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Scalar Reference Governor Bank for loosely cross-coupled MIMO systems: application to Satellite Attitude Control

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Abstract— **The control strategy of complex engineering systems, *e.g.*, in automotive, aeronautical and space applications, are oftentimes well-established and there is limited space to integrate major novelties in the design of control laws. As a consequence, the problem of managing the system constraints is addressed with overconservative and sub-optimal solutions. With a particular focus on satellite missions, this paper proposes a Reference Governor-based approach to design the optimal safe trajectory so as to exploit the full capabilities of an established closed-loop Multi-Input Multi-Output (MIMO) system subject to state and input constraints. While not interfering with its stability properties, the Governor predicts the future evolution of the closed-loop system and modifies the reference to track in case constraints are at risk. The computationally attractive Scalar Reference Governor is compared to the Vector Reference Governor, which is optimal for MIMO systems. Finally, a sub-optimal fast Governor is proposed for MIMO systems with limited coupling. Numerical simulations are run on the CNES high-fidelity simulator developed for the Microcarb mission and illustrate the advantages of the proposed methodology.**

I. INTRODUCTION

Complex engineered systems are typically asked to satisfy several stringent requirements including fuel consumption, emissions, handling qualities and performances. In addition, the physical capabilities of the actuators and the external environment define precise safety sets within which it is imperative to operate. As a consequence, designing an optimal strategy to enforce the system constraints is key. Nevertheless, integrating major novelties in the control laws of such established systems can be rare due to the cascade of complexities that would arise following a change of the dynamical behaviour of the system. For this reason, unless a strategy is already included in the closed-loop control framework, satisfying the system constraints for such systems is often achieved introducing safety margins in the trajectory design to guarantee the system to remain within the safe operating region. With a particular focus on satellite missions, these safe trajectories are usually computed on ground and are not updated onboard in real-time, ending up being sub-optimal, *i.e.*, the system performance, agility, and capabilities are not fully exploited. Motivated by improving and simplifying the satellite guidance planning, in this paper, we study the problem of constraints management for Multi-Input Multi-Output systems that present an established controller proven to accomplish the desired closed-loop behaviour in absence of constraints. The objective is to propose a computationally cheap algorithm to add between the desired target and the closed-loop system to compute safe optimal commands in real-time. In particular, the proposed solution focuses on MIMO systems that include a controller that effectively decouples the input-output channels cross-interaction.

Typically, handling constraints in MIMO systems is addressed via strategies that aim at resolving the tracking problem as well. Examples include constrained Linear Quadratic Regulator, [1], l_1-optimal control, [2], and Model Predictive Control (MPC), [3], [4], [5]. Instead, few solutions focus on the constraints management without redesigning the control legacy. For this scope, nonlinear functions, *e.g.*, saturation blocks, might help remaining within the safe operation region, but are not desirable as they might compromise the system stability. Another possibility consists of addressing the constraints management problem via Reference Governor (RG) techniques, [6]. Similar to MPC, RGs rely on solving an online optimization problem to compute an optimal signal according to the predicted system dynamics. While MPC schemes act in the place of a controller and look for the optimal control input sequence, reference governors are added between the reference signal and the closed-loop system and filter the desired reference to generate a virtual one whenever constraints violation is at risk. As a result, they do not operate to resolve the system stability and do not interfere with the closed-loop system dynamics. This feature makes RGs of great interest for all applications for which an established controller already exists, but no strategy to handle constraints is available. In addition, thanks to the limited computational effort required, reference governors, and in particular Scalar Reference Governors (SRG), are attractive in applications with relatively fast-dynamics and low computational capability, such as automotive and aerospace applications. Indeed, SRGs solve a linear programming problem and modify the reference command via one single decision variable. While this approach results optimal for Single-Input Single-Output systems (SISO), modifying the reference channel by using a unique parameter can result over conservative for MIMO systems. To overcome this limitation and add more flexibility, the optimization problem can be formulated employing as many decision variables as the number of reference inputs. As a result, this governor formulation, usually referred as Vector Reference Governors (VRG), solves a Quadratic Program (QP) and overcomes the limit of the

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SRG at the price of an increased computational cost. A recent solution merging the low computational effort of SRG and the flexibility of VRG has been proposed in [7] and [8], where it is referred as Decoupled Reference Governor (DRG). Assuming the closed-loop system is square, *i.e.*, the number of constraints corresponds to the number of reference input channels, invertible and with stable inverse, the authors design a decoupling filter so that the RG problem can be rewritten as a set of SRG problems. The required assumptions limit the applicability of the method on non-square systems since the filter cannot be computed. On a similar fashion, we propose to overcome this limitation considering non-square closed-loop systems that already present low coupling between different input-output channels. Then, the system is considered as composed of a set of SISO systems and constraints are enforced solving a set of Linear programming (LP) problems, allowing the low computational cost of SRG and the superior flexibility of VRG. In addition, as discussed in Section IV, the simple design of the solution makes it attractive for practical implementations.

The paper is organized as follows: Section II formally states the problem and clarifies the objectives. Section III describes the reference governor strategies. In Section IV, the proposed governors are added to the CNES Microcarb mission simulator and results are provided. Finally, in Section V conclusions and future perspectives are provided.

II. PROBLEM STATEMENT

Consider a closed-loop linear MIMO system for which an established control law guarantees the desired performance and stability properties. In particular, the system measured output $y(t) \in \mathbb{R}^m$ is proven to be capable of tracking the desired reference $r(t) \in \mathbb{R}^m$ in absence of state and input constraints. In addition, let the control law guarantee a low coupling between the several input-output channels. The closed-loop dynamics are then represented by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Br(t), \\ y(t) &= Cx(t) + Dr(t), \quad x(0) = x_0, \quad t \geq 0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and is assumed available for feedback, $x(0) \in \mathbb{R}^n$ is the initial state vector, and the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{m\times m}$ describe the linear time-invariant dynamics of the closed-loop system. The system state and input constraints are gathered in the variable $y_c \in \mathbb{R}^c$, where the subscript c refers to *constraints*, note that c can differ from m. Its evolution is assumed to be modelled via the matrices $C_c \in \mathbb{R}^{c\times n}$, and $D_c \in \mathbb{R}^{c\times m}$ as

$$y_c(t) = C_c x(t) + D_c r(t). \tag{2}$$

Constraints are enforced if

$$y_c(t) \in \mathcal{Y}, \quad \forall t \geq 0, \tag{3}$$

where $\mathcal{Y} \subset \mathbb{R}^c$ is a prescribed compact set defined by a set of linear inequalities, with $0 \in int\mathcal{Y}$.

Problem: Design a computationally cheap strategy that does not interfere with the closed-loop stability properties and is capable of enforcing system state and input constraints, such that (3) is satisfied.

III. PROPOSED SOLUTION

In this section, the scalar reference governor and the vector reference governor are reviewed, and their key feature that well apply to the considered scenario are highlighted. Next, the proposed strategy for loosely cross-coupled MIMO systems is detailed.

A. Scalar Reference Governor

A reference governor is an add-on control block that acts as a pre-filter between an input reference signal $r(t)$ and a closed-loop system that is proven to ensure the required performance in the absence of constraints. The scope of the governor block is to monitor the system state and input constraints and to act in case they are at risk. The general governor-based control scheme is illustrated in Fig. 1. In this paper, we assume the state $x(t)$ to be available for feedback, $\hat{x}(t) = x(t)$, and the uncertainties or system perturbations are set to zero, $w(t) = 0$. Simple extensions on systems with limited knowledge of the full state, and subject to uncertainties are possible.

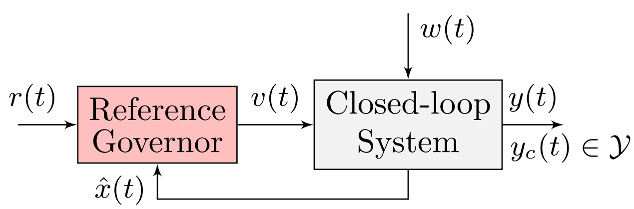

Fig. 1. Reference Governor Scheme.

The constraints enforcement is accomplished by predicting the closed-loop system trajectories on a predefined horizon and, accordingly, by modifying the current reference to track. Assuming a model for the discrete closed-loop system dynamics is available, the future system evolution is predicted employing the current reference $r(t)$ and the current state $x(t)$ as

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d v(k), \\ y_{c_d}(k) &= C_{c_d} x_d(k) + D_{c_d} v(k), \end{aligned} \tag{4}$$

where $x_d(t) \in \mathbb{R}^n$ is the discrete state vector and $x_d(k = 0) = x(t)$, $v(k) \in \mathbb{R}^m$ is the modified reference assumed to be constant along the prediction horizon, $y_{c_d}(k) \in \mathbb{R}^c$ gathers the state and input constraints specified by y_c in continuous time, and the matrices $A_d \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times m}$, $C_{c_d} \in \mathbb{R}^{c\times n}$, $D_{c_d} \in \mathbb{R}^{c\times m}$ model the closed-loop system evolution specified by (2) in continous time. State and input constraints, must be satisfied all along the prediction, *i.e.*,

$$y_{c_d}(k) \in \mathcal{Y}, \ \forall\, k \in \mathbb{Z}^+. \tag{5}$$

Then, based on the predicted trajectory, the reference governor selects the best approximation v of the desired reference $r(t)$ such that, if maintained constant, constraints are ensured to be enforced from time t onward. Assuming there exists an initial signal $v(0)$ satisfying constraints all along the system evolution, the scalar reference governor criteria for the selection of the signal $v(t)$ is formulated using the scalar parameter $\kappa(t)$:

$$v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1)) \tag{6}$$

where $0 \le \kappa(t) \le 1$. If the RG predicts that the reference $r(t)$ does not lead to constraints violation, then $\kappa(t) = 1$, and $v(t) = r(t)$. Otherwise, if constraints might be at risk, the governor selects the highest value of $\kappa(t)$ that allows constraints enforcement. According to (6), the resulting current modified reference $v(t)$ will be such that $v(t) \in [v(t-1), r(t)]$. In the extreme case, $\kappa(t) = 0$ and $v(t) = v(t-1)$. In this event, the system applies the reference $v(t-1)$, computed at the previous time step, which proved to ensure constraints enforcement from time $t-1$ onward. Safety and recursive feasibility are therefore intuitively guaranteed. To formally describe the governor problem, we introduce the maximal admissible set O_∞:

$$O_\infty = \{(\bar{v}, x(t)) : y_{c_d}(k|\bar{v}, x(t)) \in \mathcal{Y},\ \forall\ k \in \mathbb{Z}_+\}. \tag{7}$$

Then, RG computes $v(t)$ such that $(v(t), x(t)) \in P$, with $P \subseteq O_\infty \subset \mathbf{R}^m \times \mathbf{R}^n$. The set O_∞ gathers the current state $x(t)$ and all constant inputs, $v(t+k) = v(t) = \bar{v}$, such that constraints are ensured to be satisfied in the future evolution of the system. P can coincide with O_∞, but for computational reasons a tightened version $\tilde{O}_\infty$ of the set O_∞ is generally adopted so that the associated steady-state output $(D_{c_d}+C_{c_d}(I-A_d)^{-1}B_d)\bar{v}$ respects constraints with a nonzero margin $1 \gg \varepsilon > 0$. $\tilde{O}_\infty$ is then defined as $\tilde{O}_\infty = O_\infty \cap O^\varepsilon$, where $O^\varepsilon = \{(\bar{v}, x(t)) : \left(D_{c_d} + C_{c_d}(I-A_d)^{-1}B_d\right)\bar{v} \in (1-\varepsilon)\mathcal{Y}\}$. Hence, the governor computes the signal $v(t)$ such that $(v(t), x(t)) \in \tilde{O}_\infty$. Assuming that $\mathcal{Y}$ is compact, A_d is Schur, and the pair (A_d, C_{c_d}) is observable, then $\tilde{O}_\infty$ is a *finitely determined* polytope. In other words, there exists a finite index k^* such that

$$\tilde{O}_\infty = \{(\bar{v}, x(t)) : y_{c_d}(k|\bar{v}, x(t)) \in \mathcal{Y},\ k = 0, ..., k^*\} \cap O^\varepsilon. \tag{8}$$

In addition, it can be shown that $\tilde{O}_\infty$ is non-empty and *positively invariant*, which means that if $(v(t), x(t)) \in \tilde{O}_\infty$, when applying $v(t)$ to the system, then also $(v(t), x(t+1)) \in \tilde{O}_\infty$. Thanks to the positive invariance of $\tilde{O}_\infty$, $v(t) = v(t-1)$ always satisfies constraints and hence *recursive feasibility* is proven under the condition that there exist a known signal $v(0)$ such that $(v(0), x(0)) \in \tilde{O}_\infty$. See [9] for details. Then, the governor solves the following optimization problem at each time step:

$$\begin{aligned} \kappa(t) = \quad & \underset{\kappa\in[0,1]}{\text{maximize}} \quad \kappa \\ & \text{subject to} \quad (v(t), x(t)) \in \tilde{O}_\infty. \end{aligned} \tag{9}$$

In this notation, the property of *recursive feasibility* guarantees that the value of $\kappa(t) = 0$ remains a feasible solution of the optimization problem in (9) provided it is feasible at the initial time. Since only the scalar parameter $\kappa(t)$ is optimized online, the computational complexity of this approach is minimal. The reduced computational cost is a key feature of the scalar governor and makes it very interesting as compared to other predictive control techniques that might result heavier and not implementable in real-time, such as MPC schemes. In addition, the governor acts as a pre-filter between the desired set-point and the closed-loop system and does not interfere with the system stability properties. As a result, governors are particularly interesting for application that already implement an established controller but lack of an optimal technique to monitor state and input constraints. Finally, for additional computational cost reduction, under the specified conditions, the scalar reference governor optimization problem can be easily formulated to be explicitly solvable. To do so, we remark that if $\mathcal{Y}$ is a polytope, then it can be written as $\mathcal{Y} = \{y_{c_d} : Hy_{c_d} \le h\}, h > 0$. It follows that, considering a constant v along the k^* time steps of the prediction horizon, $y_{c_d}(k)$ can be predicted as $y_{c_d}(k) = C_{c_d}A_d^k x(0) + C_{c_d}(I-A_d^k)(I-A_d)^{-1}B_d v + D_{c_d}v,\ k = 0, ..., k^*$, where $x(0)$ is updated at each time step as $x(0) = x(t)$. Hence, including (6), $\tilde{O}_\infty$ can be expressed as

$$\tilde{O}_\infty = \left\{(\bar{v}, x(0)) : a + b\kappa \le c\right\} \tag{10}$$

with

$$a = \begin{bmatrix} HD_{c_d} & HC_{c_d} \\ HC_{c_d}B_d + HD_{c_d} & HC_{c_d}A_d \\ \vdots & \vdots \\ HC_{c_d}(I-A_d^{k^*})(I-A_d)^{-1}B_d + HD_{c_d} & HC_{c_d}A_d^{k^*} \\ HC_{c_d}(I-A_d)^{-1}B_d + HD_{c_d} & 0 \end{bmatrix} \begin{bmatrix} v(t-1) \\ x(t) \end{bmatrix},$$

$$b = \begin{bmatrix} HD_{c_d} \\ HB_dC_{c_d} + HD_{c_d} \\ \vdots \\ HC_{c_d}(I-A_d^{k^*})(I-A_d)^{-1}B_d + HD_{c_d} \\ HC_{c_d}(I-A_d)^{-1}B_d + HD_{c_d} \end{bmatrix} (r(t) - v(t-1)),$$

$$c = \begin{bmatrix} h \\ h \\ \vdots \\ h \\ (1-\varepsilon)h \end{bmatrix}. \tag{11}$$

Then, the SRG solution for κ is explicitly found setting

$$\begin{aligned} \kappa_U &= \min\{\min_{i:b_i>0}\{\frac{c_i - a_i}{b_i}\}, 1\} \\ \kappa_L &= \max\{\max_{i:b_i<0}\{\frac{c_i - a_i}{b_i}\}, 0\} \\ \kappa &= \begin{cases} \kappa_U & \text{if } \kappa_L \le \kappa_U \text{ and } a_i \le c_i \\ & \text{ for all } i \text{ such that } b_i = 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{12}$$

where the subscript i denotes the i^{th} row of the corresponding matrix, $i = 1, ..., k^*$.

B. Vector Reference Governor

Since (9) works with a single optimization variable, the SRG is ideal for SISO systems, $m = 1$. In the case of MIMO systems, $m > 1$, SRG might be sub-optimal as all the m

references would be simultaneously modified by the same scaling factor κ at each time step. To overcome this limitation and introduce more degrees of freedom for the optimization problem solution, Vector Reference Governors are designed. Equation (6) is rewritten as

$$v(t) = v(t-1) + K(t)(r(t) - v(t-1)) \tag{13}$$

where $K(t) = \mathrm{diag}(\kappa_i(t))$, with $i = 1, ..., m$. The m optimization variables are selected by solving a Quadratic Program (QP):

$$\begin{aligned} \underset{\kappa_i \in [0,1]}{\text{minimize}} \quad & (v(t) - r(t))^T Q (v(t) - r(t)) \\ \text{subject to} \quad & v(t) = v(t-1) + K(t)(r(t) - v(t-1)) \\ & (v(t), x(t)) \in \tilde{O}_\infty, \end{aligned} \tag{14}$$

with $Q = Q^T \succ 0$. Thanks to its superior flexibility in the choice of the modified reference $v(t)$ over the SRG, the VRG leads MIMO system to a faster convergence to the set-point r. However, despite explicit multi-parametric quadratic programming may fasten the optimization process, because of the inherently higher number of variables and the QP formulation itself, VRG remains computationally heavier than SRG. This can be critical when dealing with large-dimension systems.

C. SRG Bank for MIMO systems with low coupling

A solution merging the low computational complexity of SRG and the attractive performance of VRG is discussed for MIMO systems with low coupling effects between the input-output channels. Considering a linear MIMO system as in (1), to quantify the coupling between the input-output channels, one can rewrite the system in the form of a transfer function as $G(s) = C(sI - A)^{-1}B + D$; then, determining the input-output interactions can be achieved computing the Relative Gain Array (RGA), [10], and normalizing along its rows to infer about how each input influences each output with respect to the other inputs. For non-square systems, RGA can be computed as:

$$RGA = (G_{ss}^T)^+ . G_{ss}, \tag{15}$$

where G_{ss} is the steady-state gain matrix associated to $G(s)$, "." defines an element-wise multiplication, and the superscript "+" indicates the Moore–Penrose inverse. If there is a clear match between one input and one output, then, the corresponding element of RGA with normalized rows is close to 1 and the influence of the other inputs on that output is minimal. Alternatively, one can compute the infinite norm of the difference between $G(s)$ and the matrix $W(s)$:

$$\gamma = \| G(s) - W(s) \|_\infty , \tag{16}$$

with $W(s) = G(s)$, except on the terms supposed to be subject to low input-output interaction, which are set to zero. If γ is sufficiently small, then the coupling interactions can be considered negligible. The following RG strategy is proposed for closed-loop systems with very low coupling effects. For control purposes, such configuration is often achieved thanks to a pre-compensator that defines or enhances a dominant input-output interaction. Neglecting the coupling interactions, a MIMO system is then considered as composed of a set of SISO systems. In other words, $G(s)$ is approximated by $W(s)$. By extending this reasoning to the RG problem, enforcing the system constraints can reduce to solving a bank of m indipendent LP problems, as in (6), by means of m SRGs. Each SRG predicts the trajectories of the constrained states of the associated i^{th} SISO system and modifies one single reference r_i into v_i, with $i = 1, ...m$. The proposed methodology is schematically represented in Fig. 2.

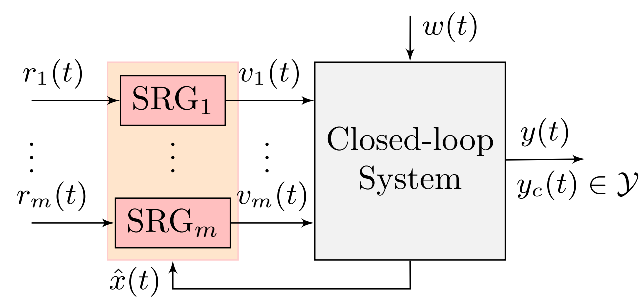

Fig. 2. Proposed Reference Governor.

IV. NUMERICAL SIMULATION

Prime examples of Multi-Input Multi-Output systems subject to performance requirements and actuators' constraints are found in satellite applications. Due to the complexity of designing the numerous and diverse subsystems composing a spacecraft, satellite missions are oftentimes conceived by re-utilizing existing satellite platforms. The great industrial advantage of 'recycling' old working designs can leave limited space for major novelties that would improve the overall system. In this context, reference governors can optimize the process of designing the missions' guidance profile that is typically more prone to introduce novelties as compared to the onboard controller. The proposed methodology is here applied on the Microcarb attitude simulator. The CNES mission Microcarb will offer a global monitoring of the CO_2 surface fluxes, which will provide an insight onto the mechanisms governing exchanges between CO_2 sources and sinks, their seasonal variability, and their evolution in response to climate change, [11]. The Microcarb Attitude and Orbit Control System employs the well-established control laws designed for the Myriad platform, on which the mission is conceived, and employs magnetorquers and up to four reaction wheels, [12]. The governor approach proposed in this paper is employed on the Microcarb attitude simulator with the scope of optimizing the guidance design so as to enforce the actuator constraints during manoeuvres while exploiting the full capabilities of the satellite. In particular, the reaction wheels are physically limited in speed and acceleration, and it is required to avoid approaching these limits to avoid their saturation and possible nonlinear undesired effects. The wheels limited capabilities are expressed in the form of hard constraints on the angular momentum $h_{RW}(t)$ and torque $t_{RW}(t)$ that can be provided to the satellite in the x, y, z directions, $i.e.$, $|h_{RW}(t)| < 0.138$ kgm^{2}s^{-1} and $|t_{RW}(t)| < 0.0048$ Nm. Assuming a linear model of the transfer between the attitude set-points and the constrained states is available, the normalized RGA ($NRGA$) matrix is computed following (15) and normalizing along the rows:

$$NRGA_{r\to[h_{RW},t_{RW}]} = \begin{bmatrix} 0.9995 & -0.0283 & -0.0166 \\ -0.0281 & 0.9996 & -0.0023 \\ -0.0177 & -0.0017 & 0.9998 \\ 0.9995 & -0.0283 & -0.0166 \\ -0.0281 & 0.9996 & -0.0023 \\ -0.0177 & -0.0017 & 0.9998 \end{bmatrix}. \tag{17}$$

In (17), the three columns correspond to the three attitude angle set-points defined by $r(t) = [\phi_r(t)\ \theta_r(t)\ \psi_r(t)]$, while the rows are related to the six constraints on h_{RW} and t_{RW} in the three directions. The $NRGA$ matrix shows that the coupling effects are limited. Following (16), this result is confirmed computing $\gamma = 7 \times 10^{-25}$. Then, the SRG bank method can be applied. Results compare the SRG and VRG with the proposed SRG bank on a simulation of 1500s. The manoeuvre considered is a large-angle slew maneuver, typically employed for reorienting sensors, antennas, and solar arrays. While essential, slew manoeuvres entail a time loss from actively fulfilling mission objectives. Reference governors offer a solution by computing online the optimal guidance that safely ensures the full utilization of actuators' capabilities, thereby minimizing slew times. Starting from $[\phi(0)\ \theta(0)\ \psi(0)] = [0\ 0\ 0]$, the attitude to reach is specified by the reference $r(t) = [\phi_r(t)\ \theta_r(t)\ \psi_r(t)] = [0.16\ -0.49\ 2.18]$rad $\approx [9\ 28\ 124]$deg. The RG optimization problem is solved every time step, $dt = 0.25$s. Due to its QP problem, the VRG is implemented using the MATLAB toolbox YALMIP, [13], while the output of the SRG and SRG bank algorithms are analytically computed following (12). The prediction horizon of the RG problems is set to be $k^* = 100$ time steps, hence the prediction horizon is $T = 25s$, and the modified reference is initialized to $v(0) = [0\ 0\ 0]$. Figure 3 shows the phase portrait of the six constrained variables on the constraint set $\mathcal{Y}$. The three solutions appropriately modify the reference so as to guarantee safe trajectories, $i.e.$, (3) is satisfied. Figures 4 and 5 show the realized attitude and the evolution of the decision variables κ, respectively. All the three governors safely slow down the satellite dynamics, that eventually reaches the reference $r(t)$. As the SRG only utilizes one decision variable κ, the three input channels are modified by the same rate. As shown, the SRG's κ reaches $\kappa = 1$ after about 1310s and the satellite converges to desired set-points after about 1320s for all the three axes. Instead, the VRG and the SRG bank benefit of a superior flexibility as the references on the three axes are modified by independent parameters. As a result, the convergence to the three set-points happen at different times during the simulations. The results underline the conservatism of the SRG. Indeed, the manoeuvre on the third axis is the most demanding in terms of torque and angular momentum since the satellite set-point on ψ is wider than on ϕ and θ. As a consequence, more torque and angular momentum are required by the wheels to rotate in this direction, and the SRG has to 'wait' 1310s before allowing the parameter κ to converge to $\kappa = 1$. As also evident in Figure 3, in the simulation where the SRG has been employed, the capabilities of the wheels are not fully exploited to rotate in the direction of ϕ and θ as two of the red trajectories remain far from the bounds of the constraint set $\mathcal{Y}$. Finally, the simulations are compared in terms of

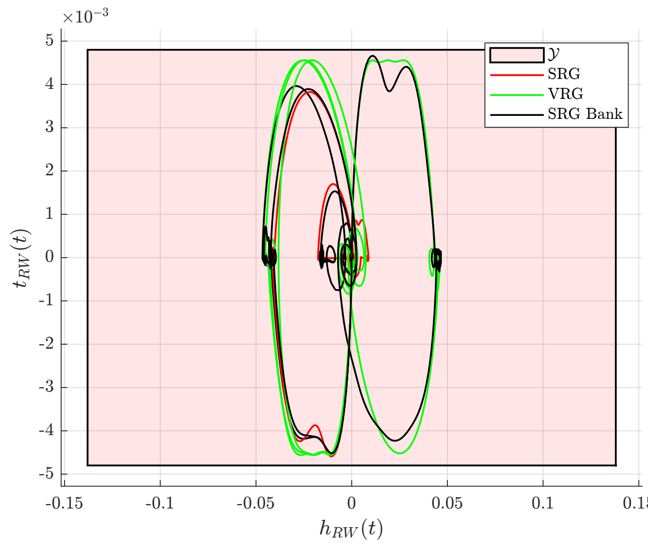

Fig. 3. Constraints evolution over $\mathcal{Y}$. Comparison of SRG, VRG, SRG Bank.

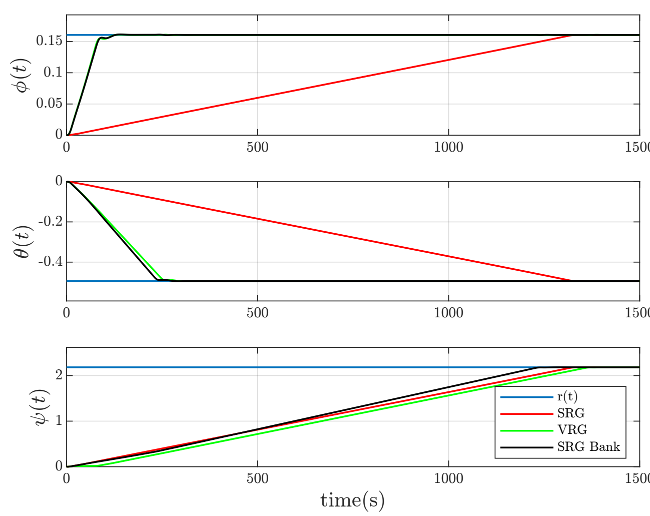

Fig. 4. Satellite Attitude. Comparison of SRG, VRG, SRG Bank.

execution time on the same hardware. Table I demonstrates how SRG Bank outperforms both SRG and VRG. While superiority against VRG is obvious, it is worth noticing that solving m smaller problems results to be more efficient than finding a single κ for the complete system.

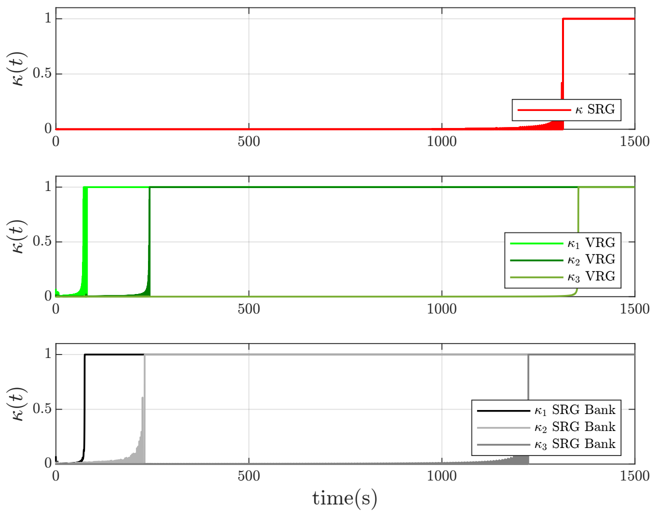

Fig. 5. Optimization parameter κ. Comparison of SRG, VRG, SRG Bank.

TABLE I

COMPUTATION TIME. COMPARISON OF SRG, VRG, SRG BANK.

Simulation Time		
SRG	**VRG**	**SRG Bank**
40 s	8701 s	13 s

V. CONCLUSIONS

In this paper, a solution to improve the constraints management of decoupled Multi-Input Multi-Output systems is proposed. Assuming available a control strategy that guarantees the desired closed-loop system performance in absence of constraints and a low cross-interaction between the input-output channels, a set of scalar reference governors predict the closed-loop system trajectories and modifies the current set-points in case the system state and input constraints are at risk. The proposed solution merges the attractive low computational cost of the scalar reference governor with the better convergence performances of the vector reference governor. Numerical simulations have been performed on the CNES high-fidelity Microcarb attitude simulator. Future works will include robustness to external perturbations and uncertainties.

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