

The dynamics within each region $r \in \mathcal{R}$ are characterized by the parameters of: the *jam density*, ρ^J_r, *critical density* ρ^C_r, *free-flow speed* u^f_r, *capacity*, $q^C_r = \rho^C_r u^f_r$, *total region length* L_r, *the average trip length* l_r and their ratio $\zeta_r = L_r/l_r$ within region $r \in \mathcal{R}$. The macroscopic modeling framework is complemented by the Macroscopic Fundamental Diagram (MFD) [9] in which the *average flow* $f_r(\rho_r(k))$ veh/h of region r is the product of the region's *density* $\rho_r(k)$ veh/km and *speed* $u_r(\rho_r(k))$ km/h at each time-step k, i.e., $f_r(\rho_r(k)) = \rho_r(k)u_r(\rho_r(k))$. The average flow of a region is derived through the asymmetric unimodal triangular MFD defined as:

$$f_r(\rho_r(k)) = \begin{cases} \dfrac{q^C_r}{\rho^C_r}\rho_r(k), \text{ if } 0 \le \rho_r(k) \le \rho^C_r, \\ w_r(\rho^J_r - \rho_r(k)), \text{ otherwise} \end{cases} \tag{2}$$

where $w_r = q^C_r/(\rho^J_r - \rho^C_r)$ is the congestion propagation speed [9]. Using the average flow we can measure the outflow of a region as follows:

$$q_r(k) = f_r(\rho_r(k))\zeta_r = u_r(\rho_r(k))\rho_r(k)\zeta_r. \tag{3}$$

Furthermore, let variable $\rho_{ord}(k)$ denote the density that currently is in $r \in \mathcal{R}$ that originated from $o \in \mathcal{O}$ destined to $d \in \mathcal{D}$ such that:

$$\rho_r(k) = \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} \rho_{ord}(k). \tag{4}$$

Similarly, the variables $q_{ord}(k)$ and $q_{orjd}(k)$ denote the *transfer flow* originating from $o \in \mathcal{O}$ towards $d \in \mathcal{D}$ and currently in $r \in \mathcal{R}$ and the corresponding flow in region $r \in \mathcal{R}$ originating from $o \in \mathcal{O}$ destined to $d \in \mathcal{D}$ that passes through neighbouring region $j \in J_r$, respectively, such that:

$$q_{ord}(k) = \frac{q_r(k)}{\rho_r(k)}\rho_{ord}(k) = u_r(\rho_r(k))\rho_{ord}(k)\zeta_r \tag{5}$$

$$q_{ord}(k) = \sum_{j\in\mathcal{J}_r} q_{orjd}(k), \tag{6}$$

$$q_r(k) = \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} q_{ord}(k). \tag{7}$$

where $\mathcal{J}_r$ is defined as

$$\mathcal{J}_r = \begin{cases} \mathcal{J}^-_r \cup \{r\}, \text{ if } r \in \mathcal{D} \\ \mathcal{J}^-_r, \text{ otherwise,} \end{cases} \tag{8}$$

and $\mathcal{J}^-_r \subseteq \mathcal{R}$ is the set of neighboring regions of region $r \in \mathcal{R}$. Interestingly, the amount of flow of vehicles that finish their trip (exit the network) in region $d \in \mathcal{D}$ at time-step k are expressed by the variable $q_{oddd}(k)$ as in that case $r = j = d$, $d \in \mathcal{D}$.

Moreover, in macroscopic modeling the maximum flow that can be transferred between neighboring regions $r \in \mathcal{R}$ and $j \in \mathcal{J}^-_r$ is limited by the inter-boundary capacity, $C_{rj}(\rho_j(k))$ which is stated as the maximum transfer flow between two adjacent neighboring regions r and $j \in \mathcal{R}$ as follows:

$$C_{rj}(\rho_j(k)) = \begin{cases} C^{\text{MAX}}_{rj}, \text{ if } \rho_j(k) \le \alpha\rho^J_j, \\ \dfrac{C^{\text{MAX}}_{rj}}{1-\alpha}(1 - \dfrac{\rho_j(k)}{\rho^J_j}), \text{ otherwise,} \end{cases} \tag{9}$$

where C^{MAX}_{rj} is the maximum inter-boundary capacity and $\alpha\rho^J_j$ is the point where the inter-boundary capacity begins to decline with $0 < \alpha < 1$, [13]. Hence, the *actual transfer flow* from $r \in \mathcal{R}$ to $j \in \mathcal{J}_r$, $\tilde{q}_{orjd}(k)$ is restricted by the remaining capacity of the neighboring regions which is mathematically expressed as follows:

$$\tilde{q}_{orjd}(k) = \min\Bigg(q_{orjd}(k),$$

$$C_{rj}(\rho_j(k))\frac{q_{orjd}(k)}{\sum_{o\in\mathcal{O}}\sum_{y\in\mathcal{D}}q_{orjy}(k)}\Bigg). \quad (10)$$

Considering all the above, the dynamics of density in region $r \in \mathcal{R}$ that originated from $o \in \mathcal{O}$ towards $d \in \mathcal{D}$, can be expressed as:

$$\begin{aligned}\rho_{ord}(k+1) =& \rho_{ord}(k) + \frac{1}{L_r}\tilde{d}_{od}(k) \\ &+ \frac{T_s}{L_r}\sum_{j\in\mathcal{J}_r}\Big(\tilde{q}_{ojrd}(k) - \tilde{q}_{orjd}(k)\Big), \quad (11)\end{aligned}$$

where, T_s denotes the simulation time-step that governs the evolution of dynamics.

C. Objective function

The multi-regional OTA problem aims to regulate departure times and determine the multi-regional routes for all vehicular flows within the road network. This is done with the goal of minimizing two objectives: (i) the On-Time Arrival (OTA) (J_{OTA}), which represents the discrepancy between the time that vehicles desire to arrive and actually arrive at the destination; and (ii) the Total Traveling Time spent (TTT) (J_{TTT}), which represents the total time spent in the network. To formally state both objective criteria let variables $S^a(k)$ and $S^b(k)$ represent the cumulative number of vehicles admitted to the network, and successfully arrive at their destination, respectively, such that:

$$S^a(k+1) = S^a(k) + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\tilde{d}_{od}(k), \quad (12)$$

$$S^b(k+1) = S^b(k) + T_s\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\tilde{q}_{oddd}(k), \quad (13)$$

where $S^a(0) = 0$ and $S^b(0) = 0$. Moreover, let variables $S^c_{od}(k)$ and $S^d_{od}(k)$ represent the cumulative number of vehicles traveling from $o \in \mathcal{O}$ and desire to arrive at $d \in \mathcal{D}$ up-to time-step k, and traveling from $o \in \mathcal{O}$ and actually arriving at $d \in \mathcal{D}$ at time-step k, respectively, such that:

$$S^c_{od}(k+1) = S^c_{od}(k) + d^D_{od}(k),\ o\in\mathcal{O}, d\in\mathcal{D}, \quad (14)$$

$$S^d_{od}(k+1) = S^d_{od}(k) + T_s\tilde{q}_{oddd}(k),\ o\in\mathcal{O}, d\in\mathcal{D}, \quad (15)$$

where $S^c_{od}(0) = 0$ and $S^d_{od}(0) = 0$. Using the above definitions, the J_{OTA} objective is expressed as the absolute value of the difference between the cumulative number of vehicles that desire to arrive at $d \in \mathcal{D}$ on or before time-step k, and those that actually arrive on time-step k, over all time-steps, such that:

$$J_{OTA} = \sum_{k\in\mathcal{K}}\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}|S^c_{od}(k) - S^d_{od}(k)|. \quad (16)$$

While, the J_{TTT}, is expressed as the sum of the difference between the cumulative number of vehicles admitted to enter the network, and successfully arrive at their destination, over all time-steps, such that:

$$J_{TTT} = \sum_{\mathcal{K}}\Big(S^a(k) - S^b(k)\Big). \quad (17)$$

Finally, the Combined Objective Criterion, (J_{COC}) is defined as the sum of the two above criteria, such that:

$$J_{COC} = J_{OTA} + J_{TTT}. \quad (18)$$

D. Problem Formulation

The OTA problem aims to regulate the admission of vehicles $\tilde{d}_{od}(k)$ and transfer flows $q_{orjd}(k)$ of each region in such a way that the J_{COC} is minimized,. The formulation of the OTA problem is mathematically expressed as follows:

$$\begin{aligned}(\mathrm{P}_1)\quad \min\ J_{COC} = \sum_{k\in\mathcal{K}}\Bigg(\Big(S^a(k) - S^b(k)\Big) + \\ \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}|S^c_{od}(k) - S^d_{od}(k)|\Bigg) \quad (19a)\end{aligned}$$

s.t. Demand and Traffic Dynamics: (1) – (15),

$$\sum_{d\in\mathcal{D}}\tilde{d}_{od}(k) \le D^{MAX}_o,\ k\in\mathcal{K}, o\in\mathcal{O}, d\in\mathcal{D}, \quad (19b)$$

$$\tilde{d}_{od}(k) \le D_{od}(k),\ k\in\mathcal{K},\ o\in\mathcal{O},\ d\in\mathcal{D}, \quad (19c)$$

$$0 \le \rho_r(k) \le \rho^J_r,\ k\in\mathcal{K},\ r\in\mathcal{R}, \quad (19d)$$

$$\sum_k d^D_{od}(k) = T_s\sum_{k\in\mathcal{K}}\tilde{q}_{oddd}(k),$$

$$k\in\mathcal{K},\ o\in\mathcal{O},\ d\in\mathcal{D}, \quad (19e)$$

$$D_{od}(K) = 0,\ o\in\mathcal{O},\ d\in\mathcal{D}, \quad (19f)$$

$$D_{od}(0) = \sum_{k\in\mathcal{K}}d^D_{od}(k),\ o\in\mathcal{O},\ d\in\mathcal{D}. \quad (19g)$$

In problem P_1, constraints (2) - (15) define the demand and traffic dynamics while constraints (19b) and (19c) sustain the total admitted external demand for all destinations smaller than, the maximum possible external demand, D^{MAX}_o and remaining external demand, $D_{od}(k)\ \forall k \in \mathcal{K}$. Constraint (19d) keeps the density of each region within its physical limits while constraints (19e) and (19f) ensure that all vehicle requests will be served within the considered time horizon. Finally, constraint (19g) ensures that the admitted demand will not surpass the total number of requests. Problem P_1 is a nonconvex nonlinear model since it involves the nonlinear functions Eqs. (2), (9) and (10) a bilinear term of Eq. (5).

III. Generation of the Pareto front

In this work, the modified Normal Boundary Intersection (NBI) method proposed in [14] is used to generate a uniformly spread and smooth sample of the Pareto Front.

A. Normal Boundary Intersection Method for the OTA problem

To simplify notation, let us denote a feasible solution to problem P_1 by the vector $\mathbf{x}$. The functions $J_{OTA}(\mathbf{x})$ and $J_{TTT}(\mathbf{x})$ represent the corresponding values of the objectives J_{OTA} and J_{TTT} that can be attained with solution vector $\mathbf{x}$. Furthermore, let J^*_{TTT} and J^*_{OTA} signify the individual minima of the two objectives of problem P_1, achieved at $\mathbf{x}^*_{TTT}$ and $\mathbf{x}^*_{OTA}$, respectively. The individual minimum of each criterion represents the minimum value that each objective can achieve, and the vectors $\mathbf{x}^*_{OTA}$ and $\mathbf{x}^*_{TTT}$ represent their solution such that, $J^*_{TTT} = J_{TTT}(\mathbf{x}^*_{TTT})$ and $J^*_{OTA} = J_{OTA}(\mathbf{x}^*_{OTA})$. To encapsulate the cross-evaluation of the objectives at these optimal solutions, we introduce the matrix $\mathbf{\Phi}$, a 2×2 matrix defined as:

$$\mathbf{\Phi} = \begin{pmatrix} J_{OTA}(\mathbf{x}^*_{OTA}), & J_{TTT}(\mathbf{x}^*_{OTA}) \\ J_{OTA}(\mathbf{x}^*_{TTT}), & J_{TTT}(\mathbf{x}^*_{TTT}) \end{pmatrix}.$$

which encapsulates the values of the objective functions of problem P_1 when evaluated at each other's individual minimum. Moreover, considering that both objective criteria are non-negative, the convex hull of the individual minimum can be defined with the use of the bilinear term $\mathbf{\Phi}\boldsymbol{\beta}$, where $\boldsymbol{\beta} = \{(b_1, b_2)|\, b_1 + b_2 = 1\}$ is a given vector parameter. Then, the NBI method can generate a representative Pareto Front of P_1 by searching for the maximum distance (i.e., t) along the normal pointing toward the origin by solving the following optimization problem,

$$(\mathrm{P}_2) \quad \max\ t \tag{20a}$$

$$\text{s.t. } \mathbf{\Phi}\boldsymbol{\beta} + t\hat{n} \geq \sum_{k\in\mathcal{K}} \Bigg(\big(S^a(k) - S^b(k)\big) + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} |S^c_{od}(k) - S^d_{od}(k)| \Bigg) \tag{20b}$$

Demand and Traffic Dynamics: (1) – (15),

Constraints: (19b) – (19g),

where, $\hat{n} = -\mathbf{\Phi}\mathbf{v}$[1] and $\mathbf{v}$ is a vector with strictly positive components (e.g., $\mathbf{v} = (1,1)^T$). Therefore, a set of uniform distributed Pareto points can be generated by evaluating a set of different equidistant $\boldsymbol{\beta}$ vectors within the problem P_2.

B. Linear relaxed Pareto generation for the OTA Problem

To generate the Pareto Frontier, multiple runs of the nonlinear nonconvex problem of P_2 should be performed. Problem P_2 contains all the nonlinear constraints of P_1.Thus, obtaining a directed solution for P_2 requires the use of nonlinear solver, which cannot offer real-time execution and optimality guarantees. To solve P_2 efficiently using standard linear solvers, this section proposes a set of linear relaxations to relax P_2 into an approximate convex program.

Lets first consider Eq. (2), which consists of two linear segments that intersect at the critical density that can be relaxed into two linear constraints by replacing the equality sign "=" with inequality "≤" such that:

$$f_r(\rho_r(k)) \leq \frac{q_r^C}{\rho_r^C}\rho_r(k), \tag{21}$$

$$f_r(\rho_r(k)) \leq w_r(\rho_r^J - \rho_r(k)). \tag{22}$$

Note that the combination of the two linear inequalities of Eq. (21) and (22) generate a superset of Eq. (2). Then, considering that the shortest travel times can be achieved only in the case that vehicles are cruising with free-flow speed, then the bilinear term in Eq. (5) can be relaxed by:

$$q_{ord}(k) \leq u_r^f \rho_{ord}(k)\zeta_r. \tag{23}$$

[1]The $\hat{n}$ is the normal direction at the point $\mathbf{\Phi}\boldsymbol{\beta}$ pointing towards the origin.

where it is true that $u_r(\rho_r(k)) \leq u_r^f$ for all densities $\rho_r(k)$. Furthermore, Eq. (10) returns the minimum between two functions and can be relaxed as follows:

$$\tilde{q}_{orjd}(k) \leq q_{orjd}(k) \tag{24}$$

$$\tilde{q}_{orjd}(k) \leq C_{rj}(\rho_j(k))\frac{q_{orjd}(k)}{\sum_{y\in\mathcal{D}} q_{orjy}(k)}. \tag{25}$$

where Eq. (25) is still nonlinear and it can be further relaxed into a linear function by taking its summation over all $\tilde{q}_{orjd}(k)$ for $d\in\mathcal{D}$ such that:

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} \tilde{q}_{orjd}(k) \leq C_{rj}(\rho_j(k)). \tag{26}$$

Similarly with Eq. (2), Eq. (9) consisted of two linear functions that intersect at the point $a\rho_j^J$ that can be relaxed into two linear constraints by replacing the equality sign "=" with the inequality "≤" such that:

$$C_{rj}(\rho_j(k)) \leq C_{rj}^{MAX}, \tag{27}$$

$$C_{rj}(\rho_j(k)) \leq \frac{C_{rj}^{\mathrm{MAX}}}{1-\alpha}\Big(1 - \frac{\rho_j(k)}{\rho_j^J}\Big). \tag{28}$$

Moreover, taking into account the shape of Eq. (9) the maximum transfer flow between two regions can be bounded from above by considering the linear inequality as follows:

$$C_{rj}(\rho_j(k)) \geq \frac{-C_{rj}^{MAX}}{\rho_j^J}(\rho_j(k) - \rho_j^J). \tag{29}$$

Moreover considering the relaxation of Eq. (26) can be combined and handled together with constraints (27)-(29) such that:

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} \tilde{q}_{orjd}(k) \leq C_{rj}^{MAX}, \tag{30}$$

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} \tilde{q}_{orjd}(k) \leq \frac{C_{rj}^{\mathrm{MAX}}}{1-\alpha}\Big(1 - \frac{\rho_j(k)}{\rho_j^J}\Big), \tag{31}$$

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} \tilde{q}_{orjd}(k) \geq \frac{-C_{rj}^{MAX}}{\rho_j^J}(\rho_j(k) - \rho_j^J), \tag{32}$$

for all $k\in\mathcal{K}$, $o\in\mathcal{O}$, $r\in\mathcal{R}$, $d\in\mathcal{D}$, $j\in\mathcal{J}_r$. In this way the Eqs. (10) and (9) can be approximated together by the convex envelop consisted of the linear segments of constraints (30)- (32). Finally, the constraint as expressed in (20b), which incorporates an absolute value, can be reformulated into an equivalent linear representation. This is achieved by introducing the slack variable $\psi_{od}(k)$, thereby allowing the replacement of (20b) with the following linear constraints:

$$\mathbf{\Phi}\boldsymbol{\beta} + t\hat{n} - \sum_{k\in\mathcal{K}}\big(S^a(k) - S^b(k)\big) \geq \sum_{k\in\mathcal{K}}\Bigg(\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\psi_{od}(k)\Bigg), \tag{33}$$

$$\psi_{od}(k) \geq \big(S^c_{od}(k) - S^d_{od}(k)\big),\ k\in\mathcal{K},\ o\in\mathcal{O},\ d\in\mathcal{D}, \tag{34}$$

$$\psi_{od}(k) \geq -\big(S^c_{od}(k) - S^d_{od}(k)\big),\ k\in\mathcal{K},\ o\in\mathcal{O},\ d\in\mathcal{D}. \tag{35}$$

Considering all the above relaxations, the problem P_2 can be approximated by a linear program by replacing constraints (2), (5), (9), (10) and (20b) with (21)-(24), (30)-(32) and (33)-(35). Hence, the mathematical formulation of the relaxed version of problem P_2 is given in the problem P_3 as follows:

$$(P_3) \quad \max\ t \tag{36}$$

$$\begin{aligned} \text{s.t. } & \text{Demand and Traffic Dynamics: } (1), (4), (6)-(8), \\ & (11)-(15), (21)-(24), \text{ and } (30)-(32), \\ & \text{Constraints: } (19b)-(19g) \text{ and } (33)-(35). \end{aligned}$$

The formulation of P_3 is a linear program that offers a relaxed solution of the original nonlinear nonconvex problem P_2. The resulting relaxation is a lower-bound of the optimal objective value of problem P_2, that may lead to infeasible solutions as may not satisfy some of the original constraints. This is due to the fact that there is a discrepancy between the actual nonlinear problem of P_2 and the approximate solution of P_3. This discrepancy can lead to poor performance and solutions far away from optimality.

C. Feasible upper-bound solution of the OTA Problem

To achieve a feasible upper-bound solution, this work introduces a rolling horizon algorithmic approach. The algorithm incrementally solves the relaxed version of problem P_3 over fixed M time intervals considering in three main steps.

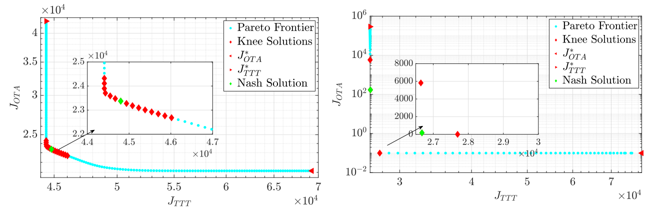
In the first step, the relaxed version of problem P_3 is solved for the time horizon including the time-steps $k = \big(M(\eta+1), \dots, |\mathcal{K}|\big)$, with η serving as the current interval index. The resulting solution is then projected onto the feasible domain of the original nonconvex problem, P_2. This projection yields feasible control decisions, by endorsing the utilization of split ratios rather than the transfer flows for the route guidance control decisions. The rationale for this preference lies in the inherent nature of split ratios as relative measures. Specifically, the split ratios $\gamma_{orjd}(k) \in [0,1]$ are defined as:

$$\gamma_{orjd}(k) = \begin{cases} \dfrac{\tilde{q}_{orjd}(k)}{\sum_{j\in\mathcal{J}_r} \tilde{q}_{orjd}(k)}, & \text{if } \sum\limits_{j\in\mathcal{J}_r} \tilde{q}_{orjd}(k) \neq 0 \\ 1/|\mathcal{J}_r|, & \text{if } \sum\limits_{j\in\mathcal{J}_r} \tilde{q}_{orjd}(k) = 0, \end{cases} \tag{37}$$

which proportionally allocate flows to adjacent regions.

The second step involves evaluating the control decisions (i.e., split ratios and admitted demand) determined in the first step, using a macroscopic simulation model that reflects the nonlinear MFD dynamics, as described by Eqs. (1)-(15). This step aims to enhance the initial solution's quality, thereby deriving feasible upper-bound control decisions, namely $\tilde{d}^*_{od}(k)$ and $\gamma^*_{orjd}(k)$, for $k = \big(M(\eta+1), \dots, M(\eta+2)\big)$.

The third and final step utilizes the simulated states from the second step to update the new initial conditions for all optimization-related variables for subsequent intervals. Thus, in each interval,the algorithm leverages the most recent simulated states to update the initial conditions for the variables $\rho_r(k)$, $\rho_{ord}(k)$, $S^a(k)$, $S^b(k)$, $S^c_{od}(k)$ and $S^d_{od}(k)$, for $k = M(\eta+2)$. This rolling horizon update is crucial to ensure that the initial values of the objective function are accurately propagated to subsequent intervals. Therefore, based on the solutions obtained from Step 2 during each iteration, we proceed to update the matrix $\mathbf{\Phi}$. This procedure is repeated throughout the entire planning horizon, advancing in increments of M time steps, and pushing the boundary of the solution space towards feasibility.



(a) (b)

Fig. 1. The Pareto front generated by applying the NBI method for : (a) the normal and (b) the congested demand scenarios.

IV. Nash Pareto optimal solution

A set of uniform distributed Pareto points can be generated by evaluating a set of different equidistant $\boldsymbol{\beta}$ vectors using the upper-bound feasible solution of Algorithm **??**. The Nash bargaining solution [11] constitutes one of the solutions that offer an appropriate trade-off among the two objective criteria J_{TTT} and J_{OTA}. Each solution over the Pareto Front is defined by the coordinates $\big(J^{PF_i}_{OTA}, J^{PF_i}_{TTT}\big)$, where, the superscript PF_i denotes the i^{th} generated point across the Pareto. Accordingly, the Nash bargaining solution is the Pareto point i.e., $\big(J^{PF^*}_{OTA}, J^{PF^*}_{TTT}\big)$, that maximizes the total benefit of both objective criteria (J_{TTT} and J_{OTA}). Hence, the Nash bargaining solution is the unique point that maximizes the product of individual net benefit gains from the threat point of the non-cooperative Nash equilibrium. The threat point is defined the worst case solution for each objective function that happens only in case that one objective is optimized without considering the other. Hence, the threat point of Total Traveling Time objective is attain at $\mathbf{x}^*_{OTA}$ where the individual minimum of the On-Time Arrival objective is achieve i.e., $J_{TTT}(\mathbf{x}^*_{OTA})$ and in the same manner the threat point of On-Time Arrival objective attain at $\mathbf{x}^*_{TTT}$, i.e., $J_{OTA}(\mathbf{x}^*_{TTT})$. Considering all the above, the Nash bargain solution selects the unique solution to the following maximization problem:

$$(P_4) \quad \max\ \big(J_{OTA}(\mathbf{x}^*_{TTT}) - J^{PF}_{OTA}\big)\big(J_{TTT}(\mathbf{x}^*_{OTA}) - J^{PF}_{TTT}\big) \tag{38}$$

$$\begin{aligned} \text{s.t. } & J^{PF}_{OTA} \leq J_{OTA}(\mathbf{x}^*_{TTT}) \\ & J^{PF}_{TTT} \leq J_{TTT}(\mathbf{x}^*_{OTA}). \end{aligned}$$

The problem P_4 returns the solutions that maximizes the product of $\big(J_{OTA}(\mathbf{x}^*_{TTT}) - J^{PF}_{OTA}\big)\big(J_{TTT}(\mathbf{x}^*_{OTA}) - J^{PF}_{TTT}\big)$ attain at the single pareto point $\mathbf{x}^*_{COC}$. The selected solution is the “best” trade-off solution as it minimizes the difference between the actual and the desired arrival time while also reducing the total time spent by all vehicles in the network.

V. Simulation Results

To evaluate the proposed framework, we considered an urban network consisted of 16 regions in which four regions

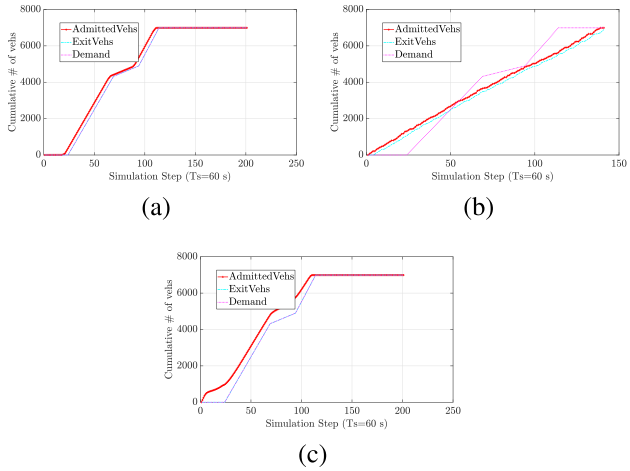


Fig. 2. The cumulative number of vehicles that are admitted to enter the network (Admitted vehicles), complete their trips (Exit vehicles), desire to arrive at their destination (Demand) as a function of time, for the congested demand scenario with: (a) J^*_{COC}; (b) J^*_{TTT}; and (c) J^*_{OTA}.

are considered as origins and four regions are considered as destinations. The traffic dynamics of each region are assumed to follow a triangular MFD model, with parameters: $\rho_r^C = 30$ veh/km, $\rho_r^J = 130$ veh/km, $L_r = 1$ km, $u_r^f = 60$ km/h, $q_r^C = 1800$ veh/h, $C_{rj}^{\text{MAX}} = 1800$ veh/h and $\alpha = 0.25$. The simulation time-step is selected equal to $T_s = 30$ s while the whole time horizon is set to $\mathcal{T} = 140$ min. Two demand scenarios are considered for evaluation purposes: (i) *normal* scenario with average demand around 2500 veh/h and (ii) *congested* with average demand around 4000 veh/h. The *normal* demand scenario refers to cases where vehicles can be served by the network without causing congestion, while the *congested* scenario refers to cases where congestion would occur without implementing any control policy.

Figures 1 (a) and (b) depict the Pareto Front generated by solving the formulation of P_3 for the normal and congested demand scenarios, respectively. Each scatter in the figures represents a Pareto solution to the approximated formulation of P_3 obtained by evaluating varying values of β ranging between $[0, 1]$. In particular, both figures depict the two special cases where only one out of the two objectives is optimized by the left- and right-pointing triangles. The left-pointing triangle indicates the case where J^*_{OTA} is minimized, while the right-pointing triangle indicates the case where J^*_{TTT} is minimized. Furthermore, the green rhombus represents the Nash Pareto solution, i.e., J^*_{COC} obtained by solving the P_4 problem, which offers the best trade-off among the two objective criteria. From the figures, we can observe that knee solutions are possible alternative solutions that can offer a good trade-off among the two criteria.

Figures 2 (a), (b), and (c) display the cumulative number of vehicles that have been admitted to enter the network (Admitted vehicles), completed their trips (Exit vehicles), and the number of vehicles that desire to arrive at their destinations (Demand) for the congested demand scenario, using the Pareto solutions of J^*_{COC}, J^*_{TTT}, and J^*_{OTA}, respectively. The comparison of the results in the Figure 2 (c) reveals that the J^*_{OTA} solution outperforms in terms of the number of vehicles that arrive at their destination on-time, but it has the poorest performance in terms of travel times reductions. Conversely, Figure 2 (b) shows that J^*_{TTT} provides better performance in terms of travel time savings but has the worst performance according to the OTA criterion, where an increase in demand results in the need to compromise the arrival times of some vehicles to maintain short travel times. Moreover, Figure 2 (a) indicate that, the J^*_{COC} solution offers a balanced trade-off between the two criteria by providing significant travel time savings while also accommodating the majority of vehicles to arrive on-time.

VI. Conclusions

This work presents the OTA problem that results in a nonconvex nonlinear multi-objective optimization program. An efficient solution is obtained through a convex relaxation method in which the NBI method is employed to generate a representative Pareto Front. Finally, the Nash bargain solution is used to select the best Pareto solution. This approach assumes that demand is known in advance, so future research will focus on developing a robust approach that can anticipate uncertainties in demand distributions.

References

[1] J. Macfarlane, "When apps rule the road: The proliferation of navigation apps is causing traffic chaos. it's time to restore order," *IEEE Spectrum*, vol. 56, no. 10, pp. 22–27, 2019.

[2] J. R. Correa, A. S. Schulz, and N. E. Stier-Moses, "Selfish routing in capacitated networks," *Mathematics of Operations Research*, vol. 29, no. 4, pp. 961–976, 2004.

[3] C. Menelaou, S. Timotheou, P. Kolios, and C. G. Panayiotou, "A linear formulation of the on-time arrival problem in multi-regional networks," in *2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC)*, 2022, pp. 3589–3594.

[4] G. Sammer, *Travel demand management and road user pricing: success, failure and feasibility*. Routledge, 2016.

[5] K. Luten, "Mitigating Traffic Congestion: The role of demand-side strategies," U.S. Department of Transport, Federal Highway Administration, Tech. Rep., 10 2004.

[6] C. Menelaou, S. Timotheou, P. Kolios, C. Panayiotou, and M. Polycarpou, "Scheduling vehicles for on-time arrival using route-reservations," in *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, 2019, pp. 1749–1754.

[7] E. D. Miller-Hooks and H. S. Mahmassani, "Least expected time paths in stochastic, time-varying transportation networks," *Transportation Science*, vol. 34, no. 2, pp. 198–215, 2000.

[8] C. F. Daganzo, "Urban gridlock: macroscopic modeling and mitigation approaches," *Transportation Research Part B: Methodological*, vol. 41, no. 1, pp. 49–62, 2007.

[9] L. Immers and S. Logghe, *Traffic flow theory*, Lecture Notes, Belgium, May 2003. [Online]. Available: https://www.mech.kuleuven.be

[10] S. Kumarage, M. Yildirimoglu, M. Ramezani, and Z. Zheng, "Schedule-Constrained Demand Management in Two-Region Urban Networks," *Transportation Science*, vol. 55, no. 4, pp. 857–882, jun 2021.

[11] K. Binmore, A. Rubinstein, and A. Wolinsky, "The nash bargaining solution in economic modelling," *The RAND Journal of Economics*, pp. 176–188, 1986.

[12] S. Sethi and G. Sorger, "A theory of rolling horizon decision making," *Annals of Operations Research*, vol. 29, no. 1, pp. 387–415, 1991.

[13] I. I. Sirmatel and N. Geroliminis, "Economic model predictive control of large-scale urban road networks via perimeter control and regional route guidance," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 4, pp. 1112–1121, April 2018.

[14] P. K. Shukla, "On the normal boundary intersection method for generation of efficient front," in *Computational Science – ICCS 2007*, Y. Shi, G. D. van Albada, J. Dongarra, and P. M. A. Sloot, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2007, pp. 310–317.