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Achievable robustness for time-varying delay systems

Juan P. R. Moreira[1,3], João L. de C. Pereira[1], René D. O. Pereira[1],
Bismark C. Torrico[1], Fabrício G. Nogueira[1] and Tito L. M. Santos[2]

***Abstract*— This work proposes a closed-loop stability analysis of unstable open-loop systems represented by first-order and second-order plus time-delay (FOPTD and SOPTD) models. This analysis considers a proportional (P) controller and a predictor-based controller for FOPTD models, while a P and a phase-lead controller are considered for SOPTD models. In all cases, the achievable robustness has been computed considering time-varying delays. Several simulations were performed, and based on the integrated absolute error, it is shown that using a predictor is a better solution for large time-delay systems. Otherwise, the results are equivalent. In addition, the effect of the left half-plane pole of the SOPTD system is analysed. Finally, a comparison is made with recently published results that employ more complex and iterative algorithms.**

***Index Terms*— time-varying delay, robust control, predictor-based control, unstable models, small-gain theorem.**

I. INTRODUCTION

Time-delay models belong to a class of functional differential equations that are of infinite dimension, in contrast to ordinary differential equations (ODEs). Time delays appear in several processes from many fields of study, such as chemistry [1], engineering [2], [3], and, particularly, communication and information technology [4], where networked systems are predominant [5]. Large time delays can deteriorate the closed-loop performance or even lead the system to instability [6].

In the literature, there are mainly two approaches to analyse the closed-loop stability of LTI systems: the frequency-domain and the time-domain analyses [7]. In the frequency-domain analysis, classical methods, including root locus (limited to rational transfer functions), and the Nyquist criterion, can be mentioned. However, in practical applications, it is common to use the small-gain theorem, also based on the frequency domain, to impose a certain degree of robustness. Moreover, it can be extended to time-varying delay systems [8]–[11].

In the case of the time-domain analysis, the Lyapunov-Krasovskii functional can be employed, and it can be expressed in the form of linear matrix inequalities (LMIs) [12]. Focusing on industrial applications, many works derived stability analysis and control tuning rules considering first-order and second-order plus time-delay (FOPTD and SOPTD) models [13]–[15].

In the case of processes with large time delays, the use of predictors can improve the closed-loop performance [9], [16]. A wide discussion of when to use predictors can be found in [17], [18].

During the last decade, many publications have emerged to deal with time-varying delay, such as [19], [20]. Additionally, the recent work in [21] proposed delay-dependent conditions combining dissipativity theory, Lyapunov–Krasovskii functionals, and an iterative algorithm. A numerical example using a SOPTD process model, found an output feedback gain, after 178 iterations, by solving a LMI problem. To derive a more straightforward solution, this work proposes a set of stability analyses for closed-loop systems with unstable FOPTD and SOPTD process models with time-varying delay. Its three main contributions are: (i) based on the small-gain theorem, an analytical stability condition is derived for systems with unstable FOPTD models and a constant feedback gain, where the upper bound of the stabilisable time-varying delay is computed; (ii) to enlarge the upper bound of the stabilisable time-varying delay, the use of a predictor is proposed and a practical rule, based on performance analysis, used to decide if the predictor is the better choice, is also proposed; and (iii) it is also derived an analytical stability condition for systems with unstable SOPTD models and constant output feedback, where, to find a feasible solution, it is considered that only one pole is in the right half plane, being also shown that a phase-lead compensator can enlarge the upper bound of the stabilisable time-varying delay for such systems.

This work is organised as follows: Section II presents the concept of achievable robustness, based on the small-gain theorem, for closed-loop systems with time-varying delay. Section III discusses closed-loop stability using unstable FOPTD models. Section IV presents how to improve the achievable robustness using a predictor. Section V shows a performance analysis and a practical rule to decide when to use a predictor. Section VI discusses stability for systems with SOPTD models with a zero. Section VII presents the results of the conducted simulations, and finally, Section VIII presents a discussion and the conclusions of the work.

II. ROBUSTNESS OF CLOSED-LOOP SYSTEMS WITH TIME-VARYING DELAY

In this section, an analysis based on the small-gain theorem presented in [10] is conducted on a closed-loop system represented in Fig. 1, where P is the time-varying delay

[1]Department of Electrical Engineering, Federal University of Ceara, Fortaleza-CE, Brazil. e-mails: juanmoreira@alu.ufc.br (Juan P. R. Moreira), jluizcp@alu.ufc.br (João L. de C. Pereira), reneolimpio@alu.ufc.br (René D. O. Pereira), bismark@dee.ufc.br (Bismark C. Torrico), fnogueira@dee.ufc.br (Fabrício G. Nogueira).

[2]Department of Electrical Engineering, Federal University of Bahia, Salvador-BA, Brazil. e-mail: tlsantos@ufba.br (Tito L. M. Santos).

[3]The files from all simulations presented in this paper are found at Github: juanmoreira20 (Juan P. R. Moreira).

NOMENCLATURE

L_{max}	upper bound of the stabilisable constant time delay for a given controller, such that $h_{max} < L_{max}$
$\hat{h}$	time-delay uncertainty for the predictor whose stability is guaranteed
$h(t)$	time-varying delay in the range $[0, h_{max})$
h_p	nominal time delay of a predictor control structure
h_{max}	upper bound of the stabilisable time-varying delay

process, C_{eq} is the equivalent controller, r is the set-point, u is the control input, and y is the process output. The purpose of this analysis is to find the upper bound of the time-varying delay whose the closed-loop system is stable.

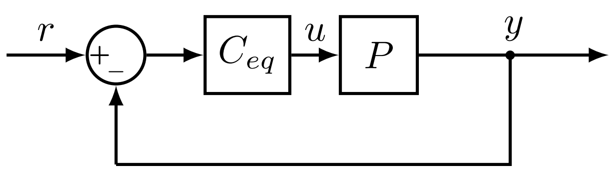

Fig. 1. Closed-loop system.

The most common robustness measures used in industrial practices are the phase margin ϕ_m and the gain margin g_m. They provide robustness against phase and gain increases of up to ϕ_m and g_m in the process, although not simultaneously. Note that ϕ_m is defined at the lowest frequency where the open-loop amplitude is 1, and g_m at the lowest frequency where the open-loop phase is -180 degrees.

Delay margin is the traditional way to define the smallest additional delay in the open-loop at the frequency of ϕ_m that results in an unstable closed-loop system. In other words, if the phase margin is ϕ_m at the frequency ω_ϕ, then the delay margin is $\Delta L_{max} = \min_i \frac{\phi_m^i}{\omega_\phi^i} > 0$ [7]. Note that the delay margin is not defined for negative delay errors since time delay cannot assume values less than zero.

However, delay margin is only valid for constant time-delay analysis. Therefore, considering time-varying delay systems, the strategy used in this work rely on the concept of robustness index from [8] and [22] and the following theorem from [10]:

Theorem 1: For the closed-loop system from Fig. 1, with continuous-time $P(s)$ and $C_{eq}(s)$, the system is stable for any time-varying delays defined by $\Delta(u) = u(t-h(t))$, with $0 \leq h(t) < h_{max}$, where $h(t)$ is the time-varying delay, and h_{max} is the upper bound of the stabilisable time-varying delay, if the following condition is satisfied

$$\left| \frac{C_{eq}(j\omega)P(j\omega)}{1+C_{eq}(j\omega)P(j\omega)} \right| < \frac{1}{h_{max}\omega}, \quad \forall \omega \in [0, \infty). \tag{1}$$

III. CLOSED-LOOP STABILITY ANALYSIS USING FOPTD MODELS

Consider an open-loop unstable FOPTD process given by

$$\begin{cases} \dot{x}(t) = ax(t) + u(t-h(t)) \\ y(t) = x(t) \\ u(t) = 0, \; t \in [-h_0, 0) \\ x(0) = x_0 = 0 \end{cases}, \tag{2}$$

where $a > 0$, t is the continuous time, $x(t)$ is the state, $y(t)$ is the output, $u(t) = 0$ is the control signal, x_0 is the value of the initial condition $x(0)$, h_0 is the value of $h(-h_0)$, and $h(t)$ is a time-varying delay, such that:

$$0 \leq h(t) < h_{max}. \tag{3}$$

To stabilise the system (2), a proportional controller can be used

$$u(t) = -k\,y(t). \tag{4}$$

Then, the closed-loop system is given by

$$\dot{x}(t) = ax(t) - kx(t-h(t)), \tag{5}$$

where $x(0) = 0$. Note that, for the case of $h(t) = 0$, the system (2) can be expressed by the figure 1 with $P(s) = \frac{1}{s-a}$ and $C_{eq}(s) = k$, such that, the following theorem can be stated:

Theorem 2: For the closed-loop system from Fig. 1, with continuous-time $P(s) = \frac{1}{s-a}$, the system can be stabilised for $C_{eq}(s) = k$ and time-varying delay defined by $\Delta(u) = u(t-h(t))$, with $0 \leq h(t) < h_{max}$, where $h(t)$ is the time-varying delay, and h_{max} is the upper bound of the stabilisable time-varying delay, if the following condition is satisfied:

$$h_{max} < \frac{1}{a}. \tag{6}$$

Proof: From the considerations in theorem (2) and the condition stated in theorem (1), it results

$$\left| \frac{kh_{max}\omega}{j\omega - a + k} \right| < 1, \quad \forall \omega \in [0, \infty). \tag{7}$$

Applying complex norm properties to (7), supposing $\exists\, M \in \mathbb{R}$, such that M is the lower bound of the condition from (1), and $0 \leq M < \infty$, it follows that:

$$\frac{(kh_{max}\omega)^2}{\omega^2 + (k-a)^2} \leq M^2 \quad \forall \omega \in [0, \infty). \tag{8}$$

Rearranging (8), one obtains:

$$\omega^2[M^2 - (kh_{max})^2] + (k-a)^2 \geq 0. \quad \forall \omega \in [0, \infty). \tag{9}$$

The inequality (9) is a sum of squares, meaning that it can only be equal to zero if $M^2-(kh_{max})^2$ and $(k-a)^2$ are also equal to zero. Thus, it results

$$M = h_{max}a. \tag{10}$$

Therefore, with (10), (9), and (7), the proof is concluded. ■

Corollary 2.1: The stability analysis performed on theorem (2) can be reduced to high frequencies only to determine the upper bound of the stabilisable time-varying delay

Proof: Borrowing the conditions found in the proof of theorem (2), as $k=a$, the high frequency effect is analysed by taking the limit:

$$\lim_{\omega\to\infty}\left|\frac{kh_{max}\omega}{j\omega-a+k}\right| = h_{max}a = M. \tag{11}$$

■

IV. Improving the achievable robustness

In this section, it is proposed to use a predictor-based control strategy to improve the maximum achievable robustness L_{max}.

$$u(t) = -k\left(\int_0^{h_p} e^{a\theta}u(t-\theta)d\theta + e^{h_pa}y(t)\right) \tag{12}$$

where h_p is the predictor constant time delay, which can be used as tuning parameter.

The following theorem is a direct result of Theorem 1 and the robustness index of [23]

***Theorem** 3:* The system in (2), feedbacked using the controller from (12), is stable for any time-varying delay, defined by $h_{max}=h_p+\hat{h}$, if the following condition is satisfied

$$\left|\frac{ke^{h_pa}}{j\omega-a+k}\right| < \frac{1}{\hat{h}\omega}, \quad \forall\omega\in[0,\infty). \tag{13}$$

where $\hat{h}$ is the predictor time-delay uncertainty defined as $\hat{h}=h_{max}-h_p$. Note that in order to guarantee stability in the range $[0,h_{max})$, then $\hat{h}>h_p$.

As in corollary (2.1), the robust stability condition is closer to being violated at high frequencies, thus

$$ke^{h_pa} < \frac{1}{\hat{h}}. \tag{14}$$

When applying the Routh-Hurwitz criterion to the delay-free system and from (14), the following condition is obtained

$$a < k < \frac{1}{e^{h_pa}\hat{h}}. \tag{15}$$

In this case, to obtain a feasible controller, then

$$a < \frac{1}{e^{h_pa}\hat{h}}, \tag{16}$$

leading to

$$\hat{h} < \frac{1}{ae^{h_pa}}. \tag{17}$$

To guarantee stability with delays $h(t)$ starting from zero, then

$$\hat{h} > h_p. \tag{18}$$

By using (17) and (18), it results

$$\hat{h} < \frac{1}{ae^{\hat{h}a}}. \tag{19}$$

In this case, the maximum achievable robustness is given by

$$\begin{cases} L_{max} & = \max\{2\hat{h}\}, \\ s.t.: & \hat{h} < \dfrac{1}{ae^{\hat{h}a}}. \end{cases} \tag{20}$$

By using numerical methods, the solution of (20) is $L_{max}=\dfrac{1.134}{a}$. Note that the stabilisable time delay using a predictor is in the range $[0,1.134]$, being a range 13.5% larger than that one for the proportional controller.

V. Performance analysis and comparison

In this section, comparative results were established by computing the integrated absolute error (IAE) multiple times across a range of L_{max} values, varying from 0.05 to 0.95, while considering the initial condition $x(0)=1$. Then, the following ratio was calculated:

$$I_p = \frac{IAE_{proportional}}{IAE_{predictor}}. \tag{21}$$

In this work, I_p is defined as a performance index. When the value of I_p is near one, both controllers exhibit similar performance. Conversely, when I_p is bigger than one, the predictor-based controller shows superior performance.

Figure 2 shows the I_p index for both cases $L=0$ and $L=0.9L_{max}$. Both controllers were computed for the same L_{max} for fair comparison in each iteration. Note that with L_{max} close to zero, both controllers have similar IAE index for $L=0$ while for $L=0.9L_{max}$, the IAE of the predictor-based controller is up to 10 % better.

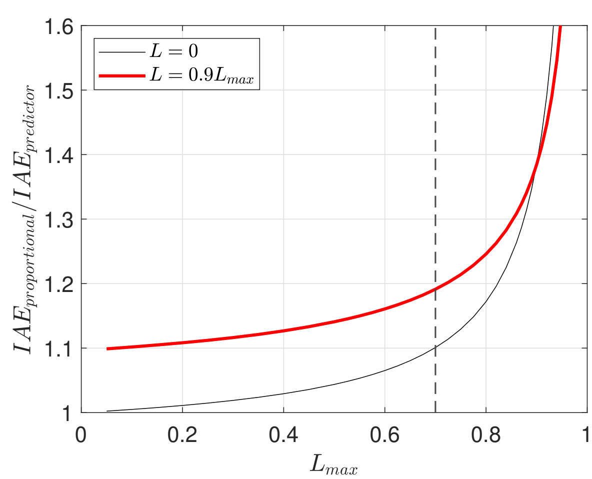

Fig. 2. Performance index for different values of L_{max}

A. Time responses comparison using FOPTD models

This section presents a comparative simulation between a proportional controller and a predictor-based proportional controller. Without loss of generality, it was chosen, plant pole $a = 1$. In this case, according to (6) and (20), the upper bound of the stabilisable time-varying delay is $L_{max} = 1$ for the proportional controller and $L_{max} = 1.134$ for the predictor-based proportional controller. In this case, for a fair comparison, both controllers were tuned with the same delay upper limit $h_{max} = 0.95$. Thus, the proportional controller gain was chosen as $k = 1.026$ while the predictor-based controller parameters were $k = 1.1308$, $h_p = 0.3h_{max} = 0.285$.

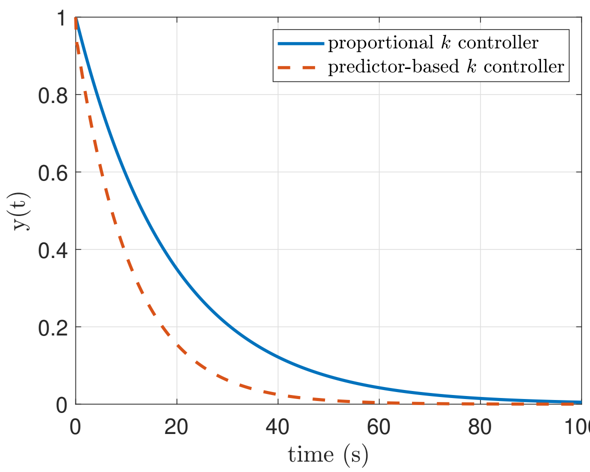

Fig. 3. Initial condition $x(0) = 1$ and $h(t) = 0$

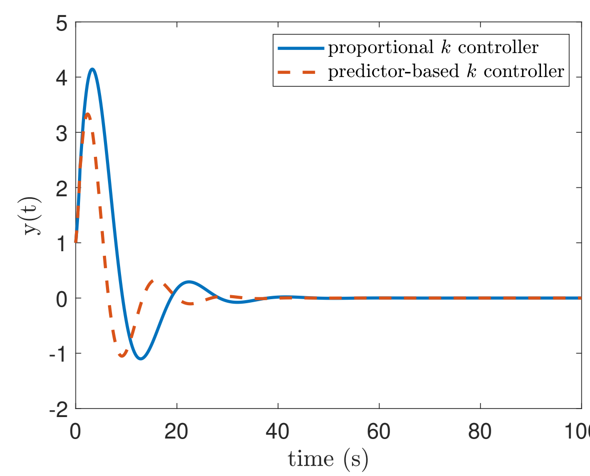

Fig. 4. Initial condition $x(0) = 1$ and $h(t) = 0.9025$

Figure 3 presents the closed-loop response for an initial condition $x_0 = 1$ and $h(t) = 0$. As can be seen, in both cases the plant output converges to zero without overshoot. Nevertheless the settling time within a range of 5% is $t_{5\%} = 32.4$ and $t_{5\%} = 56.0$ for the proportional and predictive controllers, respectively

Figure 4 presents the closed-loop response for an initial condition $x_0 = 1$ and $h(t) = 0.9h_{max} = 0.855$. The settling time for the predictor-based controller is still better than proportional controller, $t_{5\%} = 25$ against $t_{5\%} = 35$, with a smaller overshoot, which justifies the use of the predictor in both cases.

VI. Stability Analysis Using SOPTD Models with a Zero

This section analyses unstable processes represented by a delay-free second-order model with a zero, such as

$$G(s) = \frac{s+z}{(s-a)(s+p)}, \tag{22}$$

where $z, a, p \in \mathbb{R}$, such that $z > p > a > 0$.

A closed-loop system with a process represented by model (22) is Hurwitz stable when using a feedback gain in the range

$$k > \frac{ap}{z}. \tag{23}$$

However, if the process represented by (22) presents time delay, this gain k does not necessarily stabilise the process.

To guarantee stability for constant or time-varying delay, by using the robustness analysis from Section II, the following theorem can be stated:

***Theorem** 4:* A closed-loop system with process model (22) and feedback gain $k > \frac{ap}{z}$ is stable for any time-varying delay defined by $\Delta(u) = u(t - h(t))$, with $0 \leq h(t) < h_{max}$, if the following condition is satisfied:

$$\left| \frac{k[(j\omega)+z]}{(j\omega)^2 + j\omega(p-a+k) + (kz-ap)} \right| \leq \frac{1}{h_{max}\omega}, \quad \forall \omega \in [0, \infty). \tag{24}$$

It is important to mention that there is a trade-off between robustness and performance. If it is desired to improve the robustness, satisfying (23), then it is necessary that $k \to \frac{ap}{z}$. Therefore, condition (24) can be written as

$$\left| \frac{ap[(j\omega)+z]}{z(j\omega + p - a) + ap} \right| \leq \frac{1}{h_{max}}, \quad \forall \omega \in [0, \infty). \tag{25}$$

Condition (25) is closer to being violated at low frequencies. Thus, with $\omega \to 0$, the upper bound of the stabilisable time-varying delay is obtained as

$$h_{max} = \frac{1}{a} - \left(\frac{1}{p} - \frac{1}{z} \right). \tag{26}$$

Remark: As $\left(\frac{1}{p} - \frac{1}{z}\right) > 0$, the upper bound of the stabilisable time-varying delay h_{max} is worst when compared to unstable FOPTD processes. To solve this issue, it is suggested to use a phase-lead controller $C_{eq}(s) = k\frac{s+p}{s+z}$. Thus, the maximum achievable robustness will be equivalent to the one from unstable FOPTD processes.

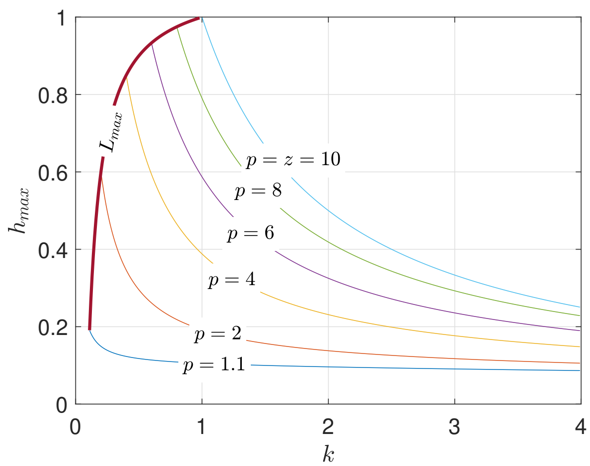

Fig. 5. Stability analysis for $z = 10$, $a = 1$

A. Numerical stability analysis

In this example, a second-order plus time delay plant, as defined in (22), is used for analysis. Then let's consider the right-half plane pole and zero equal to $a = 1$, $z = 10$

Thus, Figure 5 demonstrates the tendency for the upper bound of the stabilisable time-varying delay to increase as the value of p approaches z, cancelling the zero of the process.

Then, for theorem 4 and (26), it can be derived that stabilising condition (24) can only be violated for lower frequencies, thus, it is required that k tends to $\frac{ap}{z}$, so, it can be stated that:

$$\frac{|pa|}{|j\omega||(j\omega)+(p-a)|} < \frac{1}{h_{max}\omega}, \tag{27}$$

meaning that

$$h_{max} < L_{max} = \frac{1}{a} - \frac{1}{p}. \tag{28}$$

However, this is still a conservative condition compared to the FOPTD approach, in addition to the need to have $k = \frac{ap}{z}$, or else the condition will fail and the system will be unstable. To solve these questions, it can be used a phase lead controller in the form

$$C_{eq}(s) = \frac{k(s+p)}{(s+z)}. \tag{29}$$

In which the stable pole and zero dynamics are canceled, in a way that the closed loop can now be described using equation (5), showing that Theorem 2 is satisfied for the aforementioned system and, so as for systems with FOPTD models, it can be stated that constant and time-varying delays are bounded by (6).

VII. Simulation Results

This section provides two simulation examples for comparing methods of determining the maximum allowable delay in certain systems. This allows for the observation that which methods can be employed to define upper bound of the stabilisable time-varying delay with greater margin, in other words, in a less conservative manner.

A. Example 1

Addressing the time-varying problem of the network control system example from [4] in which the problem analysed is from a SSF perspective, and in [21], with a SOF perspective analysis, resulting in the proposed strategy achieving upper bound of the stabilisable time-varying discrete delay of 19 samples for the proposed system:

$$\dot{x(t)} = \begin{bmatrix} -0.8 & -0.01 \\ 1.00 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} u(t). \tag{30}$$

And, considering the time sample the same as in both articles as $T_s = 0.5$ and C = [0 1] as in [21], it is possible to formulate a transfer function, such that:

$$G(s) = \frac{0.1(s+4.8)}{(s+0.7887)(s-0.08875)} \tag{31}$$

Thus resulting in a second order transfer function with a zero and an unstable pole such as in [22], where $a = 0.08875, p = 0.7887, z = 4.8$, and, using (26), as follows:

$$L_{max} = \frac{1}{0.08875} - \left(\frac{1}{0.07887} - \frac{1}{4.8}\right).$$

Then, leading to:

$$C_{eq}(s) = k = 0.146, \quad L < L_{max} = 10.2081. \tag{32}$$

And, Using a phase lead controller employing the technique from (29):

$$C_{eq}(s) = \frac{0.185(s+0.07887)}{0.2083s+1}, \quad L < 11.2678. \tag{33}$$

That can be compared with the result on (32) showing that the phase lead controller improved upper bound of the stabilisable time-varying delay in 10.38%.

These comparisons are made to assess our results against those presented in [4] and [21]. The upper bound of the stabilisable time-varying-induced network delay is determined using the previously mentioned time sample, as depicted in Table I.

TABLE I

UPPER BOUND OF THE STABILISABLE TIME-VARYING DELAY.

	Controller	Delay	Discrete Delay
SSF	K = -[1.2625 1.2679]		2
SOF	K = - 0.1488		19
P	K = 0.146	10.2081	
PD	$C_{eq} = \frac{0.185(s+0.07887)}{0.2083s+1}$	11.2678	

B. Example 2

To analyse the effect of a predictor-based controller on the upper bound of the stabilisable time-varying delay, it is possible to observe the effects of a Proportional controller and the aforementioned predictor on a first order unstable plant used in the first example of [24]:

$$G(s) = \frac{3.433}{101.1s - 1}. \tag{34}$$

In which a = 1 and the system have a time constant $\tau = 101.1$. In this formulation, it is possible to compare the control technique from [6], resulting in:

$$C_{eq}(s) = k = 0.2913, \tag{35}$$
$$L < 101.1. \tag{36}$$

And, with the predictor controller technique in (20), leads to:

$$C_{eq}(s) = k = 0.2913, \tag{37}$$
$$L < 114.760. \tag{38}$$

Showing that predictor-based approach widens the domain in which the time-varying delay can be dealt without causing instability in comparison with proportional controller.

VIII. Conclusions

This work proposes and investigates the potential benefits of analytical stability conditions to determine the upper bound of the stabilisable time-varying delay, by using an analysis on frequency domain, for closed-loop systems with unstable FOPTD and SOPTD process models. Moreover, it proposes the use of predictors to further improve this upper bound in the case of time-varying delays with large variable range and a practical rule to decide when the predictor is the better choice.

The proposed methodology, despite its simplicity, is used to stabilise the system presented in [21] and surpasses its results (obtained by optimisation) for the upper bound of the stabilisable time-varying delay, also computing an analytic solution with a lower computational cost.

Possibilities for future works include comparing the proposed analytic expressions for the upper bound of the stabilizable time-varying delay with the time delay values found using the Lyapunov-Krasoviskii method. Additionally, another potential task is to generalise the observations made in this work to process models of any order.

Acknowledgment

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