2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Energy-Efficient Trajectory Optimization with Nonlinear Model Predictive Control for a Quadrotor UAV

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Abstract— **This paper proposes a control law that improves the energy efficiency of a quadrotor UAV. An optimal velocity hypothesis was followed, thanks to the correlation between ground velocity and total consumed energy. An explicit energy model is developed and employed within the trajectory optimization problems. The proposed approach comprises an optimization stage at which an energy-efficient trajectory is generated followed by a Nonlinear Model Predictive Controller that tracks the generated optimal position and velocity subtrajectories, while the open-loop optimal control problem used to generate the trajectory is dynamically constrained. The proposed control approach was extensively tested in simulation and compared to tracking obtained with standard NMPC, PID, and LQR feedback controllers. The preliminary results demonstrate the validation of our hypothesis and a noteworthy reduction in energy consumption when compared with the use of the other standard feedback controllers, a 36% of energy saving was achieved in some cases.**

Index Terms— **MPC, Trajectory Optimization, Quadrotor, Energy Efficiency**

I. Introduction

A quadrotor is an Unmanned Aerial Vehicle (UAV) that features exceptional manoeuvrability, vertical take-off and landing. Extending quadrotor endurance is one of the important features required nowadays in ubiquitous uses, especially for payload transportation. Nevertheless, endurance is mainly limited by the capacity of the onboard batteries [1]. Using batteries of higher capacity might not be a solution as these come with additional weight. Reducing the quadrotor's consumed energy has been investigated by many researchers [3-12].

Energy efficiency can be improved with better propulsion systems (motors and propellers) [2], [3], modifying/optimizing the mechanical design of the quadrotor [4], [5], or by designing control laws or algorithms that drive the quadrotor with the least possible energy consumption [6].

In terms of the algorithmic approaches, minimum-energy path generation is suggested in [7]. The authors developed an approach in a simulation to drive a quadrotor between two hovering points in which the energy consumption is at its minimum value. The quadrotor dynamics were augmented with new states representing the angular acceleration of the rotors and using them to solve an optimal problem [7]. Based on the path geometry, a different approach focusing on the quadrotor's speed profile was conducted in [8]. Significant energy saving was observed by the authors at a higher quadrotor's speed, thus an equation that provides a variable speed profile was developed to reduce the hovering time followed by a path-following controller that ensures driving the quadrotor at the desired speed [8]. An approach depending on the banking angles was adopted by [9], where the maximum rolling and pitching angles that imply the minimum energy consumption were determined practically through multiple experiments. The authors reported that nearly 5% of energy saving and adequate tracking performance have been achieved using this approach.

Although their research focuses on UAVs in general, the authors in [10] highlighted how the UAV's energy consumption is affected by its ground speed. An optimization criterion is used to implement an algorithm that finds the optimal UAV velocity that minimises the energy consumption and generates an energy-aware trajectory followed by a low-level controller. Moreover, by using a sliding modes controller, designing a trajectory that contributes to saving energy on quadrotors was studied in [11]. A performance index that reflects the input effort and the tracking error where used to minimize the jerk of the quadrotor motion and generate a smoother trajectory. A very close work to that has been conducted in [12] but by minimizing a cost function that penalizes the velocity, acceleration, jerk, and snap of the quadrotor.

Our work in this paper aligns with the domain of energy reduction driven by algorithms. We address driving the quadrotor efficiently by leveraging the usage of the nonlinear model predictive control (NMPC) to track a pre-generated energy-efficient trajectory.

The findings in [8], [6], and [10] highlighted the relation between the quadrotor's energy consumption and its velocity. Therefore, we assume that there is an optimal velocity at which the quadrotor is driven more efficiently. The recent literature that achieved an improvement in energy saving driven by algorithms is appreciated, however, these approaches either do not completely exploit the dynamics of the quadrotor, have no explicit model for energy usage, or require extensive practical experiments to determine vital parameters such as in [9].

In contrast, we optimize the flight time by solving an open-loop optimal control problem that explicitly considers the energy consumption to generate the energy-efficient trajectory. The energy consumption is predicted using an energy model based on the mechanical power of the rotors. An NMPC was adopted to track the generated trajectory to stabilize the motion of the quadrotor between two hovering points. Thus,

*This work is funded by HCED Iraq and supported by Sulaimani Polytechnic University

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this paper offers the following contribution:

- Developing an approach to generate an energy-efficient trajectory that satisfies the dynamical constraints of the quadrotor without knowing the rotor parameters by employing a parameterless energy model.
- Presenting a method to optimize the velocity of the quadrotor by optimizing the flight time as a decision variable in an open-loop optimal control problem.

The rest of this paper is structured as follows: Section II describes the hypothesis based on the relationship between the quadrotor's velocity and its energy consumption. Section III focuses on the derivation of the quadrotor UAV model, Section IV highlights the methodology of the proposed approach including the theory behind the energy model, Section V demonstrates the obtained results along with the test environment. Section VI discusses briefly the results, while Section VII concludes the findings of the paper.

II. Quadrotor Optimal Velocity

The objective of this section is to highlight the relationship between the ground speed and the consumed energy of the quadrotor. The experimental test in [8] shows that it is more efficient when the quadrotor is flying at higher velocities. However, the maximum tested velocity in this particular case was 3.61 m/s which does not give the full picture of this relationship. On the other hand, this relationship was adequately described in [6] and [10]. Figure 1 illustrates the presence of an optimal velocity range at which the consumed instantaneous power is at its minimum [6].

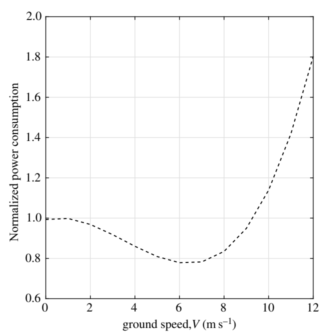

Figure 1. Power consumption correlation with quadrotor velocity [6].

A close look at Figure 1 reveals that the minimum consumption is around 4.5-6.5 m/s, and then it starts to increase as the velocity increases. This is because of the effect of the aerodynamic drag (parasitic drag) which increases at higher velocities [6]. We validated this phenomenon and the results reported in Section V. This brings us to the following assumption:

Assumption 1: There exists an optimal velocity trajectory that satisfies the dynamical constraints of the system (3) which results in performing an efficient motion between two hovering points.

III. Quadrotor Model

The dynamic modelling of the quadrotor with annotations shown in Figure 2 is taken from [13]. The quadrotor as a rigid body is aligned to its body frame, directed to the north, a y-axis is directed to the east, and a z-axis is directed down toward the earth. As an underactuated system, the quadrotor

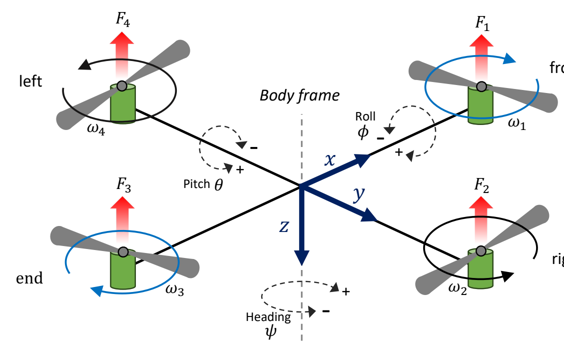

Figure 2. Quadrotor's frames coordinate systems.

has four rotors responsible for lifting its total mass denoted m, and performing the maneuverability. The thrust generated and torque developed by propeller i are denoted F_i and M_i respectively and are given by:

$$F_i = C_T \rho \omega_i^2 r^4, \quad i \in 1,2,3,4 \tag{1}$$

$$M_i = C_M \rho \omega_i^2 r^4, \quad i \in 1,2,3,4 \tag{2}$$

where C_T and C_M are respectively the thrust and torque coefficients of the rotors which are assumed identical for all rotors. ω_i is the angular velocity of the rotor i, ρ is the air density, and r is the diameter of the propeller.

If vector $\boldsymbol{u} = [F_1, F_2, F_3, F_4]^T$ is the control input of the quadrotor, let $\boldsymbol{\xi} = [x, y, z]^T$, and $\dot{\boldsymbol{\xi}} = [U, V, W]^T$ respectively define the position and velocity vectors of the quadrotor center of mass. The angular position and its rate of the quadrotor's attitude in its body axes are defined as $\boldsymbol{\eta} = [\phi, \theta, \psi]^T$, and $\dot{\boldsymbol{\eta}} = [P, Q, R]^T$, respectively. Hence, the state vector of the quadrotor nonlinear model is $\boldsymbol{x} = [x, U, y, V, z, W, \phi, P, \theta, Q, \psi, R]^T$, and the full dynamic model is given by:

$$\begin{bmatrix} \dot{x} \\ \dot{U} \\ \dot{y} \\ \dot{V} \\ \dot{z} \\ \dot{W} \\ \dot{\phi} \\ \dot{P} \\ \dot{\theta} \\ \dot{Q} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} U \\ (\sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi) \dfrac{\sum_{i=1}^{4} F_i}{m} - \dfrac{A_x}{m} U \\ V \\ (-\sin\phi \cos\psi + \cos\phi \sin\theta \sin\psi) \dfrac{\sum_{i=1}^{4} F_i}{m} - \dfrac{A_y}{m} V \\ W \\ -g + (\cos\phi \cos\theta) \dfrac{\sum_{i=1}^{4} F_i}{m} - \dfrac{A_z}{m} W \\ P \\ \dfrac{1}{I_x} \left\{ \left(I_y - I_z\right) QR - J_r \omega_T \; Q + l[F_4 - F_2] - A_r P \right\} \\ Q \\ \dfrac{1}{I_y} \left\{ \left(I_z - I_x\right) PR - J_r \omega_T \; p + l[F_3 - F_1] - A_r Q \right\} \\ R \\ \dfrac{1}{I_z} \left\{ \left(I_x - I_y\right) PQ + \dfrac{d}{b} (\sum_{i=1}^{4} (-1)^i F_i) - A_r R \right\} \end{bmatrix} \tag{3}$$

where A_r is the rotational aerodynamic drag coefficient and A_x, A_y, and A_z are the linear aerodynamic drag coefficients in the x, y, and z directions respectively. $b = C_T \rho r^4$ and $d = C_M \rho r^4$ are the lift and drag constants respectively, while l is the distance from the quadrotor's Center of Mass (CoM) to the rotor's axes. I_x, I_y, and I_z are the moment of inertia of the quadrotor about its body axes. The parameters of the quadrotor UAV used in all simulation tests in this paper are adopted from [14] and given in Table I.

By using the forward Euler discretization method, the dynamics model (3) becomes:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + T_S f(\boldsymbol{x}, \boldsymbol{u}) \tag{4}$$

where $f(\boldsymbol{x}, \boldsymbol{u}) : \mathbb{R}^{12} \times \mathbb{R}^4 \to \mathbb{R}^{12}$, T_S is sampling time at which the differential equations in (3) are discretized, and k denotes the discrete value of the variables at time kT_S.

TABLE I

QUADROTOR UAV PARAMETERS

Symbol	Description	Value (Units)
g	Acceleration due to gravity	9.806 (m/s^2)
A_r	Rotational aerodynamic drag coefficient	10e-3 $(N m.s/rad)$
m	Total mass of quadcopter UAV	0.65 $(kg.s)$
l	distance from the quadrotor's CoM motor	0.232 (m)
A_x, A_y, A_z	Linear aerodynamic drag coefficient	10e-3 (N.s/m)
J_r	Rotor inertia	4e-4 $(kg.m^2)$
I_{xx}	Moment of Inertia along x-axis	7.5e-3 $(kg.m^2)$
I_{yy}	Moment of Inertia along y-axis	7.5e-3 $(kg.m^2)$
I_{zz}	Moment of Inertia along z-axis	1.3e-2 $(kg.m^2)$
ρ	Air Density	1.293 (kg/m^3)
r	Propellers Radius	0.15 (m)
C_T	Thrust Coefficient	0.055 (N/rad^2)
C_P	Power Coefficient	0.045 $(J/s.rad^2)$
C_M	Torque Coefficient	0.024 $(N.m/rad^2)$

IV. PROBLEM FORMULATION

This paper aims to develop an algorithmic approach to drive the quadrotor efficiently between two hovering points, $\boldsymbol{\xi_0} = [x_0, y_0, z_0]^T$ to $\boldsymbol{\xi_f} = [x_f, y_f, z_f]^T$, while satisfying its dynamical constraints. To this end, an energy model has been used to predict the energy and the trajectory $\boldsymbol{x}_t^*$ is generated by solving offline an open-loop optimal control problem denoted $\mathcal{P}_{Traj.}$. Solving an optimal problem such as $\mathcal{P}_{Traj.}$ to generate a full trajectory is computationally expensive, thus the trajectory is computed offline. The quadrotor is then controlled in closed-loop, as shown in Figure 3, via employing NMPC controller that solves open-loop optimal control problem denoted $\mathcal{P}_{NMPC}$ in real-time. Before generating the trajectory, the current position $\boldsymbol{\xi_0}$ is fed from the quadrotor to $\mathcal{P}_{Traj.}$. The controller objective is tracking the energy-efficient trajectory that is offline generated and stabilises the quadrotor to achieve $\boldsymbol{\xi_f}$.

A. Energy Model

Improving the efficiency requires a good prediction of the consumed energy. In this context, some of the recent works, such as the work in [12], have used a conventional rotorcraft aerodynamic power model adopted from [15] which is a function of the total lifting thrust and the ground velocity. Another model was experimentally determined by [10] which mapped the ground velocity to the consumed power. The aforementioned models are parametric models which are complex to be used to solve nonlinear optimization problems. Therefore, we followed the lines in [16] to develop an energy model based on the mechanical power of the rotors to predict the energy while solving $\mathcal{P}_{Traj.}$ and to determine the consumed energy later in simulation. Accordingly, let's define the power developed P_i by propeller i as follows:

$$P_i = C_P \rho \omega_i^3 r^5, \quad i \in 1,2,3,4 \tag{5}$$

where C_P is the power coefficients of the rotors which is assumed identical for all rotors. Combining (1) and (5), the power developed by rotor i is given by:

$$P_i = \frac{C_P}{C_T^{1.5} \rho^{0.5} r} F_i^{1.5} = \alpha_F F_i^{1.5}, \quad i \in 1,2,3,4 \tag{6}$$

where α_F is a constant parameter. The energy consumed by rotor i, over the time interval t_0 to t_f is denoted E_i and given by:

$$E_i = \int_{t_0}^{t_f} P_i \, dt, \quad i \in 1,2,3,4 \tag{7}$$

From (6) and (7), the total energy consumption for the quadcopter is thus:

$$E_T = \alpha_F \int_{t_0}^{t_f} \sum_{i=1}^{4} F_i^{1.5} \, dt + E_C \tag{8}$$

where E_C is the total energy consumed by the onboard circuits, i.e., sensors, controller, ..., etc. For the sake of simplicity, the value of E_C is neglected. It is noteworthy that α_F is the only parameter-dependent constant which depends on the rotor specifications and the air density. In order to use the energy model (8) in generating an energy-efficient trajectory, the dynamics model (3) is augmented with an additional state E_T and its derivative given by:

$$\dot{E}_T = \alpha_F \sum_{i=1}^{4} F_i^{1.5} \tag{9}$$

The new state vector of the augmented dynamics model becomes $\tilde{\boldsymbol{x}} = [x, U, y, V, z, W, \phi, P, \theta, Q, \psi, R, E_T]^T$ and the augmented model is given in (10) by:

$$\begin{bmatrix} \dot{x} \\ \cdot \\ \cdot \\ \cdot \\ \dot{R} \\ \dot{E}_T \end{bmatrix} = \begin{bmatrix} U \\ \cdot \\ \cdot \\ \cdot \\ \frac{1}{I_z}\left\{ \left(I_x - I_y\right) PQ + \frac{d}{b}\left(\sum_{i=1}^4 (-1)^i F_i\right) - A_r R \right\} \\ \sum_{i=1}^4 F_i^{1.5} \end{bmatrix} \tag{10}$$

Since it is a constant for a given quadrotor, α_F does not affect the optimization process when the augmented system (10) is used in solving $\mathcal{P}_{Traj.}$ thus, it has been excluded from the model for optimization purposes. This offers an approach to generate the trajectory without any knowledge about the

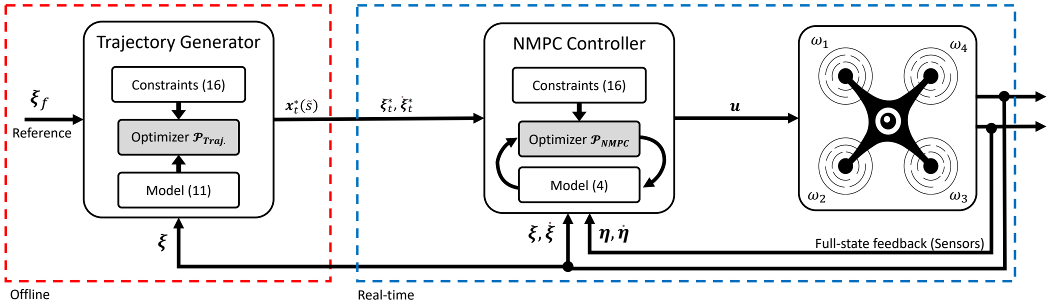

Figure 3. Overall block diagram for the proposed control approach, including both trajectory generator and the NMPC controller.

rotor parameters. By using the forward Euler discretization method, the dynamics model (10) can be rewritten as:

$$\tilde{\boldsymbol{x}}_{k+1} = \tilde{\boldsymbol{x}}_k + \frac{t_f}{h} f(\tilde{\boldsymbol{x}}, \boldsymbol{u}) \tag{11}$$

where h is the number of steps at which t_f is sampled. $f(\tilde{\boldsymbol{x}}, \boldsymbol{u}) : \mathbb{R}^{13} \times \mathbb{R}^4 \rightarrow \mathbb{R}^{13}$, $\frac{t_f}{h}$ is the sampling time at which the differential equations in (10) are discretized, and k denotes the discrete value of the variables at time $k\frac{t_f}{h}$.

B. Energy-Efficient Trajectory

The feasible, energy-efficient trajectory to reach $\boldsymbol{\xi}_f$ is addressed by solving the open-loop optimal control problem $\mathcal{P}_{Traj.}$ given in (12). This optimization problem is formulated to produce the value of the flight time t_f which is not predetermined.

$$\begin{aligned} \mathcal{P}_{Traj.} = \min_{\tilde{\boldsymbol{x}}_k, \boldsymbol{u}_k, t_f} & \sum_{k=0}^{h} \ell(\tilde{\boldsymbol{x}}_k(s), \boldsymbol{u}_k, \boldsymbol{\xi}_f) + \mu_E E_{T|k} + \mu_T t_f \\ \text{s.t.} \quad & (11), \quad t_f > 0, \quad \boldsymbol{u}_k \in \mathbb{U}, \quad \tilde{\boldsymbol{x}}_k \in \mathbb{X} \\ & \tilde{\boldsymbol{x}}_0(s) = \boldsymbol{\xi}_0, \quad \tilde{\boldsymbol{x}}_0(\backslash s) = 0 \\ & \tilde{\boldsymbol{x}}_h(s) = \boldsymbol{\xi}_f, \quad \tilde{\boldsymbol{x}}_h(\backslash s) = 0 \\ & k \in \{0, \ldots, h\}, \quad s \in \{1, 3, 5\} \end{aligned} \tag{12}$$

where $\mathbb{X}$, and $\mathbb{U}$ are the state and the control input constraint sets respectively. The notation $\tilde{\boldsymbol{x}}(s)$ denotes the elements of $\tilde{\boldsymbol{x}}$ with subscripts from s and $\tilde{\boldsymbol{x}}(\backslash s)$ denotes the elements of $\tilde{\boldsymbol{x}}$ whose subscripts are in the complement of s. μ_E and μ_T are two positive parameters that penalize the total energy and the flight time respectively, and ℓ is the performance cost function and given by:

$$\ell(\tilde{\boldsymbol{x}}_k(s), \boldsymbol{u}_k, \boldsymbol{\xi}_f) = \|\tilde{\boldsymbol{x}}_k(s) - \boldsymbol{\xi}_f\|^2_{\tilde{\boldsymbol{Q}}}, \quad \{1,3,5\} \tag{13}$$

where $\tilde{\boldsymbol{Q}}$ is a positive definite weighting matrix that penalizes the weighted Euclidean distance of the quadrotor's CoM to the destination point. The solution of $\mathcal{P}_{Traj.}$ provides an energy-efficient trajectory denoted by $\boldsymbol{x}_t^*$ which consists of all system optimal states, $\boldsymbol{\xi}_t^*$, $\dot{\boldsymbol{\xi}}_t^*$, $\boldsymbol{\eta}_t^*$, and $\dot{\boldsymbol{\eta}}_t^*$, in addition to the optimal flight time t_f required to reach $\boldsymbol{\xi}_f$.

C. NMPC Controller Design

Tracking the offline generated trajectory is assigned to a Nonlinear MPC controller. The controller's optimization problem $\mathcal{P}_{NMPC}$ given in (14), is formulated to track the optimal translational position and velocity trajectories, $\boldsymbol{\xi}_t^*$, and $\dot{\boldsymbol{\xi}}_t^*$ respectively from $\boldsymbol{x}_t^*$. The problem is solved in real time over the determined flight time t_f.

$$\begin{aligned} \mathcal{P}_{NMPC} = \min_{\boldsymbol{u}} & \sum_{k=0}^{N-1} \ell(\boldsymbol{x}_{t|k}(\tilde{s}), \boldsymbol{x}_t^*(\tilde{s}), \boldsymbol{u}_{t|k}) + V_f(\boldsymbol{x}_{t|N}(\tilde{s}), \boldsymbol{x}_t^*(\tilde{s})) \\ \text{s.t.} \quad & (4), \quad \boldsymbol{u}_k \in \mathbb{U}, \quad \boldsymbol{x}_k \in \mathbb{X}, \quad k \in \{0, \ldots, N-1\} \\ & \boldsymbol{x}_{t|0}(\tilde{s}) = \boldsymbol{x}_t \\ & \boldsymbol{x}_{t|N}(\tilde{s}) = \mathbb{X}_f, \quad \boldsymbol{x}_{t|N}(\backslash \tilde{s}) = 0 \\ & \tilde{s} \in \{1, \ldots, 6\}, \quad 0 \le t \le t_f \end{aligned} \tag{14}$$

where $\mathbb{X}_f$ is the equlity terminal constraint that selected by the designer. In this paper, the terminal constraint selected to be $[\boldsymbol{\xi}_t^*, \dot{\boldsymbol{\xi}}_t^*, 0_{1x6}]^T$. Also, ℓ and V_f are, respectively, the stage and terminal cost functions, they are given by:

$$\begin{aligned} \ell(\boldsymbol{x}_{t|k}(\tilde{s}), \boldsymbol{x}_t^*(\tilde{s}), \boldsymbol{u}_{t|k}) &= \|\boldsymbol{x}_{t|k}(\tilde{s}) - \boldsymbol{x}_t^*(\tilde{s})\|^2_{\boldsymbol{Q}} + \|\boldsymbol{u}_{t|k}\|^2_{\boldsymbol{R}} \\ V_f(\boldsymbol{x}_{t|N}(\tilde{s}), \boldsymbol{x}_t^*(\tilde{s})) &= \|\boldsymbol{x}_{t|N}(\tilde{s}) - \boldsymbol{x}_t^*(\tilde{s})\|^2_{\boldsymbol{P}} \end{aligned} \tag{15}$$

where $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{P}$ are positive definite weighting matrices that penalise the weighted Euclidean tracking error, control actions, and the terminal states respectively. The notation $\boldsymbol{x}(\tilde{s})$ denotes the elements of $\boldsymbol{x}$ with subscripts from $\tilde{s}$ and $\boldsymbol{x}(\backslash \tilde{s})$ denotes the elements of $\boldsymbol{x}$ whose subscripts are in the complement of $\tilde{s}$. The solution of $\mathcal{P}_{NMPC}$ provides the control action necessary to regulate the quadrotor about the energy-efficient state trajectory.

V. Simulation Setup & Results

In this section, the simulation environment accompanied by the achieved results is reported. Different tests were conducted, mainly to examine the consumed energy while tracking the proposed energy-efficient trajectory by the NMPC controller. To evaluate the efficiency improvement, experiments were performed to determine the consumed energy when using NMPC, PID, and LQR controllers to reach $\boldsymbol{\xi}_f$ directly without any trajectory guidance. A piecewise affine reference signal is used for the PID and the LQR controllers.

A. Simulation Setup

Both optimization problems, $\mathcal{P}_{Traj.}$, and $\mathcal{P}_{NMPC}$, were formulated using the CasADi open-source MATLAB toolbox and solved using the built-in IPOPT solver. Table II summarized the parameters used to set both problems.

TABLE II
OPTIMIZATION PROBLEMS AND NMPC CONTROLLER PARAMETERS

Parameter	Value
h	50
μ_T	1
μ_E	100
$\tilde{\boldsymbol{Q}}$	diag(1,1,1)
T_S	10 ms
N	100
$\boldsymbol{Q}$	diag(10,10,10,100,100,500)
$\boldsymbol{P}$	diag(10,10,10,100,100,500)
$\boldsymbol{R}$	diag(10,10,10,10)

The input constraints were chosen based on the physical limitations of the rotors, while the state constraints were used to ensure safe operating limits during flight as follows:

$$\begin{aligned} F_i &\in [0, \quad 14.8] \quad \mathrm{N} \\ \phi,\theta &\in (-\frac{\pi}{2},\frac{\pi}{2}) \quad rad \\ \psi &\in (-\pi,\pi) \quad rad \end{aligned} \tag{16}$$

B. Main Results

In all conducted simulation tests, rotors' signals were logged and the energy was calculated using the proposed energy model given in Section IV-A. An extensive series of simulations were conducted to validate Assumption 1 by testing the quadrotor to fly over different distances between two floating points at different velocities. The results are summarized in Figure 4.

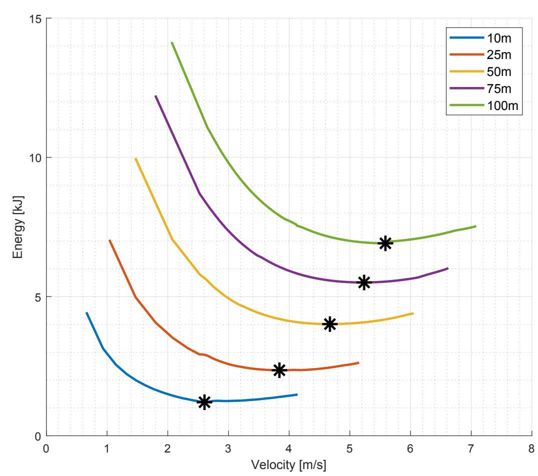

Figure 4. Velocity variation and its effect on the consumed energy

Then, from the initial absolute position $\boldsymbol{\xi}_0 = [0,0,1]^T$m, seven energy-efficient trajectories were generated to achieve seven different reference points. The NMPC controller was examined to track these trajectories. Also, NMPC, PID, and LQR controllers are used to stabilise the quadrotor to achieve the same reference points, directly without using a trajectory. To attain a fair comparison, each direct stabilizing case was conducted for the same corresponding time t_f of the proposed control approach. The energy consumption for each case accompanied by the steady-state (SS) error, flight distance d_f, is briefed in Table III. Furthermore, Figure 5 graphically summarizes the energy consumption comparison of all tested controllers for each distance case.

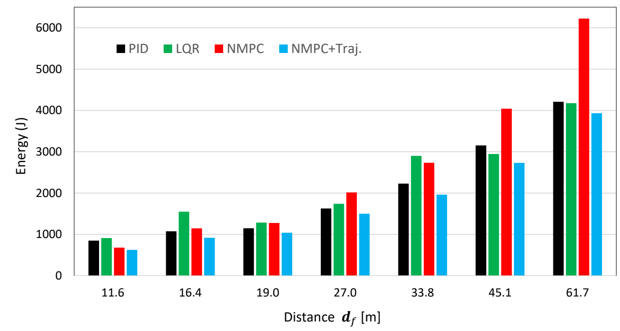

Figure 5. Comparison of energy consumption for all examined cases

In addition, the tracking behaviour and how the norm of the absolute position converges to the destination for all cases have been observed and illustrated in Figure 6. What emerges from the results reported here is that using the generated trajectory is effective in minimizing the consumed energy.

VI. Discussion

An initial objective of the paper was to validate Assumption 1 where it was hypothesised that the quadrotor could be efficiently driven at an optimal velocity. The results reveal that Figure 4 qualitatively agreed with [6] and [10]. Also, the results of this study show that the proposed approach of tracking an optimized trajectory led to significant energy saving compared to the standard controllers even with the optimal one such as LQR. One unanticipated result was that more saving was achieved for long distances. Further analysis shows that many tested cases of reaching the reference directly weren't able to reach the reference within the determined t_f, while it was enough for the proposed approach. It means that if for these cases more time is allowed to completely reach the desired destination, extra energy will be consumed and this supports the viability of the proposed approach at which the determined t_f is adequate to reach the references. Although the performance analysis of the controllers is beyond the scope of this paper, with increasing velocity, PID and LQR controllers perform better because they were not designed to track the velocity, while the NMPC tuned to penalize the error in tracking the velocity more than tracking the position, thus, at high velocities, the steady-state performance performs worse than the other controllers.

VII. Conclusions

In this investigation, the aim was to assess the proposed approach of using the NMPC to track an optimal energy-efficient trajectory. This paper has argued that energy saving could be brought off by driving the quadrotor flight at an optimal velocity. An energy model was developed and used to solve an optimal problem. The solution provides the optimal flight time between two hovering points while respecting all

TABLE III

Energy consumption and steady-state error results

Ref.	Ref. Coordinates [m]	Generated t_f [s]	Distance d_f[m]	PID		LQR		NMPC		NMPC + Trajectory	
				Energy [J]	SS Error [%]	Energy [J]	SS Error [%]	Energy [J]	SS Error [%]	Energy [J]	SS Error [%]
ξ_{f1}	[7, 7, 7]	2.36	11.58	850.15	20.10	909.74	38.00	679.09	-0.95	624.76	9.95
ξ_{f2}	[10, 5, 13]	3.37	16.40	1075.66	1.26	1550.42	-8.37	1146.67	-3.77	916.71	0.24
ξ_{f3}	[15, 10, 7]	4.03	19.00	1148.13	2.11	1285.53	20.33	1275.41	-3.10	1040.09	-3.65
ξ_{f4}	[-18, 20, 3]	5.99	26.98	1627.12	0.52	1741.34	-0.29	2015.95	0.01	1501.15	-1.95
ξ_{f5}	[30, -15, 5]	7.67	33.78	2228.95	0.10	2900.08	-0.08	2734.24	-0.04	1961.22	-0.26
ξ_{f6}	[37, 25, 7]	10.56	45.06	3153.22	-0.09	2945.76	0.44	4042.05	-0.09	2731.25	0.37
ξ_{f7}	[50, 35, 10]	14.93	61.69	4209.66	0.01	4175.46	0.02	6223.44	-0.07	3932.84	0.26

0-10% | 10-20% | 20-30% | > 30% | *Energy saving of (NMPC + Trajectory) compared to other controllers*

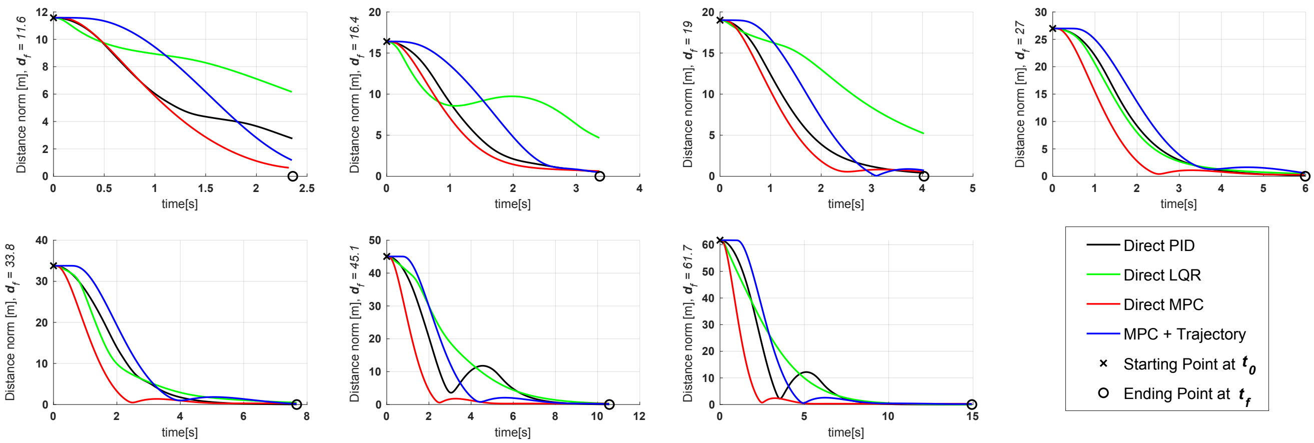

Figure 6. Convergance of the absolute position to the destination ξ_f for all examined reference points

the physical constraints, and the optimal velocity is specified as a trajectory. The performed simulation tests assess the effects of the velocity and the results confirm the validity of the hypothesis. Moreover, the results exhibit a remarkable energy saving of about 20% on average, compared to the standard reference tracking when using different feedback controllers. The limitation of the current work is that the drag effect is modelled using its first approximation, no disturbances were considered, and thrust/torque coefficients are assumed constant. Continued efforts are needed to involve these effects in future work to evaluate the practicability of this approach. Furthermore, the futurity of this approach is to examine integrating it within the distributed MPC to be used online along with the controller.

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