Enhancing PI Tuning in Plant Commissioning through Bayesian Optimization

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Abstract—This paper introduces a new approach aimed at expediting the auto-tuning of PI controllers during the commissioning phase. This approach is based on the exploitation of the process model knowledge and Bayesian optimization capabilities. The process unfolds in a sequence of steps: initially, the quality of the model is improved through the identification of unknown or uncertain parameters of the model. Subsequently, this refined model is used for searching the optimal configuration of PI controller. The outcomes obtained, encompassing the initial estimate, upper and lower bounds, and the Gaussian process mean, are then harnessed to initiate the Bayesian optimization process in the commissioning phase. By initializing adequately the Bayesian optimization, a significant reduction in the number of iterations required to reach the optimizer's optimal solution can be achieved. The approach efficiency is demonstrated through its application to a thermal plant.

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are the most commonly used controllers in industrial processes despite the availability of huge advanced controllers variants in academia. The interest to this kind of controllers is explained by the simplicity, robustness, no model knowledge required and easy to use of this kind of controllers. Numerous manual tuning methods involve monitoring the process response following adjustments to the controller setpoint are proposed. However, the PID tuning is still a tedious time consuming process and in general very difficult to result in optimal performance when it is done manually. Several auto-tuning approaches are proposed in literature where the aim is to automate the process of selecting appropriate controller parameters (for instance [1], [2], [3]). Broadly speaking, one can classify these methodologies into data-driven and modelbased approaches. For data-driven controller design (see for instance [4]), it offers an advantage in that they do not necessitate prior model knowledge. The reader can refere to the works: Virtual Reference Feedback Tuning (VRFT) [5], Fictitious Reference Iterative Tuning (FRIT) [6], Iterative Feedback Tuning (IFT) [8], one-step tuning scheme for a 2DOF control system [7] and step responsebased [11]. While data-driven PID tuning approaches offer numerous advantages, they are not without their drawbacks. These drawbacks encompass data dependency, limited generalizability to diverse operating conditions, sensitivity to noisy data, and the risk of overfitting when the process

diverges from the data distribution.

The second category is the model-based approaches where several techniques are proposed. For more detailed information, readers may consult references like [9] and its associated sources. Model-based methods exhibit their full potential when precise models are at one's disposal. Nevertheless, in practice it is difficult to have accurate and robust models because of the lack of adequate, practical uncertainty descriptions [10]. Another possible solution is to use a hybrid approach by combining data- and model-based approaches as proposed in [12]. The approach is formulated within the Bayesian optimization framework, utilizing the model plant to establish the prior mean function. While the approach holds promise, its effectiveness is contingent upon the presence of an accurate model. Without such accuracy, there is a risk of converging to a local minima, especially if a preference for exploitation over exploration dominates the optimization process. In our forthcoming paper, we will introduce a similar approach with additional enhancements.

In [13], a hybrid multi-objective optimization design method for tuning PI controllers is introduced, with a specific emphasis on reliability-based optimization scenarios. The study employs Montecarlo methods to quantitatively assess controller performance degradation resulting from unforeseen or unmodeled system dynamics. The utilization of a multi-objective framework adds a level of complexity for users, requiring proficiency in multi-objective optimization techniques. Other auto-tuning techniques have been proposed based on reinforcement learning (LR) where the controller parameters are updated using the training process (see for example [14], [15]). In contrast, [16] focuses on a training process that updates combination of value and policy functions instead of directly adjusting controller gains.

This paper introduces a Bayesian-based approach aimed at expediting the auto-tuning process of a PI controller during the commissioning phase. The approach is structured into three distinct steps. In the initial step, the primary focus lies in enhancing the quality of model predictions based on a single set of measurements. The Bayesian optimization approach (BO) is employed for the identification of unknown parameters within the existing model, with the objective of minimizing the discrepancy between real measurements and model outputs. It's worth noting that our

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approach doesn't demand a perfect model; rather, it only requires a reasonably accurate one. In the second step, the identified model is integrated with a Bayesian optimization (BO) approach to autonomously fine-tune the PI gains within a simulation setting. The fundamental concept at this stage is to capitalize on the optimal solution obtained as an initial estimate for the next step. With a reliable initial estimate in hand, it becomes unnecessary to retain the original wide ranges (i.e., upper and lower bounds) for the PI gains. Consequently, these ranges are refined, although this adjustment necessitates careful consideration, contingent upon the quality and accuracy of the identified model. Moreover, the mean (μ) and covariance of the Gaussian process identified at this stage are employed as initial values for the mean and covariance in the third step where the real plant commissioning is initiated. During this phase, the BO approach is used again and leverages the previously identified parameters (i.e, initial estimate, upper and lower bounds of the decision variables (PI gains), and the mean and covariance of the Gaussian process). This purposeful utilization of prior information accelerates the search for an optimal solution by ensuring a well-informed and appropriate initialization of the BO optimization process.

The paper is structured as follows: Section II introduces preliminary concepts and problem formulation. Section III outlines an approach for expediting PI tuning during the commissioning phase. Section IV details the application of this approach to auto-tuning a PI controller for a thermal plant. Finally, Section V offers the concluding remarks.

II. PRELIMINARY AND PROBLEM FORMULATION

In this paper, we focus on the auto tuning of PID controllers for real plants (Fig. 2). The signal y^{ref} is the reference or set point to be tracked. u and y are respectively the input and output of the process. ω represents the process perturbation and ν denotes the measurement noise. $e = y^{ref} - y$ is the tracking error.



Fig. 1. PI controller

The PI controller has the following structure:

$$u_t = K_p e(t) + K_I \int e(t) \,\mathrm{d}t \tag{1}$$

where K_p and K_i represents the proportional and integral gains. An anti-windup measure is also considered to ensure the actuator constraints.

To tune the controller gains $(K_p \text{ and } K_i)$, we need to define a cost function based on the desired tracking performances of PI controller (i.e, Overshoot, settling time, Integral

absolute error (IAE), Integral time absolute error (ITAE), etc.). In this paper, a linear combination of standardly used control key performance indicators (KPI): settling time, overshoot and IAE as defined in following equations:

$$J_{\theta} = a_1 Osh(\theta) + a_2 St(\theta) + a_3 IAE(\theta)$$
(2)

where $a_i, i = 1: 3$ are known and constant weights. θ is a vector contains the control parameters. Osh and St are the overshoot and settling time. IAE represents the Integrated Absolute Error and given by: $IAE = \int_0^\infty |e(t)| dt$.

This previous specific set of KPIs are picked to cover typical dynamic responses of interest such as settling when changing references (settling time), minimize error to reference(IAE) and limit overshoot as many thermal plants considered are particularly sensitive to very low overshoot. The weights are selected to give a quantifiable performance from step resonse, based on qualitative evaluation by plant expert.

The Auto-tuning PI controller is formulated now as blackbox multi-objective optimization problem given by:

$$\begin{split} \min_{\theta} & J_{\theta} \\ \text{s.t.} & \theta = [K_p, K_I], \theta^{up} \le \theta \le \theta^{lp} \end{split}$$
(3)

 θ^{up} and θ^{lp} are respectively the upper and lower bound of each element in the vector θ . The primary objective at this juncture is to find the optimal solution for the challenging optimization problem (3). Given the absence of a closedform expression for the objective function and its expensive evaluation (or sampling), the Bayesian optimization approach emerges as the most appropriate choice. The costly nature of objective function evaluation is attributed to the intended tuning on the actual process during the commissioning phase. In Figure 2, we illustrate the execution of the optimization framework using BO approach. Due to space limitations, we refrain from providing in-depth technical details about the BO optimization approach. Readers seeking a comprehensive explanation are encouraged to refer to the relevant literature, such as [17], for a more detailed understanding.



Fig. 2. BO-based PI controller auto-tuning, where *i* indicates a closed-loop experiment with controller parameters θ_i

Unfortunately, even if the BO approach is a promising technique its performance depends also on the tuning of some parameters such as the initial guess and the upper and lower bounds of the decision variables. But not only, it depends also on the initialization of the Gaussian process built inside the Bayesian approach. To overcome this last issue, one might initialize the mean of the Gaussian process using a mean calculated from simulation data obtained from available models as shown in [12]. However, this can accelerate the Bayesian concept only in the case of having an accurate model which is not the case in general. Besides, the authors in [12] did not tackle the issues with the initial guess and the upper and lower bounds of the decision variables where they have more impact on the acceleration of the BO. In the next section, we will propose an approach to accelerate the Bayesian optimization framework by solving all the issues mentioned above.

III. ACCELERATE THE PI TUNING IN COMMISSIONING PHASE

In this section, we propose a Bayesian-based approach for accelerating the auto-tuning of a PI controller in the commissioning phase. The approach is depicted in Fig. 3 and is structured into three distinct steps. In the first step, the main idea is to enhance the model prediction by trying to identify as accurately as possible the unknown parameters. It is both reasonable and non-limiting to assume that the model parameters are identifiable. Alternatively, it may be feasible to downsize the model, retaining only the identifiable subsystem whenever applicable. At this point, the acquisition of experimental data is needed to grasp the real dynamics of the plant. The specific quantity of experiments required is neither predetermined nor fixed; however, in most cases, a single experiment proves sufficient. In the context of this paper, we exclusively relied on a single experiment to identify the thermal plant model, as detailed in the subsequent section. The BO approach is used to identify the model parameters where the error between the real measurement and model output is minimized. Certainly, it's evident that alternative optimization solvers, such as NOMAD, or parameter estimation techniques, can also be employed in lieu of BO. Furthermore, there is no requirement for specific selections of PI gains (K_{p0}, K_{i0}) , as long as they fall within the acceptable range. It is worth noting that the objective is not to get an exceedingly precise model, but rather to achieve a model of satisfactory accuracy, suitable for utilization in the subsequent phase involving the search for optimal PI gains through the application of the BO approach. In the second



Fig. 3. PI tuning in commissioning phase

step, the identified model is combined with a BO approach

underlying concept at this stage is to leverage the optimal solution acquired as an initial estimate for the subsequent step. Given the availability of a reliable initial estimate, there is no necessity to retain the original broad ranges (i.e., upper and lower bounds) for PI gains. Consequently, these ranges are narrowed. However, this adjustment must be executed cautiously, contingent on the quality and accuracy of the identified model. In the event that the ranges become excessively narrow, and the model is not highly accurate, there is a risk that the optimal PI gains in the third step (commissioning) will fall outside the range boundaries and remain beyond the reach of the optimization approach. To address this concern, one can redefine the new ranges centered around the obtained optimal solutions by applying a percentage criterion, such as: $0.5K_{p0}^* < K_p < 1.5K_{p0}^*$. Furthermore, the identified mean (μ) and covariance of the Gaussian process at this stage can serve as initial values for the mean and covariance in the subsequent step, respectively. The real plant commissioning is initiated in the third step, where the primary aim is to autonomously fine-tune the PI controller for the physical plant. At this stage, the BO approach is once again deployed and benefits from the previously identified parameters, including the initial guess, upper and lower bounds of the decision variables (PI gains), mean, and covariance of the Gaussian process. This strategic utilization of prior information expedites the quest for an optimal solution by ensuring a suitable and well-informed initialization of the BO optimization process.

to autonomously fine-tune the PI gains in simulation. The

The approach presented in this paper operates under the assumption that a reasonably and descent model is at one's disposal. It is worth noting that the model's predictive quality can be enhanced by incorporating additional experimental data and employing domain randomization techniques for a more robust initialization of BO approach. However, it's important to recognize that this approach does entail a trade-off, as it necessitates conducting more experiments and investing additional efforts. It is understood that many industrial enterprises may be hesitant to allocate additional resources to modelling and simulation efforts due to associated costs and uncertain added value.

IV. AUTO-TUNING PI CONTROLLER FOR A THERMAL PLANT

A. Set-up

As a practical use case a thermal plant setup has been used to demonstrate and validate the tuning methods in a realistic environment, resembling on one end cooling of powertrain components, and on the other typical industrial plastic curing, and drying applications.

The setup shown in figure 4, consists of an aluminum plate split into five distinct zones each heated by a cartridge heater of 113W, which is in turn powered by a solid-state relay (SSR) applying a PWM 0-220 V DC. Next to the heating, each of the zones is also actively cooled on the back end by liquid cooling through a cooling channel using water-glycol. The valves on the entry of each of the five cooling channels

allows for opening and closing, while the speed controllable pump, enables flow control through the opened channels.



Fig. 4. Experimental setup : thermal plant

The thermal measurements are recorded by a total of nine (9) thermocouples, glued into each of the five aluminum zones, but also the boundary between each two zones. The thermocouples are calibrated to +-1C accuracy. The fluid temperature are measured by Pt100 sensors on input/output coolant lines. Additionally, pressure across the setup and the flow rate of the water glycol are measured, both for safety and for validation purposes.

All the controls and measurements are interfaced to a Beckhoff I/O stack connected over EtherCAT to a Xenomai Triphase real-time target. The base sampling time of the control loop is Ts = 1ms while the sensor measurements are logged at Tmeas = 100ms, which is deemed sufficient for a rather slow thermal process. This allows for rapid prototyping of control code in Simulink and easy integration to test environment in MATLAB.

B. Thermal plant model

As mentioned in III, a model is needed for calculating the initial condition and the upper and lower bounds of the controller gains before running the BO with the real plant in the commissioning phase. The used model is a physicsbased white box model with dynamic of heat dissipation, convection losses and heat exchange between different components. This plant model is excited by multiple disturbances and evaluated in a closed loop simulation with a feedback controller (PI).

1) Thermal Model assumptions: The thermal plant model is a second order model, consisting of two connected metal masses. The model buildup and numerical evaluation was performed in python. The thermal model under investigation includes the main modes of heat exchange between and within the components:

- One block, "mass 1 (copper)" is heated internally, through direct heat injection. this is equivalent to heating by an electric resistor inserted in this volume.
- A second block, "mass 2 (aluminium)" is cooled internally, driven by a cooling temperature and given transfer area and heat transfer coefficient. This is equivalent to

cooling through a glycol-water mixture, circulating at high throughput rate through pipes in the volume).

- The two metal blocks (aluminium and copper) are connected and do exchange (when at different temperatures) heat by conduction through an area of contact.
- Also, both blocks are in contact with surrounding air and lose or gain thermal energy through convection.

2) Model structure: The model structure is as follows: Two states are defined, representing the bulk copper temperature $(T_{node,Cu})$ and the bulk aluminum temperature $(T_{node,Alu})$. Heat injection and loss (through cooling and resistive heating) and heat losses or gains are injected directly into these states.

The node temperatures are updated every time step using a backward Euler scheme, with the combined energy inputs of each of the heat losses and gains.

The heat gains and losses from heating dT_H and cooling dT_H are a defined as:

$$dT_{H,t-1\to t} = Q_H/m/cp * dt$$

and

$$dT_{C,t-1 \to t} = Q_C/m/cp * dt$$

0

whereby the heat gain Q_H is driven directly by the PIcontroller, and

$$Q_{C} = -(h_{alu,C} * A_{alu,C} * U_{alu,C} * (T_{C,t} - T_{alu}) * dt)$$

As the two blocks have a different temperature and are physically connected along area A, a heat flow through conduction $dT_{othermat}$ will be induced. The flux $q_{Alu,Cu}$ is a function of their respective temperatures and distance of bulk temperature nodes. The joint temperature is defined as a function of to the perpendicular distances between the cooling and heating nodes and the joint:

$$T_{joint,t} = \frac{T_{cu} * k_{alu} * L_{node,alu} + T_{Alu} * k_{cu} * L_{node,cu}}{k_{alu} * L_{node,alu} + k_{cu} * L_{node,cu}}$$

The resulting heat flow $Q_{Alu,Cu}$ depends on the contact area and mass of the materials(s):

$$\begin{aligned} A_{alu,cu} * q_{Alu,Cu} &= -A_{alu,cu} * q_{Cu,Alu} \\ &= (T_{node,Alu} - T_{joint,t}) * cp_{alu} * m_{alu} * dt \end{aligned}$$

On the other hand, heat loss to the environment is modelled using conduction through a given surface A_{air} with thermal transmittance U_{air} and T_{ext} the exterior temperature:

$$dT_{air,t-1\to t} = -(T_{node,t-1} - T_{ext}) * A_{air} * U_{air}/m/cp * dt$$

The numerical values for the model parameters and state initialisation are summarised in table I. For this analysis, they are mostly kept constant, but can be made variable, and noise can be injected in the disturbance, input and output signals.

TABLE I Table with thermal model parameters

Symbol	Name	Unit	Default value
cp_{cu}	spec. heat capac. (copper)	J/kg/K	3.8E2 [18]
cp_{alu}	spec. heat capac. (alu- minum)	J/kg/K	8.97E2
k_{cu}	thermal conductivity (cop- per)	W/mK	4.95E2
k_{alu}	thermal conductivity (alu- minum)	W/mK	2.40E2
$h_{alu,C}$	heat transfer coefficient (cooling to aluminum)	$W/m^2/K$	0.05
$m_{node,cu}$	mass 1 (copper)	kg	unknown
$m_{node,alu}$	mass 2 (aluminum)	kg	unknown
$L_{node,cu}$	distance (copper node to joint)	m	0.05
$L_{node,alu}$	distance (aluminum node to joint)	m	0.05
$L_{cu,air}$	distance (copper node to air)	m	0.05
$L_{alu,air}$	distance (aluminum node to air)	m	0.05
$U_{cu,air}$	thermal transmittance (copper to air)	W/m^2K	5.0
$U_{alu,air}$	thermal transmittance (aluminum to air)	W/m^2K	5.0
$U_{alu,C}$	thermal transmittance (aluminum to cooling)	W/m^2K	0.1256
$A_{cu,air}$	area (copper to air)	m^2	0.03
$A_{alu,air}$	area (aluminum to air)	m^2	0.03
$A_{alu,cu}$	area (contact between mass 1 and 2)	m^2	0.0833
$A_{alu,C}$	area (contact between mass 2 and glycol)	m^2	0.833
$T_{Cu,init}$	initial copper temperature	$^{\circ}C$	25
$T_{Alu,init}$	initial aluminum tempera- ture	$^{\circ}C$	25
$T_{joint,init}$	initial joint temperature	$^{\circ}C$	25
dt	simulation time step	s	0.1
delay	output time delay	s	4.0
P_max	Resistor heat capacity	W	600
T_C	cooling temperature	° C	25
$T_{air,init}$	air temperature	° C	25

C. Results and benchmarking

Experiment description follows (see Figure 5). Each experiment lasts for a total of 380 seconds. It begins with a warmup to 25C for min relying on a well-tuned PI parameters (this ensures consistency in comparison between different runs), followed by a bumpless transfer of control parameters to the tested PI combination, so that the sudden change in integrator parameter does not create a large step in the output of the controller. Finally, a step reference is applied at 140 seconds and is recorded for length of 120 seconds. The experiment ends with a cool-down period of 120 seconds back to below 25C so that the next experiment can be started. For the computation of the objective function, exclusively the data within the time interval of 140 to 260 seconds is considered. Instead



Fig. 5. Experiment description

of conducting a comparison with manual tuning, we opted to employ another optimization solver, namely Nomad (Nonlinear Optimization by Mesh Adaptive Direct Search), to benchmark our findings. Nomad is a free optimization solver available in OPTI tool ^a. For solving the Bayesian optimization, we used the developed approach by [19] and publicly available at http://lis.csail.mit.edu/code/imgpo.html.

The NOMAD solver's configuration we employed was set to default, with the sole modification being the adjustment of the maximum number of iterations, which was set to 80. The considered ranges (upper and lower bounds) for the proportional gain (KP) and integral gain (KI) were defined as [4, 100] and [0.1, 3], respectively. The initial guss is chosen as $K_{p0} = 90$ and $K_{i0} = 2$. For the Bayesian optimization approach, in the first step (focused on parameter estimation) the Gaussian Process Upper Confidence Bound (GP-UCB) model is used with the following hyper-parameters (default) : $mean = 0, \ ell = 1/4, \ sf = 1 \ \text{and} \ lik = -\infty \ (\text{more}$ details about these parameters can be found in IMGPO tool manual). The unknown parameters of cupper and alluminium masses are identified using real measurement (from setup) collected with PI controller configuration: K_{p0} and K_{i0} . The considered upper and lower bounds for both masses are respectively 0.5Kg and 2Kg.

In the second step of the approach, we maintained the same configuration of the Gaussian Process for the purpose of searching for an optimal solution via the Bayesian optimizer. As described in our approach, the outcomes from step 2 served as the foundation for initializing step 3 (K_{p0}^* , K_{i0}^* , mean, and new ranges). The new ranges were set as follows: $0.5K_{p0}^* < K_p < 2K_{p0}^*$ and $0.5K_{i0}^* < K_i < 8K_{i0}^*$. The selection of these ranges depends on the quality of the model used and the need to encompass the optimal region in case the initial guess (from step 2) significantly deviates from the actual optimal solution in the commissioning phase. It represents a trade-off between achieving convergence with fewer iterations and adequately covering the optimal region

^ahttps://github.com/jonathancurrie/OPTI

of the objective function. These ranges can be further reduced if the quality of model predictions improves. For the cost function, we considered the following parameters: $a_1 =$ 4000, $a_2 = 5$, and $a_3 = 2$.

Figure 6 illustrates a comparative analysis between Nomad and the BO-based approach. The x-axis represents the cost, while the y-axis represents the number of iterations. Evidently, the BO-based approach achieves a convergence rate that is 47% faster than NOMAD, a notable performance advantage, even considering the moderate model prediction quality. There is potential for further enhancement in this percentage if additional efforts are invested in refining the model predictions. It is important to note that NOMAD still requires more iterations to reach the minimum value achieved by our approach. In fact, due to time constraints, we made the decision to halt NOMAD at 76 iterations, while the BObased approach reached a stopping point at 56 iterations.



Fig. 6. Comparison between Nomad and the proposed BO-based approach

V. CONCLUSION

This paper presented an approach for improving the convergence of PI controller auto-tuning during the commissioning phase. The approach relies on utilizing a descent process model and harnessing Bayesian optimization's capabilities. The primary objective is to enhance the accuracy of model predictions and integrate them with a Bayesian optimization approach to autonomously fine-tune the PI gains within a simulation environment. The outcomes obtained, including the initial estimate, upper and lower bounds, and Gaussian process mean, serve as the foundation for initializing the Bayesian optimization during the commissioning phase. The approach efficiency is assessed and confirmed through testing on a real thermal plant.

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