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Finite-time Consensus for a Class of Event-triggered Controllers for Multi-agent Systems

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Abstract— **In this paper, the finite-time consensus problem for single integrator multi-agent systems with fixed and undirected communication topologies is investigated. A general class of event-triggered controllers is proposed and a rigorous stability analysis is provided, showing convergence of the error signals in a finite time. An upper bound on the settling time is given. Various examples showcasing controllers falling under the class of controllers proposed in this paper are presented. Simulations are provided to verify the theoretical results.**

I. Introduction

Finite-time event-triggered consensus of multi-agent systems (MASs) has attracted significant research attention over recent years, as it is ideal for practical implementations. It combines the benefits of finite-time and event-triggered controllers, giving swift convergence and predictable performance [1] while conserving limited computational and communication resources [2], [3].

Extensive research has been conducted on proposing event-triggered consensus protocols to achieve finite-time consensus in single integrator MASs [3]–[7]. These protocols typically extend finite-time strategies to event-triggered ones. Among the aforementioned papers, only [7] uses *disagreement vectors* for convergence analysis, a technique widely used in continuous finite-time consensus [1], [8]. However, there has been limited research on the properties of MASs under *more general* finite-time event-triggered controllers. To our knowledge, [9] is among the few studies that explore an inclusive class of controllers for achieving finite-time consensus through an event-triggered approach. In contrast to other works focusing on specific controllers, this study establishes conditions sufficient to prevent Zeno behaviour (an infinite number of events occurring in a finite period of time) while not investigating control protocols for finite-time convergence.

There has been some work on finite-time consensus controllers [10], but these results cannot be directly extended to event-triggered controllers. Similarly, there have been finite-time event-triggered consensus protocols proposed for general linear [11], [12] and nonlinear MASs [13], [14], although a comprehensive study of these approaches is lacking.

In this paper, we propose a class of event-triggered controllers for single integrator MASs under fixed, undirected topologies and prove that they will reach consensus in finite-time. An upper bound on the settling time is given. To the best of our knowledge, this is the first work examining finite-time consensus for a class of event-triggered controllers. All previously mentioned works examined a single proposed controller. Yu *et al.* [9] have examined Zeno behaviour for classes of controllers, but assumed that the controllers reached finite-time consensus.

The main contributions can be summarised as follows:

1) A novel class of event-triggered controllers is proposed and a detailed stability analysis is provided.
2) An upper bound on the settling time is obtained for any controller within the proposed class.

The rest of the paper is organised as follows. Section II presents mathematical theories and results that are used in this paper. The details of the MAS and the controller class are given in Section III. The stability proof for the proposed class of controllers is presented in Section IV and a bound on the settling time is given. Section V contains several examples and simulations supporting the theoretical results. Finally, conclusions are given in Section VI.

II. Preliminaries

In this section, we review some relevant algebraic graph theory and a Lyapunov-based finite-time stability theorem, and present some useful mathematical results.

A. Algebraic Graph Theory

The interaction topology of a multi-agent system consisting of n agents can be described by an undirected graph $\mathcal{G}(\mathcal{A}) = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$. The graph is composed of a set of nodes $\mathcal{V} = \{1, 2, \cdots, n\}$ where each node represents each agent, an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ where each edge represents a communication link between nodes, and an adjacency matrix $\mathcal{A} = [a_{ij}]$ where $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. It is assumed that there are no self loops in the $\mathcal{G}(\mathcal{A})$, i.e. $a_{ii} = 0$ for $i \in \mathcal{V}$. If agents i and j can communicate, then they are called neighbours. The set of neighbours for agent i is given by $\mathcal{N}_i = \{j : (i, j) \in \mathcal{E}, j \neq i\}$. The degree of each vertex is the number of its neighbouring vertices and the degree matrix of $\mathcal{G}(\mathcal{A})$ is $D = \text{diag}\{d_1, d_2, ..., d_n\}$ where d_i is defined as the number of vertices neighbouring vertex i. The Laplacian matrix of graph $\mathcal{G}(\mathcal{A})$ is given by $L = D - \mathcal{A}$. For an undirected graph, both L and $\mathcal{A}$ are symmetric matrices. If there exists a path between all pairs of nodes in graph $\mathcal{G}$, the graph is connected. If $\mathcal{G}(\mathcal{A})$ is connected, then the Laplacian matrix L has a simple eigenvalue 0, and $\mathbf{1}_n$ is the corresponding eigenvector, where $\mathbf{1}_n$ is the n-dimensional column vector with all elements being 1. The eigenvalues of L are denoted as $0 = \lambda_1(L) \leq \lambda_2(L) \leq ... \leq \lambda_n(L)$.

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B. Lemmas

Lemma 1. *[15] Consider the system $\dot{x} = f(x)$, where $f : \mathcal{D} \to \mathbb{R}^n$ is continuous on an open neighbourhood $\mathcal{D} \subseteq \mathbb{R}^n$ of the origin and $f(0) = 0$. Suppose there exists a continuous function $V : \mathcal{D} \to \mathbb{R}$ such that the following conditions hold:*

- *V is positive definite*
- *There exist real numbers $c > 0$ and $\alpha \in (0, 1)$, and an open neighbourhood $\mathcal{V} \subseteq \mathcal{D}$ of the origin such that*

$$\dot{V}(x) + c(V(x))^{\alpha} \leq 0, \qquad x \in \mathcal{V}\backslash\{0\}.$$

Then the origin is finite-time stable, and the settling time T is given by

$$T(x_0) \leq \frac{(V(x_0))^{1-\alpha}}{c(1-\alpha)}.$$

Lemma 2. *[16] If $x_i \geq 0$ and $0 < p < 1$, then*

$$\left(\sum_{i=1}^{n} x_i\right)^p \leq \sum_{i=1}^{n} x_i^p \leq n^{1-p}\left(\sum_{i=1}^{n} x_i\right)^p$$

Lemma 3. *[17] If $\mathbf{1}_n^T x = 0$ and L is the Laplacian of a connected, undirected graph, then*

$$x^T L x \geq \lambda_2(L) x^T x. \tag{1}$$

III. Problem Formulation

Consider a leaderless MAS with n identical agents, where the communication topology is modelled by the connected, undirected graph $\mathcal{G}$. The dynamics of each agent i is given by

$$\dot{x_i}(t) = u_i(t), \qquad i = 1, ..., n, \tag{2}$$

where $x_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the state and control input of agent i, respectively. The vectors x and u denote $x = [x_1, x_2, ..., x_n]^{\top}$ and $u = [u_1, u_2, ..., u_n]^{\top}$.

The control protocol is given by

$$u_i = -\sum_{j=1}^{n} a_{ij} f(x_i(t_k^i) - x_j(t_{k'}^j)) \tag{3}$$

where t_k^i is the last event time of agent i, and $t_{k'}^j$ is the last event time of agent j. The function f cannot be the trivial function $f = 0$, and must fulfil the following conditions:

- f is continuous,
- $f(0) = 0$,
- There exists some $1 \leq c < 2$ such that

$$x_i f(x_i - x_j) + x_j f(x_j - x_i) \geq |x_i - x_j|^c. \tag{4}$$

Examples of possible functions f are given in Section V.

Remark 1. *The conditions of the function f are critical to ensuring that convergence is reached in finite time. The function f must be continuous, so that $x_i(t)$ is continuously differentiable everywhere. If this is true, and $f(0) = 0$, then the system requirements for Lemma 1 are met. Condition (4) will be used in the convergence analysis, but can be expressed as follows if f is odd.*

$$\mathit{sign}(x - y) f(x - y) \geq |x - y|^{c-1}. \tag{5}$$

In the standard consensus problem explained in [17] and for a balanced graph, the average of the state vector x, given by $\bar{x}$, is an invariant quantity. In particular, leaderless MASs converge to the average position $\bar{x}$. Note that all undirected graphs are balanced. Therefore the agent states can be expressed as

$$x = \bar{x}\mathbf{1} + \delta \tag{6}$$

where $\delta \in \mathbb{R}^n$ satisfies $\sum_i^n \delta_i = 0$. The vector δ is called the *disagreement vector*. Using the disagreement vector, the closed-loop equation can be written as

$$\dot{\delta_i} = -\sum_{j=1}^{n} a_{ij} f(\delta_i(t_k^i) - \delta_j(t_{k'}^j)). \tag{7}$$

The measurement error is defined as

$$\begin{aligned} e_i(t) = &\sum_{j=1}^{n} a_{ij} f(\delta_i(t_k^i) - \delta_j(t_{k'}^j)) \\ &- \sum_{j=1}^{n} a_{ij} f(\delta_i(t) - \delta_j(t)). \end{aligned} \tag{8}$$

Finally, the event trigger condition is given by

$$|e_i(t)| \leq \frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} |\delta_i(t)|^{c-1} \tag{9}$$

where c is defined in (4), the matrix D is defined as $D = [d_{ij}] \in \mathbb{R}^{n \times n}$, $d_{ij} = a_{ij}^{\frac{2}{c}}$, and $0 < \sigma < 1$. While $\lambda_2(L(D))$ and $\bar{x}$ are global information, both values are constant and determined before the controller is applied. It is assumed that all agents have this information.

IV. Convergence Analysis

Theorem 1. *For the MAS described in (2) under the proposed controller (3) and the event condition (9), consensus is achieved in a finite time for all initial conditions.*

Proof. Throughout this proof, the dependence on t is omitted for ease. By rearranging (8), we obtain

$$\sum_{j=1}^{n} a_{ij} f(\delta_i(t_k^i) - \delta_j(t_{k'}^j)) = \sum_{j=1}^{n} a_{ij} f(\delta_i - \delta_j) + e_i \tag{10}$$

and by substituting (10) in the closed-loop equation (7), we have

$$\dot{\delta_i} = -\sum_{j=1}^{n} a_{ij} f(\delta_i - \delta_j) - e_i. \tag{11}$$

Consider the following Lyapunov candidate,

$$V = \frac{1}{2}\delta^{\top}\delta = \frac{1}{2}\|\delta\|^2 = \frac{1}{2}\sum_{i=1}^{n} \delta_i^2 \tag{12}$$

where $\|\delta\|$ denotes the Euclidean norm of δ. Taking the time derivative of the Lyapunov candidate and substituting the closed-loop equation (11) yields

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} \delta_i \dot{\delta}_i \\
&= \sum_{i=1}^{n} \delta_i \Big(-\sum_{j=1}^{n} a_{ij} f(\delta_i - \delta_j) - e_i \Big) \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_i a_{ij} f(\delta_i - \delta_j) - \sum_{i=1}^{n} \delta_i e_i.
\end{aligned} \tag{13}
$$

For the ease of analysis let

$$
\dot{V}_1 = -\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_i a_{ij} f(\delta_i - \delta_j) \tag{14}
$$

and

$$
\dot{V}_2 = -\sum_{i=1}^{n} \delta_i e_i \tag{15}
$$

so that $\dot{V} = \dot{V}_1 + \dot{V}_2$.

First, consider $\dot{V}_1$. The following equation holds for all undirected graphs,

$$
\begin{aligned}
\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_i a_{ij} f(\delta_i - \delta_j) &= \sum_{j=1}^{n}\sum_{i=1}^{n} \delta_j a_{ji} f(\delta_j - \delta_i) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \delta_j a_{ji} f(\delta_j - \delta_i) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} \delta_j a_{ij} f(\delta_j - \delta_i)
\end{aligned} \tag{16}
$$

as $a_{ij} = a_{ji}, \ \forall i,j = 1,\cdots,n$ for undirected graphs. Therefore (14) can be expressed as

$$
\begin{aligned}
\dot{V}_1 &= -\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_i a_{ij} f(\delta_i - \delta_j) \\
&= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \Big(\delta_i a_{ij} f(\delta_i - \delta_j) + \delta_j a_{ij} f(\delta_j - \delta_i) \Big) \\
&= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \Big(\delta_i f(\delta_i - \delta_j) + \delta_j f(\delta_j - \delta_i) \Big).
\end{aligned} \tag{17}
$$

From the property of the function f given in (4), Equation (17) can be written as

$$
\begin{aligned}
\dot{V}_1 &\le -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} |\delta_i - \delta_j|^c \\
&= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \Big(a_{ij}^{\frac{2}{c}} (\delta_i - \delta_j)^2 \Big)^{\frac{c}{2}} \\
&\le -\frac{1}{2}\Big(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{\frac{2}{c}} (\delta_i - \delta_j)^2 \Big)^{\frac{c}{2}}
\end{aligned} \tag{18}
$$

where the last inequality results from Lemma 2. Let D be defined as the adjacency matrix given by $D = [d_{ij}] \in \mathbb{R}^{n\times n}$, $d_{ij} = a_{ij}^{\frac{2}{c}}$. Therefore, $L(D)$ is the graph Laplacian matrix of $\mathcal{G}(D)$. Then $\dot{V}_1$ can be written as

$$
\dot{V}_1 \le -\frac{1}{2}\Big(2\delta^\top L(D)\delta \Big)^{\frac{c}{2}}. \tag{19}
$$

From the definition of δ, $\mathbf{1}_n^T\delta = 0$ and so Lemma 3 can be applied.

$$
\begin{aligned}
\dot{V}_1 &\le -\frac{1}{2}\Big(2\lambda_2(L(D))\delta^\top\delta \Big)^{\frac{c}{2}} \\
&= -\frac{1}{2}\Big(2\lambda_2(L(D)) \times 2 \times \frac{1}{2}\delta^\top\delta \Big)^{\frac{c}{2}} \\
&= -\frac{1}{2}\Big(4\lambda_2(L(D)) \Big)^{\frac{c}{2}} V^{\frac{c}{2}}.
\end{aligned} \tag{20}
$$

Next, $\dot{V}_2$ must be bounded as follows

$$
\begin{aligned}
\dot{V}_2 &= -\sum_{i=1}^{n} \delta_i e_i \\
&= -\delta^\top e \\
&\le \| -\delta^\top e \| \\
&\le \|\delta\|\|e\| \\
&= \frac{\|\delta\|\|e\| V^{\frac{c}{2}}}{V^{\frac{c}{2}}} \\
&= \frac{\|\delta\|\|e\| V^{\frac{c}{2}}}{\frac{1}{2}^{\frac{c}{2}} \|\delta\|^{\frac{2c}{2}}} \\
&\le 2^{\frac{c}{2}} \|\delta\|^{1-c} \|e\| V^{\frac{c}{2}}.
\end{aligned} \tag{21}
$$

Substituting the event trigger condition (9) into the previous inequality yields

$$
\begin{aligned}
\|e\| &= \Big(\sum_{i}^{n} |e_i|^2 \Big)^{\frac{1}{2}} \\
&\le \Bigg(\sum_{i}^{n} \Big(\frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} |\delta_i(t)|^{c-1} \Big)^2 \Bigg)^{\frac{1}{2}} \\
&= \Bigg(\Big(\frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} \Big)^2 \sum_{i}^{n} \big(|\delta_i|^{c-1} \big)^2 \Bigg)^{\frac{1}{2}} \\
&= \Bigg(\Big(\frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} \Big)^2 \Bigg)^{\frac{1}{2}} \Big(\sum_{i}^{n} \big(|\delta_i|^{c-1} \big)^2 \Big)^{\frac{1}{2}} \\
&= \frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} \Big(\sum_{i}^{n} |\delta_i|^{2(c-1)} \Big)^{\frac{1}{2}} \\
&= \frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}} n^{(2-c)}} n^{1-(c-1)} \Big(\sum_{i}^{n} |\delta_i|^2 \Big)^{\frac{1}{2}(c-1)} \\
&= \frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}}} \|\delta\|^{c-1}
\end{aligned} \tag{22}
$$

Therefore,

$$
\begin{aligned}
\dot{V}_2 &\le 2^{\frac{c}{2}} \|\delta\|^{1-c} \times \frac{\frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}}{2^{\frac{c}{2}}} \|\delta\|^{-(1-c)} \times V^{\frac{c}{2}} \\
&\le \frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}} V^{\frac{c}{2}}.
\end{aligned} \tag{23}
$$

As $\dot{V} = \dot{V}_1 + \dot{V}_2$, $\dot{V}$ can be bounded using (20) and (23) as follows.

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}(4\lambda_2(L(D)))^{\frac{c}{2}}V^{\frac{c}{2}} + \frac{1}{2}\sigma(4\lambda_2(L(D)))^{\frac{c}{2}}V^{\frac{c}{2}} \\ &\leq -\frac{(1-\sigma)}{2}(4\lambda_2(L(D)))^{\frac{c}{2}}V^{\frac{c}{2}}.\end{aligned} \tag{24}$$

It is clear that

$$\dot{V} + \frac{(1-\sigma)}{2}(4\lambda_2(L(D)))^{\frac{c}{2}}V^{\frac{c}{2}} \leq 0 \tag{25}$$

as required by Lemma 1 and therefore the MAS reaches consensus in finite time for all initial conditions. Furthermore, using the same lemma, the settling time T is given by

$$T \leq \frac{1}{\eta(1-\frac{c}{2})}V(x_0)^{(1-\frac{c}{2})} \tag{26}$$

where $\eta = \frac{(1-\sigma)}{2}(4\lambda_2(L(D)))^{\frac{c}{2}}$. It should be noted that the choice of controller only affects the value of c, so different controllers using the same c value will give the same upper bound for the settling time. □

V. Simulations

Simulations are given to verify the theoretical proof of Theorem 1. We present four functions that satisfy the conditions for f and simulate the MAS subject to controller (3) using these example functions. The controllers are all applied to the same leaderless MAS described by (2) with five agents and communication topology given by Figure 1. The value of event trigger threshold is $\sigma = 0.6$ and the initial positions of the agents are given by $x_0 = [2, 1, -1, 4, -5]^\top$. As the controllers are all applied to the same system, the results can be directly compared.

A. Example 1

First consider the function

$$f = \text{sig}(y-z)^\alpha \tag{27}$$

where $\text{sig}(x)^\alpha = \text{sign}(x)|x|^\alpha$ and $0 < \alpha < 1$. The function (27) satisfies condition (4) as shown below.

$$\begin{aligned}&y\text{sig}(y-z)^\alpha + z\text{sig}(z-y)^\alpha \\ &\quad= y\text{sign}(y-z)|y-z|^\alpha + z\text{sign}(z-y)|z-y|^\alpha \\ &\quad= y\text{sign}(y-z)|y-z|^\alpha + \\ &\qquad z\text{sign}(-(y-z))|-(y-z)|^\alpha \\ &\quad= y\text{sign}(y-z)|y-z|^\alpha + \\ &\qquad z(-\text{sign}(y-z)|y-z|^\alpha) \\ &\quad= |y-z|^\alpha\text{sign}(y-z)(y-z) \\ &\quad= |y-z|^{\alpha+1}\end{aligned} \tag{28}$$

where $c = \alpha + 1$.

The system was simulated using the following controller,

$$u_i = -\sum_{j=1}^{n} a_{ij}\text{sig}(x_i(t_k^i) - x_j(t_{k'}^j))^\alpha \tag{29}$$

where $\alpha = 0.3$. The trajectories are shown in Figure 2. The settling time upper bound was found by substituting the system and controller parameters into (26), yielding $T \leq 12.9607$, which is reflected in the simulated trajectories.

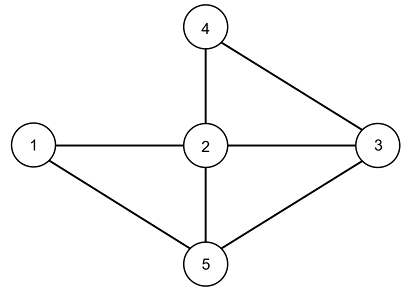

Fig. 1: Communication topology for simulated examples

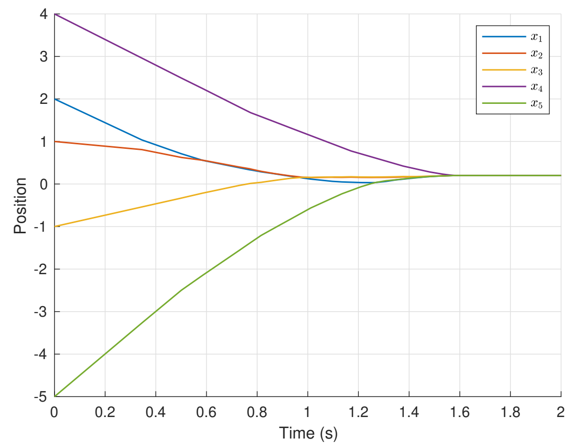

Fig. 2: Simulated trajectories under controller (29)

B. Example 2

Now consider the function

$$f = \begin{cases} y-z & (y-z) < -1, (y-z) > 1 \\ \text{sig}(y-z)^{\frac{1}{5}} & -1 \leq y-z \leq 1 \end{cases} \tag{30}$$

It is clear that f is continuous and $f(0) = 0$. The function is odd, so (5) may be used. This function fulfils the requirements as

$$\text{sign}(y-z)f(y-z) \geq |y-z|^{1.5-1} \tag{31}$$

for all $y, z \in \mathbb{R}$. Equation (31) is proved below.

First consider the case $(y-z) < -1$ or $(y-z) > 1$, where is is clear that $\text{sign}(y-z)(y-z) = |y-z| \geq |y-z|^{\frac{1}{2}}$ for $|y-z| > 1$. Then consider the case where $-1 \leq y-z \leq 1$. Then

$$\begin{aligned}\text{sign}(y-z)\text{sig}(y-z)^{\frac{1}{5}} &= |y-z|^{\frac{1}{5}} \\ &\geq |y-z|^{\frac{1}{2}}\end{aligned} \tag{32}$$

for $|y-z| \leq 1$.

The same MAS was simulated, and the trajectories are shown in Figure 3. For this controller, the settling time is limited by $T \leq 11.0054$.

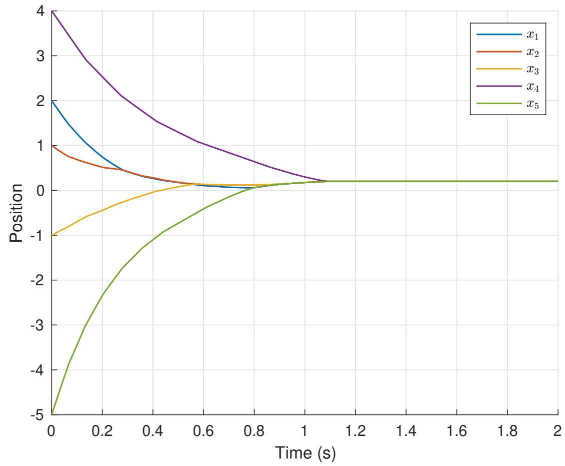

Fig. 3: Simulated trajectories under controller using function (30)

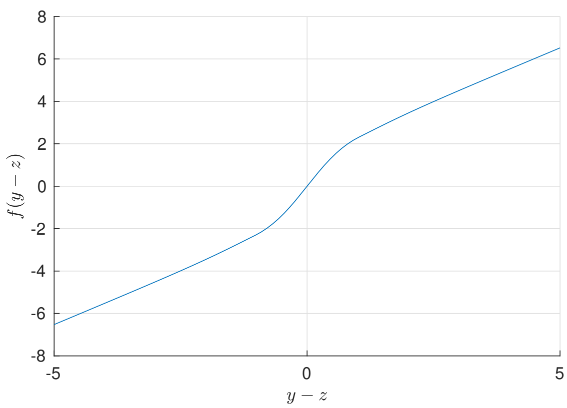

Fig. 4: A graph of (33)

C. Example 3

Now consider the function

$$f = \begin{cases} |y-z|\tanh(y-z) - (3\tanh 1 - \tanh 1) & (y-z) < -1 \\ 3\tanh(y-z) & -1 \leq y-z \leq 1 \\ |y-z|\tanh(y-z) + (3\tanh 1 - \tanh 1) & (y-z) > 1 \end{cases} \tag{33}$$

From Figure 4, f is continuous and $f(0) = 0$. It can be shown that

$$\mathrm{sign}(y-z)f(y-z) \geq |y-z|^{1.9-1} \tag{34}$$

for all $y, z \in \mathbb{R}$. This is similar to the proof for (31) and is shown graphically in Figure 5. Once more, the system under a controller using (33) as f was simulated, with results given in Figure 6. The bound on the settling time is given by $T \leq 20.2428$.

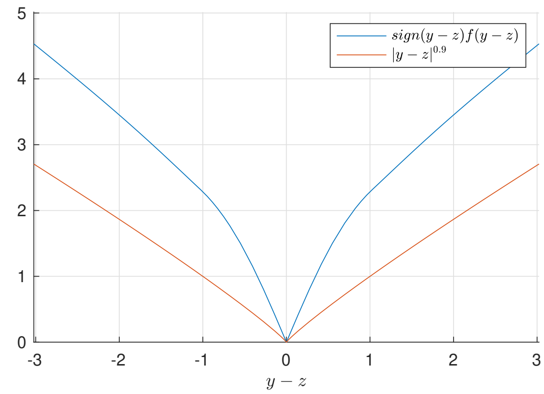

Fig. 5: A graph comparing $\mathrm{sign}(y-z)f(y-z)$ and $|y-z|^{1.9-1}$ where f = (33)

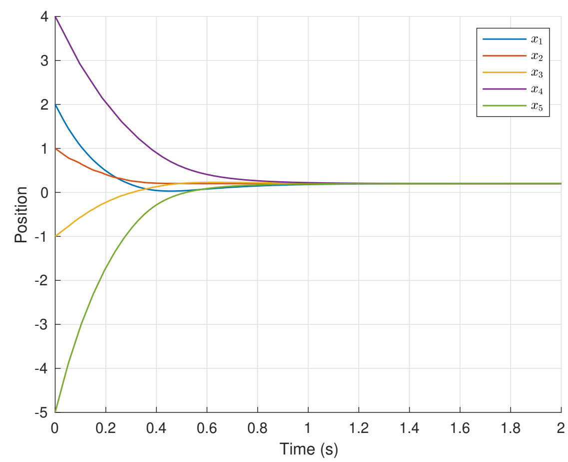

Fig. 6: Simulated trajectories under controller using function (33)

D. Example 4

Finally consider the function

$$f = \begin{cases} x - (3\ln 2 - 1) & (y-z) < -1 \\ 3\mathrm{sign}(y-z)\ln(|y-z|+1) & -1 \leq y-z \leq 1 \\ x + (3\ln 2 - 1) & (y-z) > 1 \end{cases} \tag{35}$$

Similarly to (33), it can be shown that

$$\mathrm{sign}(y-z)f(y-z) \geq |y-z|^{1.9-1}. \tag{36}$$

for all $y, z \in \mathbb{R}$. Once again, this is similar to the proof for (31) and can be seen in a graph. The system under this controller was simulated, with results given in Figure 7. The settling time was found to satisfy $T \leq 20.2428$, which is the same as the previous example because the same value of c was used as previously mentioned.

It is clear from Figures 2, 3, 6, and 7 that the system reaches consensus under all the simulated controllers within the bound of the settling times given in (26). The system always converges to the same position, which is the average

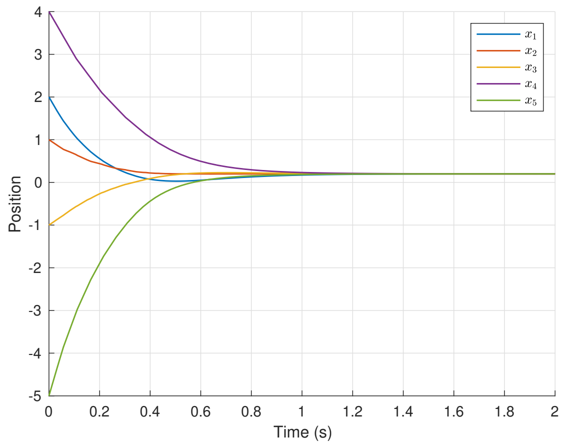

Fig. 7: Simulated trajectories under controller using function (35)

of the initial agent positions. While the bound on the settling time depends on only the value of c from the function f, from the simulations it can be seen that the system reaches consensus faster if the value of $|u_i|$ is greater, particularly for larger values of $|x_i(t_k^i) - x_j(t_{k'}^j)|$. Figures 2 and 3 demonstrate this as the system converges more quickly under the controller using (30) and

$$|\text{sig}(y-z)^{0.3}| \leq \begin{cases} |y-z| & (y-z) < -1, (y-z) > 1 \\ |\text{sig}(y-z)^{\frac{1}{5}}| & -1 \leq y-z \leq 1 \end{cases} \tag{37}$$

for all $y, z \in \mathbb{R}$.

VI. Conclusion

In this paper, we proposed a class of event-triggered controllers for single-integrator MASs with undirected communication topologies. The class of controllers was shown to reach consensus in finite-time, and an upper bound on the settling time was obtained. Numeric simulations were presented to demonstrate the effectiveness of the proposed controllers. Future work includes extending the controller class to higher dimension systems and investigating a class of controllers for general linear and nonlinear MASs.

References

[1] L. Wang and F. Xiao, "Finite-time consensus problems for networks of dynamic agents," *IEEE Transactions on Automatic Control*, vol. 55, no. 4, pp. 950–955, 2010.

[2] D. V. Dimarogonas and K. H. Johansson, "Event-triggered control for multi-agent systems," in *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*. IEEE, 2009, pp. 7131–7136.

[3] Y. Zhu, X. Guan, X. Luo, and S. Li, "Finite-time consensus of multi-agent system via nonlinear event-triggered control strategy," *IET Control Theory & Applications*, vol. 9, no. 17, pp. 2548–2552, 2015.

[4] H. Zhang, D. Yue, X. Yin, S. Hu, and C. xia Dou, "Finite-time distributed event-triggered consensus control for multi-agent systems," *Information Sciences*, vol. 339, pp. 132–142, 2016.

[5] B. Hu, Z.-H. Guan, and M. Fu, "Distributed event-driven control for finite-time consensus," *Automatica*, vol. 103, pp. 88–95, 2019.

[6] J. Fu, Y. Wan, and T. Huang, "Event-triggered finite-time practical consensus of multiagent systems with general directed communication graphs," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 17, pp. 7255–7277, 2020.

[7] X. Wang, J. Li, J. Xing, and R. Wang, "A novel finite-time average consensus protocol based on event-triggered nonlinear control strategy for multiagent systems," *Journal of Inequalities and Applications*, vol. 2017, pp. 1–13, 2017.

[8] X. Lin and Y. Zheng, "Finite-time consensus of switched multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 7, pp. 1535–1545, 2016.

[9] H. Yu and T. Chen, "On zeno behavior in event-triggered finite-time consensus of multiagent systems," *IEEE Transactions on Automatic Control*, vol. 66, no. 10, pp. 4700–4714, 2020.

[10] Y. Shang, "Finite-time consensus for multi-agent systems with fixed topologies," *International Journal of Systems Science*, vol. 43, no. 3, pp. 499–506, 2012.

[11] C. Du, X. Liu, W. Ren, P. Lu, and H. Liu, "Finite-time consensus for linear multiagent systems via event-triggered strategy without continuous communication," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 1, pp. 19–29, 2019.

[12] Z. Cao, C. Li, X. Wang, and T. Huang, "Finite-time consensus of linear multi-agent system via distributed event-triggered strategy," *Journal of the Franklin Institute*, vol. 355, no. 3, pp. 1338–1350, 2018.

[13] L. Ji, D. Lv, S. Yang, X. Guo, and H. Li, "Finite time consensus control for nonlinear heterogeneous multi-agent systems with disturbances," *Nonlinear Dynamics*, vol. 108, no. 3, pp. 2323–2336, 2022.

[14] Y. Zhang, J. Wang, and G. Chen, "Adaptive event-based finite-time consensus of nonholonomic multi-agent systems subject to unknown disturbances," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 13, pp. 7391–7407, 2022.

[15] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM Journal on Control and optimization*, vol. 38, no. 3, pp. 751–766, 2000.

[16] W. J. Lick, *Difference equations from differential equations*. Springer Science & Business Media, 2012, vol. 41.

[17] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on automatic control*, vol. 49, no. 9, pp. 1520–1533, 2004.