# A new control-oriented METANET model to encompass service stations on highways

Ayda Kamalifar, Carlo Cenedese, Michele Cucuzzella, Antonella Ferrara

*Abstract*— In this paper, we propose the METANET with service station (METANET-s) model, a second-order macroscopic traffic model that, compared to the classical METANET, incorporates the dynamics of service stations on highways. Specifically, we employ the (so-called) store-and-forward links to model the stop of vehicles and the possible queue forming in the process of merging back into the highway mainstream. We explore the capability of the METANET-s to capture well both traffic back propagation and capacity drops, which are typically caused by the presence of vehicles joining again the mainstream traffic from the service station. Therefore, capturing these effects is crucial to improving the model's predictive capabilities. Finally, we perform a comparative analysis with the Cell Transmission Model with service station (CTM-s), showcasing that the METANET-s describes the traffic evolution much better than its first-order counterpart.

#### I. INTRODUCTION

Nowadays, transportation experts face the complex challenge of balancing the urgency for sustainable solutions and the rising traffic demand due to the growing urbanization [1]. Traffic models and simulations play a fundamental role in addressing these challenges [2]. They empower experts to attain a profound understanding of the real issues within transportation systems, identify opportunities for enhancement, predict and measure the impacts of infrastructure developments [3]. Essentially, these tools enable informed decision-making and control systems contributing to the endeavor for a more sustainable and efficient transportation network [4].

Macroscopic first-order traffic models have a long and rich history, starting from the seminal Lighthill-Whitham-Richards (LWR) model [5], and the Cell Transmission Model (CTM), which corresponds to the discretized version of the LWR [6]. The CTM has been successfully applied in many applications, including simulation of large-scale motorway networks [7], dynamic traffic assignment [8], and traffic estimation [9]. A recent noteworthy extension of the CTM is the CTM-s, firstly proposed in [10], [11]. Specifically, the CTM-s endows the classical CTM with the dynamics

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necessary to model the presence of a Service Station (ST). The rapid grow of electric vehicles creates a shift in the use of STs by drivers. These facilities provide not only the possibility of charging and/or refueling but also improved ancillary services that have the effect of increasing the time spent by drivers during a stop [12]. Moreover, policymakers aim to have a service station every 60 km in the whole EU by 2030 [13], [14]. Therefore, the CTM-s has been proven to be a useful tool for planning [11] and studying the effect of STs on the evolution of traffic [10], [15]. Nonetheless, first-order traffic models, including CTM-s, exhibit one main limitation. Due to the lack of knowledge on the traffic velocity, these models are not able to capture complex traffic phenomena such as capacity drops or have to rely on convoluted tricks to somehow capture them [15].

To overcome this limitation, higher-order models can be adopted. In [16], the METANET model is introduced to describe also the speed evolution, enabling to capture instabilities such as stop-and-go waves, and capacity drops. In [17], [18], the METANET has been shown to describe the traffic flow much better than first-order models. In [19], [20], further refinements have been made to the METANET model to improve traffic state estimation and prediction. Finally, control strategies like model predictive control, ramp-metering, and Variable Speed Limit (VSL) have been explored in [21], [22], [23].

The main contributions of this paper are the following: *i)* we develop the METANET-s, a novel highway traffic network model that is capable of capturing the dynamics of STs on highways; *ii)* we incorporate the physical constraints of STs into the model, ensuring that the number of vehicles at a ST does not exceed the ST capacity; *iii)* we use fundamental concepts of the classical METANET ensuring that the model can be easily implementable by practitioners to predict traffic evolution and design decision strategies; *iv)* we show via numerical simulations that the METANET-s can describe complex phenomena such as capacity drops much better than the CTM-s.

## II. PRELIMINARIES ON THE CLASSICAL METANET MODEL

In this section, we introduce the key components of the classical METANET model as a cornerstone over which we build in the next section the proposed METANET-s model. In the following we adopt a similar notation as in [24].

The METANET model discretizes time into intervals of length  $T \in \mathbb{R}$  and indexed by  $k \in \mathbb{N}$ . The highway stretch is divided into  $N \in \mathbb{N}$  *links*. Each link  $m \in \mathbb{N} := \{1, \ldots, N\}$ 

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Fig. 1. Link discretization of a freeway stretch in METANET model

is divided into  $N_m$  sections composed of  $\lambda_m$  lanes of length  $L_m$ , which are used to better describe the variation of the link features. The set of all the sections of link  $m$  is denoted by  $N_m := \{1, \ldots, N_m\}$ ; see Fig. 1. Every two adjacent links in N are connected by *nodes* (blue dots in Fig. 1). We denote the total number of nodes by  $P$  and the corresponding set by  $P := \{1, ..., P\}$ . For each section  $i \in \mathbb{N}_m$  of link  $m \in \mathbb{N}$ , the following variables are defined:

- *density*  $\rho_{m,i}(k)$  [veh/km lane], the number of vehicles in the section  $i$  during the time interval  $k$  which divided by the length of the link  $L_m$  and by the number of lanes  $\lambda_m$ ;
- *mean speed*  $v_{m,i}(k)$  [km/h] of the vehicles moving along the section  $i$ ;
- *flow*  $q_{m,i}(k)$  [veh/h lane], the number of vehicles exiting section *i* through a lane  $\lambda_m$  during k and divided by T.

The METANET model can operate in two different modes: the *non destination-oriented*, and the *destination-oriented*. In the former, the traffic assignment—specifically the behavior of drivers choosing their routes—is not taken into account, while in the latter it is [25]. In the upcoming section, we employ the destination-oriented operation mode to distinguish between the behavior of vehicles that stop at the ST and those that do not.

To properly define the destination-oriented mode, the following additional variables are introduced:

- *partial traffic density*  $\rho_{m,i,j}(k)$  [veh/km lane], the density of vehicles in  $i \in \mathbb{N}_m$  with destination  $j \in J_m$ , where  $J_m \subseteq \mathbb{N}$  is the set of destination links that can be accessed from link  $m \in \mathbb{N}$ ;
- *composition rate*  $\gamma_{m,i,j}(k) \in [0,1]$ , is the fraction of traffic with destination  $j \in J_m$ .

In the METANET model the transportation network is described as a collection of nodes, describing junctions or bifurcations, connected by links. For the readers' convenience, we briefly introduce next the dynamics associated with the links and nodes (see e.g. [26] for further details).

# *A. The links*

The links in the METANET can be of four different types serving specific purposes and endowed with unique features. We discuss them hereafter.

*1) Freeway links:* are employed for homogeneous freeway stretches where the dynamics read as

$$
\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m} \left[ q_{m,i-1}(k) - q_{m,i}(k) \right]
$$
\n(1a)

$$
q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)
$$
 (1b)

$$
v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau} \left[ v(\rho_{m,i}(k)) - v_{m,i}(k) \right] + \frac{T}{L_m} v_{m,i}(k) \left[ v_{m,i-1}(k) - v_{m,i}(k) \right] - \frac{VT[\rho_{m,i+1}(k) - \rho_{m,i}(k)]}{\tau L_m[\rho_{m,i}(k) + K]} \tag{1c}
$$

$$
v(\rho_{m,i}(k)) := \bar{v}_m \, \exp\bigg[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{cr},m}}\right)^{a_m}\bigg],\qquad(1d)
$$

where (1a) is derived from the conservation principle and (1b) from the definition of the flow, while (1c) empirically describes the average speed evolution.  $v_{m,i-1}(k)$  and  $\rho_{m,i+1}(k)$  in (1c), indicates that the speed evaluation in section i of link  $m$  is influenced by the speed and density of previous and next sections, i.e.,  $i - 1$  and  $i + 1$ , respectively. The function defined in (1d) describes the desired speed given the current density of section  $i$ . A brief description of the parameters in (1a)-(1d) is reported in Table I. Extra terms can be incorporated into (1c) to model lane reductions and merging phenomena near on-ramps. For more details see [27]. Moreover, the partial densities for each destination  $j \in J_m$  read as

$$
\rho_{m,i,j}(k+1) = \rho_{m,i,j}(k) + \frac{T}{L_m \lambda_m} \left[ \gamma_{m,i-1,j}(k) q_{m,i-1}(k) - \gamma_{m,i,j}(k) q_{m,i}(k) \right]
$$
\n(2a)

$$
\gamma_{m,i,j}(k) = \frac{\rho_{m,i,j}(k)}{\rho_{m,i}(k)}.
$$
\n(2b)

In the case that  $i = 1$ , the composition rate and flow entering the first section of link  $m$  is equal to the composition rate and the flow exiting the previous link, i.e.,  $\gamma_{m-1,N_{m-1},j}(k)$ and  $q_{m-1,N_{m-1}}(k)$ , respectively.

*2) Store-and-forward (saf) links:* differently from the freeway links, they include neither density and speed dynamics nor link discretization. They are queue models represented by their maximum capacity  $q_{\text{max},s}(k)$ , queue length  $w_s(k)$ . For each saf link  $s \in \mathbb{N}^{\text{saf}} \subseteq \mathbb{N}$ , the incoming flow of vehicles creates a queue. The evolution of the queue length

<b>Parameters</b>	<b>Descriptions</b>	<b>Units</b>
$\overline{v}_m$	Free-flow speed	[km/h]
$\rho_{cr,m}$	Critical density per lane	[veh/km lane]
$a_m$	Exponent parameter	- 1
$\tau$	Relaxation time	[h]
	Anticipation constant	$[km^2/h]$
	Numerical stability parameter	[veh/km]

TABLE I STATIC PARAMETER IN THE METANET MODEL.

is attained via the conservation principle, i.e.,

$$
w_s(k+1) = w_s(k) + T [q_{\text{in},s}(k) - q_s(k)] \tag{3}
$$

Then, the queuing vehicles are directed to the succeeding downstream link after a certain time delay. The dynamics of the outflow for  $s \in \mathbb{N}^{\text{safe}}$  are as follow

$$
q_s(k) = \min\left[q_{\text{in},s}(k) + \frac{w_s(k)}{T}, q_{\text{max},s}(k)\right],\tag{4}
$$

where  $q_{\text{in},s}(k)$  [veh/h] is the entering flow and  $q_{\text{max},s}(k)$  is the maximum outflow of the saf link which depends on the density of the downstream link  $d \in \mathbb{N}$  and is defined as

$$
q_{\max,s}(k) = \begin{cases} Q_s & \text{if } \rho_{d,1}(k) < \rho_{\text{cr,d,1}} \\ Q_s R(k) & \text{otherwise,} \end{cases}
$$

where  $Q_s$  is the maximum flow of  $s \in \mathbb{N}^{\text{safe}}$  and  $R(k)$  is the portion of it that can enter link d. This fraction is defined as  $R(k) \coloneqq \frac{\rho_{\max,d,1} - \rho_{d,1}(k)}{\rho_{\max,d,1} - \rho_{d,1}(k)}$  $\rho_{\max,d,1}^{\text{max},d,1-\rho_{d,1}(k)}$ , where  $\rho_{\max,d,1},\rho_{\text{cr},d,1}$  and  $\rho_{d,1}(k)$ are, respectively, the jam density, critical density and actual density of the first section of the downstream link d, hence discriminating whether the whole demand can enter or not. In the destination-oriented mode, the notion of the partial queues for each destination  $j \in J_s$  reachable from link s has to be introduced. For more details see [25].

*3) Origin links:* serve as entry points for the traffic demand  $d_o(k) \in \mathbb{R}$ . Each origin link  $o \in \mathbb{N}$  is modelled as a saf link [27].

*4) Destination links:* are the exit points for the traffic flow. Traffic conditions in these links are strongly influenced by the conditions in the next link and are assumed to be not congested if there is no available measurement.

## *B. The nodes*

At locations corresponding to junctions, bifurcations, merging on-ramps, and diverging off-ramps, nodes play an essential role in representing the interconnections between various links within the network. For each node  $p \in P$ , the set of entering and exiting links are denoted by  $I_p \subseteq N$ and  $O_p \subseteq \mathbb{N}$ , respectively. Then,  $Q_p(k)$  is the total traffic flow entering node  $p$  during  $k$ . Similarly, the flow leaving  $p$ through a link  $d \in O_p$  is denoted by  $q_{\text{in},d}(k)$ . Additionally, the *turning rate*  $\beta_p^d(k)$ , indicates the portion of traffic flow  $Q_p(k)$  which leaves node p at period k via link  $d \in O_p$ . The relation among these quantities is described as follows:

$$
Q_p(k) = \sum_{m \in I_p} q_{m,N_m}(k)
$$
 (5a)

$$
q_{\text{in},d}(k) = \beta_p^d(k) Q_p(k), \tag{5b}
$$

where  $q_{m,N_m}(k)$  is the flow exiting the last section of link  $m \in I_n$  and  $q_{\text{in},d}(k)$  that entering the first section of the exiting link  $d$  (see e.g. [25] for further details).

# III. THE METANET WITH SERVICE STATION (METANET-S)

In this section, the classical METANET model is endowed with the dynamics of a ST. Figure 2 is a visual aid for the structure of the METANET-s and the used notation. The freeway stretch is partitioned into  $N$  freeway links, interconnected by  $P$  nodes. Without loss of generality, we suppose that each link is subdivided into only one section, i.e.,  $N_m = 1$ ,  $\forall m \in \mathbb{N}$ . An origin link  $o \in \mathbb{N}$  is used to inject into the network the traffic demand  $d_o(k) \in \mathbb{R}$ . Vehicles can exit the mainstream through the link  $s_1 \in \mathbb{N}$ to reach the ST, which is represented by a saf link. A queue of vehicles stay for a certain amount of time  $\delta$  at the ST to use ancillary services such as resting and refueling. After spending  $\delta$  amount of time at the ST, drivers attempt to merge back into the mainstream. Then, if the flow exiting the ST, i.e.,  $q_{st}(k)$ , exceeds the capacity  $q_{max,st}(k)$ , a queue of vehicles forms at the exit point of the ST. Thus, a coupling should be considered between the number of vehicles entering the ST and the number of vehicles attempting to exit in the next time intervals. Furthermore, a constraint should be considered for the entering flow to the ST in order to avoid traffic congestion in the entry point.

#### *A. METANET-s dynamics*

The origin link  $o \in \mathbb{N}$  receives as input the traffic demand  $d_o(k)$  and thus the exiting flow is defined as

$$
q_o(k) = r_o(k) \min \left[ d_o(k) + \frac{w_o(k)}{T}, q_{\max,o}(k) \right],
$$
 (6)

where  $r_o(k)$  is the metering rate for the link o (if it is not applied, then  $r_o(k) = 1$  for all  $k \in \mathbb{N}$ ). The maximum flow of the origin link depends on the density of the first section of the first downstream link, i.e.,  $\rho_{m_1}(k)$ , hence

$$
q_{\max,o}(k) = \begin{cases} Q_o & \text{if } \rho_{m_1}(k) < \rho_{\text{cr},m_1} \\ Q_o R(k) & \text{otherwise.} \end{cases}
$$
 (7)

The permit function  $R(k) := \frac{\rho_{\max,m_1} - \rho_{m_1}(k)}{2}$  $\frac{\rho_{\max,m_1}-\rho_{m_1}(\kappa)}{\rho_{\max,m_1}-\rho_{\text{cr},m_1}}$  influences the outflow of the origin link  $q_o(k)$  based on the congestion level of the exiting link  $m_1$ . If link  $m_1$  is dense or congested, the capacity of the origin link  $q_{\text{max},o}(k)$  is reduced. As the capacity decreases, a queue of vehicles forms at the origin link o, resulting in

$$
w_o(k+1) = w_o(k) + T \bigg[ d_o(k) - q_o(k) \bigg].
$$
 (8)



Fig. 2. Depiction of the METANET-s model. The freeway links (pink rectangles) represent the mainstream and on-ramp/off-ramp of service station which are connected by the nodes (black dots), an origin link (blue rectangle), a destination link (brown rectangle), and the ST is modeled via a saf link (green rectangle).

Density and speed dynamics for link  $m_1$  are introduced similarly to (1a)-(1d). In order to identify the portion of the flow  $q_{m_1}(k)$  entering the mainstream and the ST, it is necessary to consider partial densities as in (2a), i.e.,

$$
\rho_{m_1,s_1}(k+1) = \rho_{m_1,s_1}(k) + \frac{T}{L_{m_1}\lambda_{m_1}} \left[ \gamma_{o,s_1}(k)q_o(k) - \gamma_{m_1,s_1}(k)q_{m_1}(k) \right]
$$
(9)

$$
\rho_{m_1,m_2}(k+1) = \rho_{m_1,m_2}(k) + \frac{T}{L_{m_1}\lambda_{m_1}} \bigg[\gamma_{o,m_2}(k)q_o(k) - \gamma_{m_1,m_2}(k)q_{m_1}(k)\bigg], \tag{10}
$$

where  $\gamma_{o,s_1}(k)$ ,  $\gamma_{m_1,s_1}(k)$ ,  $\gamma_{o,m_2}(k)$ , and  $\gamma_{m_1,m_2}(k)$  represent the composition rates at time period  $k$  based on  $(2b)$ and satisfy the following constraints

$$
\gamma_{o,s_1}(k) + \gamma_{o,m_2}(k) = 1 \tag{11a}
$$

$$
\gamma_{m_1, s_1}(k) + \gamma_{m_1, m_2}(k) = 1.
$$
 (11b)

The vehicles entering the ST through the link  $s_1$  during k, stop at the ST for  $\delta$  amount of time before trying to merge back into the mainstream. Then, the number of vehicles at the ST evolves as

$$
\ell_{st}(k+1) = \ell_{st}(k) + T\bigg[q_{s_1}(k) - q_{st}(k)\bigg],\qquad(12)
$$

where,  $q_{s_1}(k)$  and  $q_{st}(k)$  denote the entering and exiting flow of the ST in the time interval k, respectively. After  $\delta$ time intervals, vehicles attempt to exit the ST to merge back into the mainstream. However, if the flow  $q_{st}(k)$  exceeds the maximum capacity  $q_{\text{max,st}}(k)$ , a queue forms at the exit point of the ST, including vehicles waiting for merging back into the mainstream in the next time intervals. Let  $w_{st}(k)$  denote the queue length, its dynamics can be expressed as

$$
w_{\rm st}(k+1) = w_{\rm st}(k) + T \bigg[ q_{s_1}(k-\delta) - q_{\rm st}(k) \bigg]. \tag{13}
$$

Then, the outflow of the ST can be computed as

$$
q_{\rm st}(k) = r_{\rm st}(k) \min\bigg[q_{s_1}(k-\delta) + \frac{w_{\rm st}(k)}{T}, q_{\rm max, st}(k)\bigg], \quad (14)
$$

where  $q_{\text{max,st}}(k)$  indicates the capacity of the ST, which is directly influenced by the density of the next link, i.e.,

 $\rho_{s_2}(k)$ , as follows

$$
q_{\text{max,st}}(k) = \begin{cases} Q_{\text{st}} & \text{if } \rho_{s_2}(k) < \rho_{\text{cr},s_2} \\ Q_{\text{st}}R(k) & \text{otherwise.} \end{cases}
$$
(15)

Now, it is necessary to prevent the formation of a potential overloaded queue of vehicles at the entrance of the ST during each time interval  $k$ . Inspired by stationary queuing model of traffic flow networks [28], we apply a manipulation of the flow entering the ST, i.e.,  $q_{s_1}(k)$ , based on the dynamics of the ST. Then, let  $\ell_{\text{max,st}}$  denote the maximum number of vehicles that can stop at the ST, which is computed according to the length of the ST and the average length of a vehicle. Then, the flow entering the ST, can be constrained by the space available at the ST, i.e.,

$$
q_{s_1}(k) = \min\bigg[\rho_{s_1}(k)v_{s_1}(k), \frac{\ell_{\max, st} - \ell_{st}(k)}{T}\bigg].
$$
 (16)

Note that the proposed METANET-s has been designed to be easily integrated with various traditional and advanced control strategies aimed at preventing or reducing traffic congestion. For example, a ramp-metering mechanism [29] based on model predictive control approaches can be designed to regulate the flow of vehicles merging back into the mainstream, i.e.,  $q_{st}(k)$ . Another possibility is to design incentive-based traffic control policies to incentivize drivers to exit the main stream and stop at the ST to take advantage of services at discounted prices, such as charging electric vehicles at a cheaper price [12]. In such a framework, gametheoretical approaches can be used to design control policies for  $\gamma_{m_1,s_1}(k)$  and  $\delta$ .

#### IV. SIMULATION RESULTS

We consider a freeway stretch consisting of  $N =$ 11 links organized as in Figure 2, hence  $N :=$  $\{o, st, s_1, s_2, m_1, \ldots, m_7\}$ . The origin link  $o \in \mathbb{N}$  is entry point for the incoming demand  $d_o(k)$ , while  $\{m_1, \ldots, m_7\}$ describe the mainstream. The traffic condition after the destination link  $m<sub>7</sub>$  is assumed to be uncongested. Furthermore, the length of the ST is 1 [km], while the length of the links  $\{o, s_1, s_2, m_1, \ldots, m_7\}$  are equal to 0.3 [km] with  $\lambda_m = 3$ lanes. The links  $\{s_1, st, s_2\}$  describe the ST, where  $s_1$  and  $s<sub>2</sub>$  are the off and on-ramps respectively, while st is the saf link modeling the ST.  $\rho_{\text{max}}$ ,  $\rho_{\text{cr}}$ , and  $\bar{v}_m$  are equal to 30 and 20 [veh/km], and 102 [km/h], respectively. The time interval



Fig. 3. Back propagation phenomenon resulting from congestion, hence  $\rho_{s_2} > \rho_{cr,s_2}$  at the off-ramp of the ST  $s_2$ . (a) the flow of ST  $q_{st}(k)$  and density of  $s_2 \rho_{s_2}(k)$ , (b) flow of on-ramp  $s_1 q_{s_1}(k)$ , queue length of vehicles in ST  $\ell_{st}(k)$ , (c) flow of  $m_1 q_{m_1}(k)$ , density of link  $s_1 \rho_{s_1}(k)$ , (d) the flow and density in  $m_2$ , i.e.,  $q_{m_2}(k)$ ,  $\rho_{m_2}(k)$ .

in the simulations is  $T = 0.01$  [s] and we simulate a total of 1.6 [h], hence  $k \in [0, 6 \times 10^4]$ . To mimic a classic peak hour demand,  $d_o(k)$  is assumed to be the following piece-wise linear demand function

$$
d_o(k) = \max\left(500, -\frac{1}{9}|k - 30000| + 2500\right). \tag{17}
$$

Of the total demand, 30% of it stops at the ST, and the remainder 70% remains on the mainstream.

#### *A. Traffic congestion back propagation in METANET-s*

Here, we show that METANET-s can describe the back propagation of a traffic congestion. As shown in Figure 3.a, at  $t = 0.7$  [h] a congestion occurs in the link  $s_2$ , hence  $\rho_{s_2}(k) > \rho_{\text{cr},s_2}$ . This congestion leads to a decrease in the available capacity of the ST  $q_{\text{max,st}}(k)$  as per (15). As a result, the number of vehicles at the ST  $\ell_{st}(k)$  increases, see Figure 3.b. Then the saturation in (16) activates to limit  $q_{s_1}(k)$ . Due to the reduced capacity of the ST  $q_{\text{max,st}}(k)$ , the total number of vehicles in the ST  $\ell_{st}(k)$  peaks at 1.3 [h], showing a significant backlog of vehicles in  $s_1$  waiting to enter to the ST, see  $\rho_{s_1}(k)$  in Figure 3.c.

# *B. Capacity drop in METANET-s comparison with CTM-s*

We compare now the ability of METANET-s to describe capacity drops compared to the CTM-s. In Figure 2, the flow and density of links  $m_4$ ,  $m_5$ , and  $m_6$  are represented. Initially,  $q_{m_5}(k)$  peaks around 1213 [veh/h lane] at approximately 0.5 [h]. Then, there is a significant capacity drop, making it fall at around 1100 [veh/h lane], hence a 9.5% reduction, see Figure 4.b. As shown in Figure 4.c the evolution of  $q_{m_6}(k)$  is similar to  $q_{m_5}(k)$ .

In Figure 4.a, we plot  $q_{m_4}(k)$ . Due to the propagation of a shock wave, there is a drop in traffic flow in link  $m_4$ . This drop is accompanied by a rapid increase in density, indicating that congestion was forming as vehicles approached the bottleneck.

Noticeably, simulation results in Figures 4.a-c show that CTM-s, implemented as in [10], is not able to capture the presence of the capacity drop and highly overestimates the flow among links. This behavior is consistent with our other numerical simulation allowing us to conclude that METANET-s has better-predicting capabilities than CTM-s in case of congestion.

#### V. CONCLUSION

Inspired by the classical METANET model, in this paper we have developed a novel traffic model: the METANET-s. It is a second-order macroscopic traffic model incorporating service stations dynamics. The capability of the METANET-s to model speed dynamics excels in capturing intricate traffic phenomena, such as capacity drops, thereby addressing the limitations of the recently proposed CTM-s. Including a constraint on the flow entering the ST allows us to take into account the ST capacity. Furthermore, the METANETs has been developed to aid in the design of innovative control strategies aimed to incentivize drivers to stop at STs to prevent and/or reduce traffic congestion.

## **REFERENCES**

- [1] M. Angelidou, C. Politis, A. Panori, T. Bakratsas, and K. Fellnhofer, "Emerging smart city, transport and energy trends in urban settings: Results of a pan-european foresight exercise with 120 experts," *Technological Forecasting and Social Change*, vol. 183, p. 121915, 2022.
- [2] A. Kotsialos and M. Papageorgiou, "The importance of traffic flow modeling for motorway traffic control," *Networks and Spatial Economics*, vol. 1, pp. 179–203, 2001.
- [3] S. Siri, C. Pasquale, S. Sacone, and A. Ferrara, "Freeway traffic control: A survey," *Automatica*, vol. 130, p. 109655, 2021.
- [4] A. A. Ganin, M. Kitsak, D. Marchese, J. M. Keisler, T. Seager, and I. Linkov, "Resilience and efficiency in transportation networks," *Science Advances*, vol. 3, no. 12, p. e1701079, 2017.



Fig. 4. Capacity drop captured in the METANET-s in comparison with the flow variations captured by the model of CTM-s at On-ramp merging location: flow and density trajectories for links  $m_4, m_5$ , and  $m_6$  in METANET-s and flow captured by CTM-s ( $\phi(k)$ ) of cells  $m_4, m_5$ , and  $m_6$  respectively in (a), (b), and (c).

- [5] M. J. Lighthill and G. B. Whitham, "On kinematic waves ii. a theory of traffic flow on long crowded roads," *Proceedings of the Royal Society of London. Series a. Mathematical and Physical Sciences*, vol. 229, no. 1178, pp. 317–345, 1955.
- [6] C. F. Daganzo, "The cell transmission model, part ii: Network traffic," *Transportation Research Part B: Methodological*, vol. 29, no. 2, pp. 79–93, 1995.
- [7] J. Lebacque, "The godunov scheme and what it means for rst order tra c ow models," in *Proceedings of the 13th International Symposium on Transportation and Trafc Theory, Lyon, France, July*, vol. 2426, 1996.
- [8] J. Lebacque, C. Buisson, J. Lesort, and H. Mongeot, "The strada model for dynamic assignment," in *Intelligent Transportation: Realizing the Future. Abstracts of the Third World Congress on Intelligent Transport SystemsITS America*, 1996.
- [9] C. Alecsandru, A. Quddus, K. C. Huang, B. Rouhieh, A. R. Khan, and Q. Zeng, "An assessment of the cell-transmission traffic flow paradigm: Development and applications," Tech. Rep., 2011.
- [10] C. Cenedese, M. Cucuzzella, A. Ferrara, and J. Lygeros, "A novel control-oriented cell transmission model including service stations on highways," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, 2022, pp. 6278–6283.
- [11] C. Cenedese, M. Cucuzzella, A. C. Ramusino, D. Spalenza, J. Lygeros, and A. Ferrara, "Optimal service station design for traffic mitigation via genetic algorithm and neural network," *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 1528–1533, 2023, 22nd IFAC World Congress.
- [12] C. Cenedese, M. Cucuzzella, J. M. A. Scherpen, S. Grammatico, and M. Cao, "Highway traffic control via smart e-mobility – part i: Theory," 2021.
- [13] Council and the European Parliament. (2019) Fit for 55 package. [Online]. Available: https://www.consilium.europa.eu/en/policies
- [14] C. Cenedese, P. Stokkink, N. Geroliminis, and J. Lygeros, "Incentivebased electric vehicle charging for managing bottleneck congestion," *European Journal of Control*, vol. 68, p. 100697, 2022, 2022 European Control Conference Special Issue.
- [15] C. Cenedese, M. Lucchini, M. Cucuzzella, A. Ferrara, and J. Lygeros, "On the effect of capacity drops in highways with service stations," in *2023 31st Mediterranean Conference on Control and Automation (MED)*, 2023, pp. 478–483.
- [16] A. Messmer and M. Papageorgiou, "Metanet: A macroscopic simulation program for motorway networks," *Traffic Engineering & Control*, vol. 31, no. 9, 1990.
- [17] M. Cremer and M. Papageorgiou, "Parameter identification for a traffic flow model," *Automatica*, vol. 17, no. 6, pp. 837–843, 1981.
- [18] S. Fan and B. Seibold, "Data-fitted first-order traffic models and their

second-order generalizations: Comparison by trajectory and sensor data," *Transportation Research Record*, vol. 2391, no. 1, pp. 32–43, 2013.

- [19] Y. Wang, X. Yu, S. Zhang, P. Zheng, J. Guo, L. Zhang, S. Hu, S. Cheng, and H. Wei, "Freeway traffic control in presence of capacity drop," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 3, pp. 1497–1516, 2021.
- [20] Y. Wang, M. Zhao, X. Yu, Y. Hu, P. Zheng, W. Hua, L. Zhang, S. Hu, and J. Guo, "Real-time joint traffic state and model parameter estimation on freeways with fixed sensors and connected vehicles: State-of-the-art overview, methods, and case studies," *Transportation Research Part C: Emerging Technologies*, vol. 134, p. 103444, 2022.
- [21] A. Ferrara, G. P. Incremona, and G. Piacentini, "A hierarchical mpc and sliding mode based two-level control for freeway traffic systems with partial demand information," *European Journal of Control*, vol. 59, pp. 152–164, 2021.
- [22] R. C. Carlson, I. Papamichail, M. Papageorgiou, and A. Messmer, "Optimal mainstream traffic flow control of large-scale motorway networks," *Transportation Research Part C: Emerging Technologies*, vol. 18, no. 2, pp. 193–212, 2010.
- [23] ——, "Optimal motorway traffic flow control involving variable speed limits and ramp metering," *Transportation science*, vol. 44, no. 2, pp. 238–253, 2010.
- [24] C. Pasquale, S. Sacone, S. Siri, and A. Ferrara, "A new micro-macro" metanet model for platoon control in freeway traffic networks," in *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2018, pp. 1481–1486.
- [25] A. Ferrara, S. Sacone, and S. Siri, *Freeway traffic modelling and control*. Springer, 2018, vol. 585.
- [26] A. Kotsialos, Y. Pavlis, F. Middelham, C. Diakaki, G. Vardaka, and M. Papageorgiou, "Modelling of the large scale motorway network around amsterdam," *IFAC Proceedings Volumes*, vol. 31, no. 20, pp. 349–354, 1998.
- [27] A. Kotsialos, M. Papageorgiou, C. Diakaki, Y. Pavlis, and F. Middelham, "Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool metanet," *IEEE Transactions on intelligent transportation systems*, vol. 3, no. 4, pp. 282–292, 2002.
- [28] T. Van Woensel and N. Vandaele, "Modeling traffic flows with queueing models: a review," *Asia-Pacific Journal of Operational Research*, vol. 24, no. 04, pp. 435–461, 2007.
- [29] V. L. Knoop, H. Taale, M. Meulenberg, P. B. van Erp, and S. P. Hoogendoorn, "Ramp metering with real-time estimation of parameters," in *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2018, pp. 3619–3626.