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Bearing-only Formation Control of Multi-agent Systems using a Signed Protocol

Jeslin Jacob M., Ajul Dinesh and Ameer K. Mulla

***Abstract*— This paper presents a new signed protocol for bearing-only formation control of homogeneous multi-agent system consisting of single integrator agents. The agents communicate over an undirected interaction topology, and the desired formation is specified by inter-neighbour bearings. A bearing-only controller that moves the agent in a direction normal to the desired bearings based on the location of the agent with respect to desired bearing vector is presented. To uniquely specify the centroid and scale of the formation, a leader-follower configuration is analyzed, along with the leaderless case. Stability and convergence of the multi-agent system with the proposed controller are analyzed using Lyapunov techniques. It is shown that, using the proposed distributed bearing-only formation control, the formation converges to the desired bearing-rigid formation. As the proposed controller uses sign function rather than absolute magnitude, it requires accurate inter-neighbour bearing measurements *only* near its desired bearing direction. This eliminates the requirement for accurate sensors during controller implementation. Simulation results validate the effectiveness of the proposed controller for formation control with and without leader agents.**

***Index Terms*— Multi-agent systems, Bearing-rigid formation, Bearing-only control, Sign-based control.**

I. Introduction

Multi-agent system consists of multiple interacting agents with identical or non-identical dynamics. These agents can be ground vehicles, robots, aerial vehicles, or even subsystems of a larger system. With advancements in distributed control, multi-agent systems find applications in a variety of fields like agriculture [1], surveillance, defense [2], crowd monitoring, search and rescue missions [3], etc.

Formation control is a class of cooperative control of multi-agent systems in which the agents attempt to align themselves to form a desired formation shape [4]. This formation shape can be specified using absolute positions, inter-neighbour distances, or inter-neighbour bearings [5]. Formation control using absolute positions allows for all possible manoeuvres of agents, but, requires precise position sensors for control. Distance-based formation control relaxes this need for absolute position measurements, by relying on relative distance measurements. As scaling of the formation alters the inter-neighbour distance, distance-based formations can perform only translation and rotation manoeuvres [6]. In contrast, when the desired formation is specified using inter-neighbour bearing vectors, the formation can perform both translation, scaling and unified rotation, as these manoeuvres do not alter the inter-neighbour bearing vectors.

In this work, we introduce a distributed bearing-only control law for a multi-agent system to achieve a formation specified using inter-neighbor bearings. To uniquely specify a formation using bearing vectors, bearing rigidity theory was introduced in [7], analogous to the notion of distance rigidity in distance-based controls. The concepts of bearing rigidity and infinitesimal bearing rigid formations are detailed in [8]. Formation control based on bearings can be achieved using bearing-only or bearing-based control laws. The former directly utilize inter-neighbour bearing vectors and are therefore nonlinear, while the latter uses the projection of relative displacements and is therefore linear in nature [8].

Considering leader-follower configuration, a bearing-based control law for translation and formation scaling of single integrator agents is presented in [9] for an undirected interaction topology. Bearing-based control laws are also discussed for multi-agent systems consisting of single integrator, double integrator, and unicycle agents communicating over a directed interaction graph in [10] and [11]. The work in [12] presents a bearing-based formation tracking approach when the interaction among the agents is both directed and time-varying. Since these bearing-based control laws use the projection of relative displacement vector, the agents need to be equipped with precise distance sensors like sonar or radar, which can be bulky and have limited fields of view. In contrast, bearing-only control laws rely solely on relative bearing vectors and hence they can be implemented using low-cost onboard cameras, which are both lighter and offer a wider field of view.

Considering nonlinear agent dynamics, the work in [13] introduced bearing-only control for a group of UAVs with bilateral high-level control for the leaders. In [14], a bearing-only consensus and formation control law for single integrator agents is presented, using gradient flow techniques. In [15], bearing-only control for a multi-agent system consisting of single integrator agents is discussed, and this is extended to double integrator agents and unicycles in [16]. Furthermore, in [17], a unified bearing-only control law for a heterogeneous multi-agent system, including single integrator, double integrator, and unicycle agents, is presented. However, all these control laws require accurate measurements of inter-neighbour bearings throughout.In contrast to existing bearing-only controllers, the controller presented in this work uses a sign function to select the desired direction

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of agents' motion. Since a sign function is used instead of absolute bearing magnitude, the controller only requires accurate measurement of the bearing vectors in the vicinity of the desired bearing vector. This eliminates the need for too accurate sensors.

In this work, we address the bearing-only formation control problem of a homogeneous multi-agent system comprising n agents with single integrator dynamics, communicating over an undirected graph. A novel bearing-only formation control law based on a signed protocol is proposed, which is applicable to both leader-less and leader-follower scenarios. We analyze the convergence of the multi-agent system to achieve the desired formation shape in both the cases. In the case of leader-follower configuration, by specifying the location of leaders, convergence of the formation up to a unique centroid and formation scale.

The main contributions of this paper are twofold:

1) A novel bearing-only formation controller for a single integrator multi-agent system is presented, which only requires absolute bearing measurements near the desired bearing vector.
2) The convergence of the formation in a leader-follower configuration using the proposed bearing-only control for followers is validated, when the desired formation is specified in terms of position and scale, using a minimal number of leader agents.

The rest of the paper is organized as follows. Section II provides preliminaries on graph theory and bearing-rigidity concepts, along with the problem definition. Section III presents the proposed bearing-only controller for formation control in both leader-less and leader-follower cases, along with convergence analysis. The results are validated through simulations in Section IV, followed by conclusions and future directions in Section V.

II. Preliminaries and Problem formulation

Consider a homogeneous multi-agent system consisting of n-single integrator agents $a_1, a_2, \dots, a_n$. Dynamics of agent a_i is given by,

$$\dot{p}_i = v_i, \quad i \in \{1, 2, \dots n\}. \tag{1}$$

Here p_i, $v_i \in \mathbb{R}^d$ represent position and velocity of the agent a_i. Let $\mathbf{p} = \begin{bmatrix} p_1^T & p_2^T & \cdots & p_n^T \end{bmatrix}^T \in \mathbb{R}^{nd}$ and $\mathbf{v} = \begin{bmatrix} v_1^T & v_2^T & \cdots & v_n^T \end{bmatrix}^T \in \mathbb{R}^{nd}$ respectively denote the position and velocity configuration of the multi-agent system.

A. Graph Theory Preliminaries

The multi-agent system in (1) communicate over an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where, agents $a_1, a_2, \dots, a_n$ forms the vertex set $\mathcal{V}$ and each communicating pair of agents forms the edge set $\mathcal{E}$.

An edge $(a_i, a_j) \in \mathcal{E}$ indicates that a_i can measure relative bearing of a_j with respect to a_i. For an undirected graph $\mathcal{G}$, $(a_i, a_j) \in \mathcal{E} \implies (a_j, a_i) \in \mathcal{E}$. The set of neighbours of agent a_i is denoted by $\mathcal{N}_i$ and is defined as, $\mathcal{N}_i = \{a_j : (a_i, a_j) \in \mathcal{E}\}$. An oriented graph is a graph obtained by assigning directions to the edges of a given graph. For an oriented graph with m edges, the incidence matrix $H \in \mathbb{R}^{m \times n}$ is defined as,

$$[H]_{kj} = \begin{cases} 1 & \text{if } a_j \text{ is terminal node of } k^{\text{th}}\text{edge} \\ -1 & \text{if } a_j \text{ is starting node of } k^{\text{th}}\text{edge} \\ 0 & \text{if } a_j \notin k^{\text{th}}\text{edge} \end{cases}. \tag{2}$$

A formation $\mathcal{G}(\mathbf{p})$ is obtained by assigning position configuration $\mathbf{p}$ to the agents in $\mathcal{V}$, communicating over $\mathcal{G}$. Let $(a_i, a_j) \in \mathcal{E}$ be the k^{th} edge of the graph $\mathcal{G}$. The edge vector for this communicating pair of agents is defined as,

$$e_k = e_{i,j} = p_j - p_i. \tag{3}$$

Using incidence matrix H, we have,

$$\mathbf{e} = \mathbf{H}\mathbf{p} \tag{4}$$

where $\mathbf{e} = \begin{bmatrix} e_1^T & e_2^T & \cdots & e_m^T \end{bmatrix}^T$ and $\mathbf{H} = H \otimes I_d$. For an edge $e_k := (a_i, a_j) \in \mathcal{E}$, the bearing vector g_{ij} is defined as,

$$g_k = g_{i,j} = \begin{cases} \frac{e_{i,j}}{\|e_{i,j}\|} & \text{if } \|e_{i,j}\| \neq 0 \\ \mathbf{0}_d & \text{otherwise} \end{cases} \tag{5}$$

where, $\mathbf{0}_d \in \mathbb{R}^d$ denotes the vector of all zeros. As $e_{i,j} = -e_{j,i}$, we have the bearing vectors satisfying $g_{i,j} = -g_{j,i}$. Further, as can be inferred from (5), the bearing vector $g_{i,j}$ is a unit vector pointing from a_i to a_j.

Orthogonal projection matrix for a vector $g_{i,j}$ is defined as,

$$P_{ij} = I_d - g_{i,j} g_{i,j}^T. \tag{6}$$

It is used to obtain the projection of any vector onto the orthogonal component of the vector $g_{i,j}$, as shown in Fig. 1. Here P_{ij}^* is the orthogonal projection matrix corresponding to desired bearing $g_{i,j}^*$.

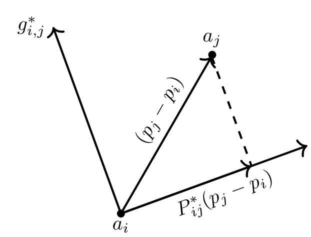


Fig. 1: Effect of orthogonal projection matrix

B. Bearing-rigid Formations

The uniqueness conditions for a formation specified using inter-neighbour bearing vectors are given by *bearing rigidity theory* and are stated as follows.

Definition 1. *[8] A formation $\mathcal{G}(\mathbf{p}^*)$, with desired position configuration of agents $\mathbf{p}^*$ and interaction topology $\mathcal{G}$, is said to be* infinitesimally bearing rigid *if any allowable infinitesimal motion (motion that preserves inter-neighbour bearings) of the formation corresponds only to translation and scaling of the formation, and does not change the shape of the formation.*

Let $\mathcal{L}_g^* \in \mathbb{R}^{nd \times nd}$ represents the bearing Laplacian matrix with desired bearings, defined as,

$$[\mathcal{L}_g^*]_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i} P_{ij}^* & \text{if } i = j \\ -P_{ij}^* & \text{if } i \neq j,\ a_j \in \mathcal{N}_i \ . \\ \mathbf{0}_d & \text{otherwise} \end{cases} \quad (7)$$

Using this bearing Laplacian matrix, the rank condition for infinitesimal bearing rigidity is given by [18],

$$rank(\mathcal{L}_g^*) = dn - d - 1.$$

When the agents in the multi-agent system achieve the desired bearing-rigid formation $\mathcal{G}(\mathbf{p}^*)$, we have $P_{ij}^*(p_j - p_i) = \mathbf{0}_d \ \forall (a_i, a_j) \in \mathcal{E}$ and hence, $\sum_{j \in \mathcal{N}_i} P_{ij}^*(p_j - p_i) = \mathbf{0}_d$, which in matrix form is given by,

$$\mathcal{L}_g^* \mathbf{p} = \mathbf{0}_{dn}. \quad (8)$$

C. Leader-Follower configuration

Consider a multi-agent system of n-agents in leader-follower configuration. Let the first n_l agents be leaders, and the remaining n_f agents be followers, such that $n = n_l + n_f$. The set of leaders and followers are respectively denoted by $L = \{a_1, a_2, \ldots, a_{n_l}\}$ and $F = \{a_{n_l+1}, a_{n_l+2}, \ldots, a_n\}$. With this, the position configuration of the agents $\mathbf{p}$ and the bearing Laplacian matrix $\mathcal{L}_g^*$ for the formation can be subdivided as,

$$\mathbf{p} = \begin{bmatrix} \mathbf{p}_l \\ \mathbf{p}_f \end{bmatrix} \quad \text{and} \quad \mathcal{L}_g^* = \begin{bmatrix} \mathcal{L}_{ll} & \mathcal{L}_{lf} \\ \mathcal{L}_{fl} & \mathcal{L}_{ff} \end{bmatrix}.$$

where, $\mathbf{p}_l \in \mathbb{R}^{dn_l}$, $\mathbf{p}_f \in \mathbb{R}^{dn_f}$, $\mathcal{L}_{ll} \in \mathbb{R}^{dn_l \times dn_l}$, $\mathcal{L}_{lf} = \mathcal{L}_{fl}^T \in \mathbb{R}^{dn_l \times dn_f}$ and $\mathcal{L}_{ff} \in \mathbb{R}^{dn_f \times dn_f}$. When the leader agents maintain the desired leader-to-leader bearing, the bearing rigidity conditions for the multi-agent system simplifies to $rank(\mathcal{L}_{ff}) = dn_f$ [9]. Thus, when leader configuration satisfies $\mathbf{p}_l = \mathbf{p}_l^*$, using (8), the corresponding unique configuration of the followers is given by,

$$\mathbf{p}_f^* = -\mathcal{L}_{ff}^{-1} \mathcal{L}_{fl} \mathbf{p}_l^*. \quad (9)$$

D. Problem Formulation

Consider a homogeneous multi-agent system of n-agents with dynamics (1), communicating over an undirected graph $\mathcal{G}$. The formation shape is specified using inter neighbour bearings $g_{i,j}^*, \forall (a_i, a_j) \in \mathcal{E}$.

Assumption 1. *The desired formation $\mathcal{G}(\mathbf{p}^*)$ is infinitesimally bearing rigid.*

First, we present our sign-based distributed bearing-only controller for formation stabilization in leader-less case.

Problem 1. (Bearing-only formation stabilization) *For the multi-agent system in* (1) *communicating over $\mathcal{G}$, obtain distributed bearing-only control law for agents that depends on direction of current bearing with respect to desired bearing, to achieve the unique desired formation $\mathcal{G}(\mathbf{p}^*)$ specified in terms of inter-neighbour bearings $g_{i,j}^*$, i.e.,*

$$\lim_{t \to \infty} g_{i,j}(t) = g_{i,j}^* \quad \forall \ (a_i, a_j) \in \mathcal{E}.$$

Further, to specify the centroid and scale of the formation, we consider a leader-follower configuration, where we consider a minimal set of leader agents.

Problem 2. (Bearing-only formation stabilization in leader-follower configuration) *For the multi-agent system in* (1) *communicating over $\mathcal{G}$, when leader agents satisfy $\mathbf{p}_l = \mathbf{p}_l^*$, obtain distributed bearing-only control law for followers that requires only the direction of current bearing with respect to desired bearing, to achieve the unique configuration $\mathbf{p}_f^*$ in* (9) *specified by inter-neighbour bearings $g_{i,j}^*$.*

To solve the formation stabilization problems, we make the following standard assumptions:

Assumption 2. *No agents collide with each other during the evolution of the formation, and the configuration of the agents does not satisfy $g_{ij} = -g_{ij}^*$, as it leads to flip ambiguity.*

III. Bearing-only formation control

In this section, we introduce a bearing-only control law for agents to stabilize the formation in both leader-less and leader-follower configurations. Unlike existing control laws that require precise calculations of the bearing vector, our approach presents a novel bearing-only controller using a sign function, which only requires precise calculation of the bearing vector in the vicinity of the desired bearing vector $g_{i,j}^*$.

A. Bearing-only Formation Stabilization in Leader-less Configuration

Consider a bearing rigid formation $\mathcal{G}(\mathbf{p}^*)$, of n-single integrator agents communicating over an undirected graph $\mathcal{G}$. The formation is specified using desired inter neighbour bearings $g_{ij}^*,\ \forall (a_i, a_j) \in \mathcal{E}$. We propose a novel bearing-only control law for agent a_i given by,

$$v_i = \sum_{j \in \mathcal{N}_i} sgn(g_{i,j^*}^{\perp T}\ g_{i,j}) g_{i,j^*}^{\perp}. \quad (10)$$

Here, $sgn(.)$ represents the sign function defined as,

$$sgn(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \ . \\ 0 & \text{if } x = 0 \end{cases} \quad (11)$$

According to this control law, agent a_i consider the location of a_j with respect to the desired bearing vector $g_{i,j}^*$ and takes the corresponding control strategy which would be either $g_{i,j^*}^{\perp}$ or $-g_{i,j^*}^{\perp}$. The graphical representation of the control action of a_i, based on its position with respect to a_j is shown in Fig. 2. Here, the blue circle denoted by a_i^* represents the desired configuration of agent a_i, which maintains the desired bearing $g_{i,j}^*$ with respect to agent a_j. The desired bearing vector $g_{i,j}^*$ and its normal components $g_{i,j^*}^{\perp}$ and $-g_{i,j^*}^{\perp}$ are also marked in blue colour.

As can be inferred from Fig. 2 and control law (10), an agent a_i moves normal to the desired bearing vector, in order to minimize the component of the actual bearing vector in

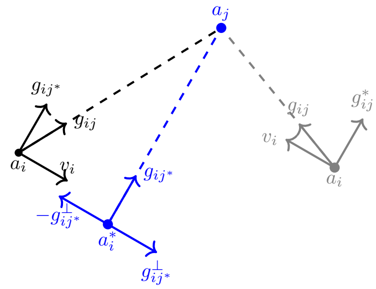


Fig. 2: Graphical interpretation of the proposed bearing-only control law

the direction normal to desired bearing vector. For instance, when a_i takes the position shown by the gray circle, the corresponding control action is shown by the gray control vector v_i, which is the same as $-g^{\perp}_{i,j^*}$, the normal vector to $g^*_{i,j}$ that lies in the same side as $g_{i,j}$. Similarly, when a_i takes the position shown by the black circle, the corresponding control action v_i (black color) is the same as $g^{*\,\perp}_{i,j}$, which is the normal vector to $g^*_{i,j}$ that lies in the same side as that of $g_{i,j}$.

As the strategy of the agent remains constant as long as the agent is on the same side as the desired bearing, precise measurement of the actual bearing vector is required only when the current bearing approaches the desired bearing. This eliminates the requirement for precise sensors. Furthermore, when the current bearing aligns with the desired bearing, the velocity input to the agent becomes zero, which further prevents any deviation from the actual bearing.

Theorem 1. *For the multi-agent system of n-agents with dynamics* (1)*, communicating over an undirected graph $\mathcal{G}$, using bearing-only control law for the agents given by* (10)*, the multi-agent system asymptotically converges to the desired formation $\mathcal{G}(\mathbf{p}^*)$ specified using inter-neighbour bearings $g^*_{ij}\forall(a_i, a_j) \in \mathcal{E}$.*

Proof. Let $\mathbf{p}^*$ represent the desired configuration of the agents satisfying the desired bearing vectors $g^*_{i,j}\ \forall(a_i, a_j) \in \mathcal{E}$. Considering position error for a_i given by $\delta_{p_i} = p_i - p^*_i$, we have the formation error δ_p defined as,

$$\delta_p = \mathbf{p} - \mathbf{p}^* \tag{12}$$

where, $\mathbf{p}^* = \begin{bmatrix} p^*_1, & p^*_2, & \dots, & p^*_n \end{bmatrix}^T$. Using incidence matrix H and the control function (10), the velocity configuration $\mathbf{v} = \begin{bmatrix} v^T_1 & v^T_2 & \cdots & v^T_n \end{bmatrix}^T$ is given by,

$$\dot{\mathbf{p}} = \mathbf{v} = -\mathbf{H}^T M \mathbf{g}^{*\perp}. \tag{13}$$

Here, $M = diag(sgn(g^{*\perp T}_k g_k)) \in \mathbb{R}^{md \times md}$ and $\mathbf{g}^{*\perp} = \begin{bmatrix} g^{*\perp}_1 & g^{*\perp}_2 \cdots & g^{*\perp}_m \end{bmatrix}^T \in \mathbb{R}^{md}$, where g_k and $g^{*\perp}_k$ respectively denote the actual bearing vector of k^{th} edge and the unit vector normal to the desired bearing vector of k^{th} edge.

Here, we consider a Lyapunov function as in [19] and [20] for the sign based controller. Using Lyapunov candidate function $V = \frac{1}{2}\|\delta_p\|^2$, we have,

$$\dot{V} = \delta^T_p \dot{\delta}_p.$$

With δ_p as in (12) and considering formation stabilization problem ($\dot{\mathbf{p}}^* = 0$), the time derivative of V turns out to be,

$$\dot{V} = \delta^T_p \mathbf{v}.$$

Substituting for the velocity configuration $\mathbf{v}$ from (13), we obtain,

$$\begin{aligned} \dot{V} &= -\delta^T_p \mathbf{H}^T M \mathbf{g}^{*\perp} \\ &= -(\mathbf{p} - \mathbf{p}^*)^T \mathbf{H}^T M \mathbf{g}^{*\perp} \\ &= -\mathbf{e}^T M \mathbf{g}^{*\perp} + \mathbf{e}^{*T} M \mathbf{g}^{*\perp}. \end{aligned} \tag{14}$$

The last equality follows from (4).

Since e^*_k and $g^{*\perp}_k$ are normal to each other, the second term in (14) turns out to be,

$$\mathbf{e}^{*T} M \mathbf{g}^{*\perp} = \sum_{k=1}^{m} sgn(g^{*\perp T}_k g_k) e^{*T}_k g^{*\perp}_k = 0.$$

Now, consider the first term in (14). It follows that,

$$\begin{aligned} \mathbf{e}^T M \mathbf{g}^{*\perp} &= \sum_{k=1}^{m} sgn(g^{*\perp T}_k g_k) e^T_k g^{*\perp}_k \\ &= \sum_{k=1}^{m} \underbrace{sgn(g^{*\perp T}_k g_k)}_{I} \underbrace{(g^T_k g^{*\perp}_k)}_{II} \|e_k\| \geq 0, \end{aligned} \tag{15}$$

since terms I and II are of same sign and $\|e_k\| > 0$. Substituting these terms in (14), we have,

$$\dot{V} = -\mathbf{e}^T M \mathbf{g}^{*\perp} \leq 0$$

Here, $\dot{V} = 0$ if $g_{ij} = g^*_{ij}$ and $\dot{V} < 0$ otherwise. Thus, using control law (10) for the agents, the multi-agent system asymptotically converges to the desired formation $\mathcal{G}(\mathbf{p}^*)$. This completes the proof of Theorem 1. □

Further, for the leader-less formation stabilization, we show the invariance of centroid and formation scale.

Consider the centroid of the formation, $\bar{p} \in \mathbb{R}^d$ defined as,

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i = \frac{1}{n}(1_n \otimes I_d)^T \mathbf{p}. \tag{16}$$

With this, the formation scale $\alpha \in \mathbb{R}$ is given by,

$$\alpha = \sum_{i=1}^{n} \|p_i - \bar{p}\|^2 = \|p - 1_n \otimes \bar{p}\|^2. \tag{17}$$

Using the proposed bearing-only control law (10) for agents, the behaviour of the centroid and formation scale is given by the following lemma.

Lemma 1. *With the proposed bearing-only control* (10) *for agents, under Assumption 1 and 2, the formation centroid $\bar{p}$ is invariant and the formation scale decreases as long as $g_{ij} \neq g^*_{ij}\forall(a_i, a_j) \in \mathcal{E}$. Furthermore, the position of the agents and the edge vectors of the formation are upper bounded by a positive scalar.*

Proof. Using (16), we have $\dot{\bar{p}} = \frac{1}{n}(1_n \otimes I_d)^T \dot{\mathbf{p}} = 0$, since $(1_n \otimes I_d)\mathbf{H}^T = 0$. This implies that the centroid of the formation is invariant under the proposed control action.

Further, using (17), the rate of change of formation scale is given by,

$$\dot{\alpha} = 2(\mathbf{p} - 1_n \otimes \bar{p})^T \dot{\mathbf{p}} = -2(\mathbf{p} - 1_n \otimes \bar{p})\mathbf{H}^T M g^{*\perp}.$$

Substituting $\bar{p}$ from (16) and using the fact that $(1_n \otimes I_d)^T \mathbf{H}^T = 0$, we have,

$$\dot{\alpha} = -2\mathbf{p}^T \mathbf{H}^T M \mathbf{g}^{*\perp} = -2\mathbf{e}^T M \mathbf{g}^{*\perp} \leq 0.$$

The last inequality follows from (15). Therefore, $\dot{\alpha} = 0$ if $\mathbf{g} = \mathbf{g}^*$ and $\dot{\alpha} < 0$ otherwise, which implies that the formation scale decreases until the desired formation is achieved.

As $\dot{\alpha} \leq 0$, we have, $\alpha(0) \geq \alpha(t)$. Therefore, it turns out that,

$$\sqrt{\alpha(0)} \geq \|\mathbf{p} - 1_n \otimes \bar{p}\| \geq \|\mathbf{p}\| - \|1_n \otimes \bar{p}\|.$$

Thus, $\|\mathbf{p}\| \leq \sqrt{\alpha(0)} + \|1_n \otimes \bar{p}\|, \ \forall t$ and hence, $\|\mathbf{e}\| \leq \|\mathbf{H}\|\|\mathbf{p}\| \leq \|\mathbf{H}\|(\sqrt{\alpha(0)} + \|1_n \otimes \bar{p}\|)$. Therefore, using control law (10) for the agents, both the position and edge vector of the formation are upper bounded by a positive scalar. $\square$

B. Bearing-only formation control in leader-follower configuration

Consider the homogeneous multi-agent system of n-agents with dynamics (1) in leader-follower configuration, communicating over an undirected graph $\mathcal{G}$. Here, the leaders are controlled externally to define the centroid and scale of the formation. A minimum of two leaders are required to specify both centroid and formation scale. The leader configuration satisfies $\mathbf{p}_l = \mathbf{p}_l^*$, which maintains the desired bearing between the leaders. By specifying the leader configuration $\mathbf{p}_l^*$, the position and scale of the formation gets predefined.

In the following, we present a bearing-only control law for followers to drive the multi-agent system to the desired formation $\mathcal{G}(\mathbf{p}^*)$, specified using inter-neighbour bearings $g_{ij}^* \ \forall (a_i, a_j) \in \mathcal{E}$.

Corollary 1. *Consider the n-agent multi-agent system with n_l leaders and $n_f = n - n_l$ followers. When leaders satisfy the desired configuration $\mathbf{p}_l = \mathbf{p}_l^*$, using control law* (10) *for followers, the multi-agent system asymptotically stabilizes to the desired formation $\mathcal{G}(\mathbf{p}^*)$ specified by inter-neighbour bearings.*

Proof. Consider the position error in formation δ_p, given by (12). As leader configuration satisfies $\mathbf{p}_l = \mathbf{p}_l^*$, the position error δ_p takes the form $\delta_p = \begin{bmatrix}\delta_{p_l}^T & \delta_{pf}^T\end{bmatrix}^T = \begin{bmatrix}0_{n_l d}^T & \delta_{pf}^T\end{bmatrix}^T$. With the control law (10) for followers, the velocity configuration of the multi-agent system is given by,

$$\mathbf{v} = -\begin{bmatrix}0 & 0 \\ 0 & I_{n_f d}\end{bmatrix} \mathbf{H}^T M \mathbf{g}^{*\perp}. \tag{18}$$

Consider the Lyapunov candidate function as $V = \frac{1}{2}\|\delta_p\|^2$. Using (18), the derivative of the Lyapunov function turns out to be,

$$\dot{V} = \delta_p^T \dot{\mathbf{p}} = -\delta_p^T \begin{bmatrix}0 & 0 \\ 0 & I_{n_f d}\end{bmatrix} \mathbf{H}^T M \mathbf{g}^{*\perp}. \tag{19}$$

As $\delta_p^T \begin{bmatrix}0 & 0 \\ 0 & I_{n_f d}\end{bmatrix} = \begin{bmatrix}0_{n_l d}^T & \delta_{pf}^T\end{bmatrix} = \delta_p^T$, we have,

$$\dot{V} = -\delta_p^T \mathbf{H}^T M \mathbf{g}^{*\perp} < 0.$$

The last inequality follows from the proof of Theorem 1. Thus, using control law (10) for followers, when the leader configuration satisfies $\mathbf{p}_l = \mathbf{p}_l^*$, the multi-agent system asymptotically stabilizes to the desired formation specified using inter neighbour bearing vectors and leaders' configuration $\mathbf{p}_l$. Here, the position and scale of the formation can be pre-specified based on $\mathbf{p}_l^*$. This completes the proof of Corollary 1. $\square$

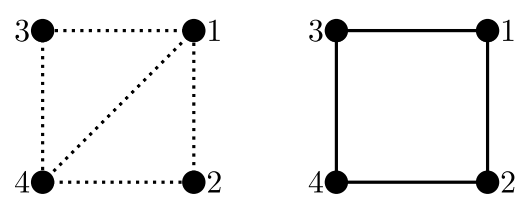
Remark 1. *In the leader-follower configuration, the leaders' configuration $\mathbf{p}_l$ satisfies the desired leader-to-leader bearings. Further, as leader agents do not take control actions based on follower configuration, the communication graph can be assumed to be mixed, with leader-to-follower interaction as a directed edge while the interaction within followers as undirected.*

IV. Simulation Results

The results of the proposed bearing-only formation control are validated through simulations, considering a multi-agent system of four agents. The initial position configuration of the agents is selected as,

$$p_1(0) = \begin{bmatrix}2 & 5\end{bmatrix}^T, \ p_2(0) = \begin{bmatrix}2 & 0\end{bmatrix}^T,$$
$$p_3(0) = \begin{bmatrix}3 & -5\end{bmatrix}^T, \ \text{ and } \ p_4(0) = \begin{bmatrix}-4 & 2\end{bmatrix}^T.$$

The communication among the agents is selected as shown in Fig. 3a. The objective of the multi-agent system is to form a square formation, as shown in Fig. 3b, using the proposed bearing-only control law for the agents.

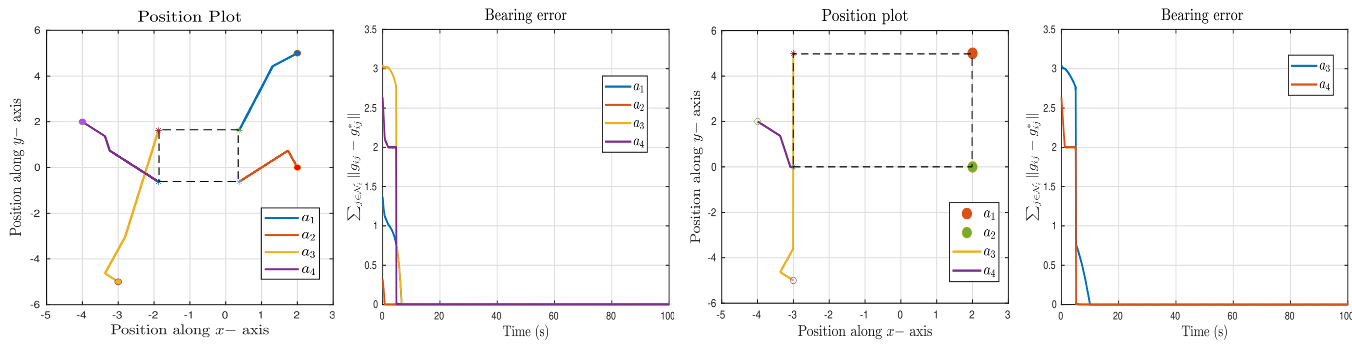


(a) Communication topology (b) Formation configuration

Fig. 3: Communication topology and desired configuration of agents in the multi-agent system

In leader-less formation control case, using control law (10) for the agents, the agents in the multi-agent system approach each other, as predicted by Lemma 1, until the desired formation shape is achieved. The agents move from their initial configuration to the desired configuration and remain in that configuration, as can be inferred from the position and bearing error plots in Fig. 4a.

Further, by considering a_1 and a_2 as leaders, we aim for formation stabilization at a particular point and to a predefined formation scale defined by leaders' configuration. The position plot and the corresponding bearing error plot for the agents are shown in Fig. 4b.



(a) Leaderless configuration

(b) Leader-follower configuration

Fig. 4: Position and Bearing error plot of agents with control law (10)

As can be inferred from the figures, the bearing error converges to zero. Consequently, the follower agents move from their initial configuration to the desired configuration, specified by both the leader configuration and inter-neighbour bearings, and they remain in that configuration.

V. Conclusions

This paper presents a bearing-only control law for formation control of a homogeneous multi-agent system consisting of single integrator agents. The proposed bearing-only controller is based on a sign function protocol that moves the agent normal to the desired bearing based on the direction of current bearing vector with respect to the desired bearing vector. In comparison to existing bearing-only control laws, the proposed control law is easy to implement, as the control action takes values that switch within a finite set of inputs. The convergence of the multi-agent system with the proposed control laws is analyzed mathematically and verified through simulation, for formation stabilization scenarios. The proposed approach can be extended to account for time-varying and directed interaction typologies, incorporating more complex agent dynamics, and addressing robustness against disturbances. Further, works are in progress to analyze finite time convergence with the specified controller.

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