2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Adaptive Observer with Parametric Uncertainty in the System Dynamics and Output: An Initial Excitation Approach

Atul Katiyar, Sayan Basu Roy and Shubhendu Bhasin

Abstract— **In this paper, an adaptive observer is designed, which addresses the problem of joint estimation of state and the unknown parameters that emerge in the system dynamics as well as the output equation of a multi-input multi-output (MIMO) plant. Conventional adaptive observers ensure parameter convergence when the input-output signals are significantly 'energy-rich' to satisfy the persistence of excitation (PE). The proposed work develops a two-layered filter adaptive observer architecture based on initial excitation (IE), which relaxes the stringent PE condition in terms of excitation requirement and online verifiability. The unknown initial condition of the state is strategically appended with the vector of the unknown parameter, which essentially results in a higher dimensional estimator. This extended parameter estimator proves to be instrumental in ensuring uniformly global exponential stability (UGES) of the estimation error, applicable in a delayed sense. As far as the authors are aware this is the first work where a 'relaxed' excitation condition is utilized for simultaneous estimation of states and the unknown parameters which appear both in the state dynamics and the output. To validate the efficacy of the performance of the adaptive observer proposed, a simulation study has been undertaken on a remotely piloted aircraft model.**

I. INTRODUCTION

An adaptive observer is a recurrent technique used to simultaneously estimate the states which are immeasurable and the parameters that are unknown. To deal with an uncertain SISO LTI system, an adaptive observer technique was proposed in [1]. Using minimal realization, an alternative approach was introduced in [2]. In [3], [4] an alternative non-minimal system representation was employed for the synthesis of adaptive observer for single-input single-output (SISO) systems. Another design was developed in [5], aiming to ensure that the parameter estimates would exponentially converge to the true parameters at an adjustable rate. Building upon the findings presented in [2], an equiobservable canonical system form was introduced in [6] for multi-input, multi-output (MIMO) systems. In [7], a straightforward and computationally efficient scheme for adaptive observers was introduced for MIMO systems with linear time-invariant (LTI) dynamics. This approach was further extended to MIMO systems with linear time-varying (LTV) dynamics in [8], [9].

Atul Katiyar is with the Electrical Engineering Department, MJP Rohilkhand University, Bareilly, U.P., India and the EE Department, Indian Institute of Technology Delhi, New Delhi 110016, India atulkatiyar4u@gmail.com.

Sayan Basu Roy is with the ECE Department, Indraprastha Institute of Information Technology Delhi. New Delhi, India sayanetce@gmail.com

Shubhendu Bhasin is with the Department of Electrical Engineering, Indian Institute of Technology Delhi, New Delhi, India sbhasin@ee.iitd.ac.in

In each of the adaptive observer designs mentioned above, the unknown parameters appear in the state dynamics only. The problem of uncertainty in the output, due to the difficulty of the appearance of unknown parameters in output feedback term, was lately addressed in the 1980s in [10]. Subsequently, various adaptive observer designs with uncertainty in output relation were proposed in [11]–[13]. Algorithm to simultaneously estimate state and unknown parameters both in state and output relation was first developed in [9] where a global convergent adaptive observer was proposed, and later followed by a high gain adaptive observer in [14] for MIMO nonlinear systems. Recently, using the method of decoupling parameter estimation and state observation, an adaptive observer is proposed in [15].

The convergence of parameters in all the above-cited designs of adaptive observer hinges around the requirement that the input signal must be an "energy-rich" probing signal at an appropriate frequency to ensure the persistence of excitation (PE) (refer to chapter 6 of [16]). Furthermore, this signal needs to maintain a substantial richness of trajectory throughout the entire operational time. Nonetheless, it is impractical to consistently observe if the signal is persistently exciting, as this requirement depends on the signal's future behavior. With a motive to relax stringent PE condition, a contemporary approach of initial excitation (IE) condition [17]–[21] based switched adaptive observer was brought up and highlighted in [22], [23] for SISO LTI systems followed by a robust adaptive observer [24] for MIMO LTI systems (subjected to unmodeled bounded disturbances), which relaxes the strict PE condition. This work builds upon the authors' earlier work [22]–[25] and tackles a more intricate problem by developing an IE-based adaptive observer for a category of MIMO LTI systems where systems involve unknown parameters in both the system dynamics and the output relationship while also relaxing the stringent PE condition. The proposed adaptive observer employs an estimator with a higher dimensionality than the parameter space, as discussed in [24], [26]. In this approach, the initial states that are not directly accessible are tactfully considered as extra unknown parameters and are appended with the vector of unknown system parameters. This incorporation enables the utilization of an IE-based single-switching mechanism, ensuring uniformly global exponential stability (UGES), applicable in a delayed context. As far as authors are aware this work is the first of its kind where actuator and sensor faults are simultaneously dealt with, under a relaxed excitation condition (IE).

A Lyapunov-based analysis establishes closed-loop switched system stability ensuring that the extended parameter estimator (beyond the dimension of the parameter space) provides uniformly global exponential stability (UGES), in a delayed context. Simulation results reinforce the advantages of the suggested algorithm

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Model Description

A MIMO LTI system is considered which is affected by parametric uncertainty in both system dynamics and output relation that appears in linear regression form as follows

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) + \phi(u, y)\theta\,, \qquad z(t_0) = z_0 \\ y(t) &= Cz(t) + \varphi(u, y)\theta \end{aligned} \tag{1}$$

where $z(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^m$, $u(t) \in \mathbb{R}^l$ denote state, output and input vectors, respectively. Linear regression terms $\phi(u, y)\theta$, $\varphi(u, y)\theta$ represent uncertainty in the state dynamics and output, respectively where $\theta \in \mathbb{R}^p$ is a constant vector representing the unknown regression coefficient, $\phi(u, y) \in \mathbb{R}^{n\times p}$ and $\varphi(u, y) \in \mathbb{R}^{m\times p}$ which are typically the function of known signals (u, y), represent the time-varying regressor matrices. Also, the triplet (A, B, C) is observable and controllable.

B. Objective

Incorporating information from both the input $u(t)$ and output $y(t)$, the objective is to devise an adaptive observer capable of jointly estimating the system state $z(t)$ and the unknown parameters θ that feature in both the system dynamics and the output relationship given by (1).

Assumption 1: It is assumed that the system input $u(t)$ stabilizes the system in such a way that $u(t), z(t) \in \mathcal{L}_\infty$, i.e., $u(t) \in U$ and $z(t) \in Z$ where $U \subset \mathbb{R}^l$, $Z \subset \mathbb{R}^n$ are compact sets.

It's important to emphasize that this is a customary assumption within the realm of adaptive observer design [5], [27], [28].

C. Preliminary Definitions and Concepts

Many of the conventional adaptive observer designs necessitate the persistence of excitation (PE) condition for the regressor to ensure parameter convergence [1], [29], [5], [7], [30], [31]. To address the influence of initial conditions on the regressor, the concept of uniform Persistence of Excitation (u-PE) is introduced, as detailed in [32]. This definition is specifically formulated for the pair (ξ, f), where $\xi(t, z)$ represents the regressor while $f(t, z)$ characterizes the system's dynamics, and it is articulated as follows

$$\dot{z} = f(z, t),\ z(t_0) = z_0,\ t \in [t_0, \infty),\ t_0 \geq 0 \tag{2}$$

Definition 1: A function $\xi(z, t) \in \mathbb{R}^{q\times p}$, where $q > p > 0$, is uniformly persistently exciting (u-PE) w.r.t. $f(z, t)$ (in (2)) if, $\exists\, T_{PE}, \gamma_{PE}, > 0, \forall (z_0, t_0) \in \mathbb{R}^n \times \mathbb{R}_{\geq 0}$, such that all corresponding solutions satisfy

$$\int_t^{t+T_{PE}} \xi(r,\, z(r))\xi^T(r,\, z(r))\, dr \geq \gamma_{PE}\, I_q, \qquad \forall t \geq t_0$$

where t_0 corresponds to the initial time, and $z_0 = z(t_0)$ denotes the initial state.

The subsequent proposition indicates that the PE signal necessitates an infinite amount of energy

Proposition 1: $\Xi_{PE} \cap \Xi_{\mathcal{L}_2} = \emptyset$, where Ξ_{PE} is the space of PE signals and $\Xi_{\mathcal{L}_2}$ represents the space of square-integrable signals.

Proof: For proof refer to [24]. ■

Unlike the Persistence of Excitation (PE) condition, the adaptive observer proposed operates under a much relaxed initial excitation (IE) requirement on the regressor [17]–[21].

The formal definition of IE condition can be referred in [18] and stated as

Definition 2: A function $\xi(z, t) \in \mathbb{R}^{q\times p}$, where $q > p > 0$, is uniformly initially exciting (u-IE) w.r.t. $f(z, t)$ (in (2)) if, $\exists\,, T_{IE},\, \gamma_{IE} > 0$, $\forall (z_0, t_0) \in \mathbb{R}^n \times \mathbb{R}_{\geq 0}$, such that all corresponding solutions satisfy

$$\int_{t_0}^{t_0+T_{IE}} \xi(r,\, z(r))\xi^T(r,\, z(r))\, dr \geq \gamma_{IE} I_q$$

Compared to the PE condition, it can be established that IE signals are proven to possess finite energy, as given in the subsequent proposition.

Proposition 2: $\Xi_{IE} \cap \Xi_{\mathcal{L}_2} \neq \emptyset$, where Ξ_{IE} is the space of IE signals.

Proof: For proof refer to [24]. ■

III. ADAPTIVE OBSERVER

A two-tier filter architecture is proposed for the design of adaptive observer, which is subsequently discussed.

A. First tier filtering

Using simple manipulation, the dynamic system mentioned in (1) can be rewritten as

$$\begin{aligned} &\dot{z}(t) = [A - LC]z(t) + Bu(t) + Ly(t) + \underbrace{(\phi(t) - L\varphi(t))}_{\rho(t)}\theta,\ z(t_0) = z_0 \\ &y(t) = Cz(t) + \varphi(t)\theta \end{aligned} \tag{3}$$

where the feedback gain $L \in \mathbb{R}^{n\times m}$ is selected in a manner that ensures $(A - LC)$ to be Hurwitz and $\rho(t)$ is considered to be a unified version of the regressors appearing in state dynamics and the output relation of (1). For the synthesis of adaptive observer, it is now possible to view the estimation of $z(t)$ as reliant on two external signals [8], denoted as $\rho(t)\theta$ and $Bu(t) + Ly(t)$, therefore, to streamline the design process, the solution of the state equation in (3) can be decoupled as

$$z(t) = z_\theta(t) + z_{u,y}(t) \tag{4}$$

where $z_\theta(t)$ and $z_{u,y}(t)$ are the contribution to the state dynamics arising from the signals $\rho(t)\theta$ and $Bu(t) + Ly(t)$, respectively and their respective dynamics are given as

$$\dot{z}_\theta(t) = [A - LC]z_\theta(t) + \rho(t)\theta,\ z_\theta(t_0) = z_{(\theta)0} \tag{5}$$

$$\dot{z}_{u,y}(t) = [A - LC]z_{u,y}(t) + Bu(t) + Ly(t),\ z_{u,y}(t_0) = z_{(u,y)0} \tag{6}$$

and in line with (4), $z(t_0) = z_\theta(t_0) + z_{u,y}(t_0)$. The observer for $z_\theta(t)$ can be given as

$$\dot{\hat{z}}_\theta(t)=[A-LC]\hat{z}_\theta(t)+\rho(t)\hat{\theta}+\mu(t),\ \hat{z}_\theta(t_0)=\hat{z}_{(\theta)0} \quad (7)$$

and following similar lines, observer for $z_{u,y}(t)$ is given by

$$\dot{\hat{z}}_{u,y}(t)=[A-LC]\hat{z}_{u,y}(t)+Bu(t)+Ly(t),\ \hat{z}_{u,y}(t_0)=\hat{z}_{(u,y)0} \quad (8)$$

where $\hat{\theta}(t)$ denotes the parameter estimate, and $\mu(t)$ serves as an auxiliary signal to account for the disparity between the true parameter θ and its estimate $\hat{\theta}(t)$. This signal holds significant importance in the formulation of the estimation law and will be devised subsequently.

The estimated state vector $\hat{z}(t)$ can be obtained as $\hat{z}(t) = \hat{z}_\theta(t) + \hat{z}_{u,y}(t)$. Further, combining (7) and (8) yields

$$\dot{\hat{z}}(t)=[A-LC]\hat{z}(t)+Bu(t)+Ly(t)+\rho(t)\hat{\theta}+\mu(t),\ \hat{z}(t_0)=\hat{z}_0 \quad (9)$$

where $\hat{z}(t_0) = \hat{z}_\theta(t_0)+\hat{z}_{u,y}(t_0)$ and the state estimation error ($\tilde{z}(t) \triangleq \hat{z}(t)-z(t)$) dynamics can be obtained by subtracting (3) from (9) as

$$\dot{\tilde{z}}(t)=[A-LC]\tilde{z}(t)+\rho(t)\tilde{\theta}+\mu(t),\ \tilde{z}(t_0)=\hat{z}_0-z_0 \quad (10)$$

where $\tilde{\theta}(t) \triangleq \hat{\theta}(t)-\theta$ represents the parameter estimation error.

To develop the adaptive law, another auxiliary signal $\Omega(t) \in \mathbb{R}^n$, which is a linear combination of $\tilde{\theta}(t)$ and $\tilde{z}(t)$, is defined as [8]

$$\Omega(t) = \tilde{z}(t) - \Psi_f(t)\tilde{\theta}(t) \quad (11)$$

By taking the derivative of (11) and employing (10), the dynamics of Ω can be expressed as

$$\begin{aligned}\dot{\Omega}(t) &=(A-LC)\tilde{z}(t)+\rho(t)\tilde{\theta}(t)-\dot{\Psi}_f(t)\tilde{\theta}(t)-\Psi_f(t)\dot{\tilde{\theta}}(t) +\mu(t)\\ &=(A-LC)(\Omega(t)+\Psi_f(t)\tilde{\theta}(t))+\rho(t)\tilde{\theta}(t)-\dot{\Psi}_f(t)\tilde{\theta}(t)\\ &-\Psi_f(t)\dot{\tilde{\theta}}(t) +\mu(t)\\ &=(A-LC)\Omega(t)+[(A-LC)\Psi_f(t)+\rho(t) -\dot{\Psi}_f(t)]\tilde{\theta}(t)\\ &-\Psi_f(t)\dot{\tilde{\theta}}(t) +\mu(t)\end{aligned} \quad (12)$$

It can be observed that proper choice of variables $\mu(t)$ and $\Psi_f(t)$ facilitates the solution of error dynamics mentioned in (11). One such choice for $\mu(t)$ is

$$\mu(t) = \Psi_f(t)\dot{\hat{\theta}}(t) \quad (13)$$

since $\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t)$, the dynamics of $\Psi_f(t)$ is given as

$$\dot{\Psi}_f(t) = (A-LC)\Psi_f(t)+\rho(t), \quad \Psi_f(t_0) = 0 \quad (14)$$

where A, L, C and $\rho(t)$ are known and therefore, it becomes feasible to compute $\Psi_f(t)$ online. The dynamics of the matrix signal $\Psi_f(t)$, as defined in equation (14), correspond to a filtered version of the signal $\rho(t)$ and constitute the first-layer filtering in the adaptive observer design. The aforementioned choice of $\mu(t)$ and $\Psi_f(t)$ simplifies the dynamics of error-like auxiliary signal Ω as

$$\dot{\Omega}(t) = (A - LC)\Omega(t),\ \Omega(t_0) = \Omega_0 \quad (15)$$

Here, $\Omega_0 = \tilde{z}_0$, since $\Psi_f(t_0) = 0$. Further, the solution to (15) is presented as

$$\Omega(t)=exp\{(A-LC)(t-t_0)\}\Omega_0,\ t \geq t_0 \quad (16)$$

Since the matrix $(A-LC)$ is Hurwitz, $\Omega(t)$ is exponentially converging. Therefore, (10) signifies the necessity of devising an appropriate update law such that the convergence of $\tilde{\theta}(t)$ determines the convergence of $\tilde{z}(t)$. The introduction of $\Omega(t)$ is pivotal in the subsequent analysis of stability.

B. Linear Parametrization

The advantage of this proposed design is rooted in its utilization of a linear parametrization. This approach involves the incorporation of not only the unknown system parameters but also the unknown initial conditions, forming an extended unknown parameter vector [26]. This extended parameter vector is instrumental in enabling the development of the IE-based adaptive observer.

Upon multiplying both sides of equation (11) by C and subsequently rearranging, we derive the following expression

$$C\tilde{z}(t) = C\Psi_f(t)\tilde{\theta}(t) + C\Omega(t) \quad (17)$$

Further, by utilizing (16) and the output equation of (1) in (17), the following relationship can be achieved

$$\underbrace{C\hat{z}(t)}_{\hat{y}(t)}-\underbrace{(y(t)-\varphi(t)\theta)}_{Cz(t)}=C\Psi_f(t)\underbrace{(\hat{\theta}(t)-\theta)}_{\tilde{\theta}(t)}+Cexp\{(A-LC)(t-t_0)\}\Omega(t_0) \quad (18)$$

By substituting $\mu(t_0) = \tilde{z}(t_0) = \hat{z}(t_0) - z(t_0)$ into (18) and rearranging, the subsequent expression is derived

$$\begin{aligned}&\underbrace{\hat{y}(t)-y(t)-C\Psi_f(t)\hat{\theta}(t) - Cexp\{(A-LC)(t-t_0)\}\hat{z}_0}_{g(t)} =\\ &-\varphi(t)\theta - C\Psi_f(t)\theta - Cexp\{(A-LC)(t-t_0)\}z_0,\ t \geq t_0\end{aligned} \quad (19)$$

Ultimately, (19) can be reformulated into a Linear in Parameters (LIP) form as follows.

$$g(t)=\underbrace{[-(\varphi(t) + C\Psi_f(t)) \quad - Cexp\{(A-LC)(t-t_0)\}]}_{\zeta(t)}\underbrace{\begin{bmatrix}\theta\\ z_0\end{bmatrix}}_{\Theta},\ t \geq t_0 \quad (20)$$

where $g(t) \in \mathbb{R}^m$, $\zeta(t) \in \mathbb{R}^{m\times(p+n)}$ are functions that rely on known signals and therefore, can be computed online, $\Theta \in \mathbb{R}^{p+n}$ includes the vector containing both the uncertain system parameters θ and the unknown initial state z_0. Thus, the first-layer filtering is of utmost importance in attaining the linearity in the unknown parameters (θ, z_0) and this, in turn, simplifies the development of a stable adaptive observer, as elaborated in subsection D of section III.

C. Second tier Filtering

The following low-pass filter dynamics are proposed which would avert the PE restriction, inspired from [5], [18].

$$\dot{\Psi}_{ff} = - c_f\Psi_{ff} + \zeta^T\zeta, \quad \Psi_{ff}(t_0) = 0 \quad (21)$$

$$\dot{u}_f = - c_f u_f + \zeta^T g, \quad u_f(t_0) = 0 \quad (22)$$

where $c_f > 0$ is a positive scalar gain, $\Psi_{ff}(t) \in \mathbb{R}^{(p+n)\times(p+n)}$ is the double-filtered regressor and $u_f(t) \in \mathbb{R}^{p+n}$. Hence, the motivation of second-tier filtering is to construct a square matrix regressor which upon fulfilling the IE condition, facilitates the rank sufficiency condition.

Solving (21) and (22) and performing straightforward manipulations, the resulting relationship plays a pivotal role in the formulation of the adaptive law.

$$u_f(t) = \Psi_{ff}(t)\Theta,\ t \geq t_0 \quad (23)$$

Upon integrating (21), the expression for the square matrix $\Psi_{ff}(t)$ is derived as follows

$$\Psi_{ff}(t) = \underbrace{exp\{-c_f t\}}_{\geq 0} \int_{t_0}^{t} \underbrace{exp\{c_f r\}}_{\geq exp\{c_f t_0\} \geq 1} \underbrace{\zeta^T(r)\zeta(r)}_{\geq 0} dr, \; t \geq t_0 \tag{24}$$

Corollary 1: The double-filtered regressor $\Psi_{ff}(t)$ is a function which is positive semi-definite over time i.e. $\Psi_{ff}(t) \geq 0, \, \forall t \geq t_0$.
Further, consider the following assumption.

Assumption 2: The regressor $\zeta(y,t)$ is u-IE, as defined in Definition 2, with respect to the right-hand side of (1), (9), (21), (22) and (26) for some T_{IE}, $\gamma_{IE} > 0$.

The condition for IE can be seen as a measure of the richness of information pertaining to parameters unknown, present during the initial time frame. It serves as the fundamental basis for the forthcoming developments in the proposed work.

Remark 1: A necessary and sufficient condition for the regressor $\zeta(y,t)$ to be IE is that $\Psi_{ff}(t_0 + T_{IE})$ is positive definite (PD) and $\Psi_{ff}(t) > 0, \forall t \in [t_0 + T_{IE}, t_f]$, for any $t_0 + T_{IE} < t_f < \infty$ (refer lemmas 4 and 5 in [18]). Consequently, the condition of IE for the regressor $\zeta(t,y)$, as stated in Assumption 2, can be validated by continuously evaluating the determinant of $\Psi_{ff}(t)$ online. If the determinant is positive, it indicates that the IE condition on the regressor $\zeta(t,y)$ is met.

When Assumption 2 holds, the IE information can be obtained through the filter output described in (21). However, a limitation of this closed-loop filter output is the exponential decay of IE information as

$$\Psi_{ff}(t) \geq \gamma_{IE} \, exp\{-c_f(t-t_0)\} I_p, \, \forall \, t \geq \, t_0 + T_{IE} \tag{25}$$

where c_f is the forgetting factor.

Remark 2: Under the IE assumption, transient information within the initial time-window captures adequate information about the unknown parameters. The closed-loop filter dynamics, as presented in (21), capture the requisite information, assuming the IE condition (Assumption 2) is satisfied. However, it is important to note that it exhibits a gradual information decay with an exponential decay rate denoted as c_f in (25). To address the attenuation of this information, a novel switching mechanism has been integrated into the parameter estimator design, as detailed in the following subsection.

D. Design of Parameter Estimation

The parameter update law is designed as

$$\dot{\hat{\Theta}} = -\Gamma_\Theta (F_1 + F_2 + sF_{IE}) \tag{26}$$

where F_1, F_2 and F_{IE} are given as

$$F_1 \triangleq c_1 \zeta^T(\zeta\hat{\Theta} - g), \tag{27}$$

$$F_2 \triangleq c_2 \Psi_{ff}^T(\Psi_{ff}\hat{\Theta} - u_f) \tag{28}$$

$$F_{IE} \triangleq c_{IE}(Q\hat{\Theta} - P), \tag{29}$$

where the gains c_1, c_2, c_{IE} are positive constants and $\Gamma_\Theta \in \mathbb{R}^{(p+n)\times(p+n)}$ is a positive-definite symmetric adaptation gain matrix, constant signals $Q \in \mathbb{R}^{(p+n)\times(p+n)}$ and

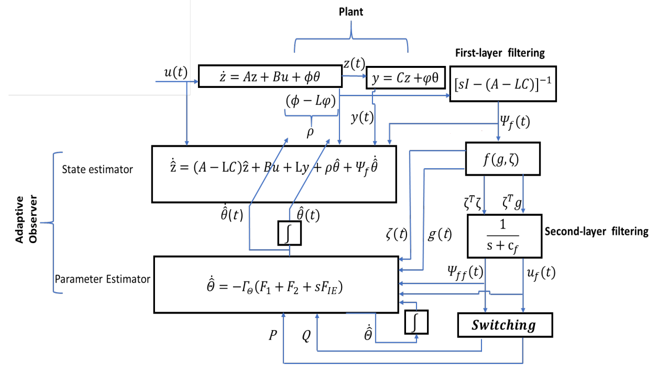

Fig. 1: Block diagram for IE-based adaptive observer

$P \in \mathbb{R}^{p+n}$ correspond to the stored values of Ψ_{ff} and u_f, respectively at $t = t_0 + T_{IE}$ (i.e. the moment when the IE condition is met) and defined as

$$P \triangleq u_f(t_0 + T_{IE}), \, Q \triangleq \Psi_{ff}(t_0 + T_{IE}), \tag{30}$$

The switching signal $s(t) \in \mathbb{R}$ which is piecewise-constant is defined as

$$s(t) = \begin{cases} 0 & for \; t \in [t_0, t_0 + T_{IE}) \\ 1 & else \end{cases} \tag{31}$$

Remark 3: The term $t_0 + T_{IE}$, representing the time when the IE condition is met (as outlined in Definition 2), is determined online by monitoring when the regressor $\Psi_{ff}(t)$ becomes positive-definite (or full-rank). It is important to note that thereafter ($t_0 + T_{IE} \leq t < \infty$), $\Psi_{ff}(t)$ remains positive-definite [18].
Fig. (1) depicts a block diagram illustrating the algorithm for the proposed adaptive observer.

E. STABILITY ANALYSIS

Theorem 1: For the system described in (1), assuming that the triplet (A, B, C) is both controllable and observable, and given that Assumption 1 is satisfied, the parameter update law outlined in (26)-(29) ensures the global uniform Lyapunov stability of the parameter estimation error $\tilde{\Theta}(t)$. Furthermore, if Assumption 2 regarding $\zeta(t,y)$ is met with some γ_{IE} and $T_{IE} > 0$, the parameter estimation error undergoes exponential convergence to zero i.e.

$$||\tilde{\Theta}(t)|| \leq \alpha_1 ||\tilde{\Theta}(t_0 + T_{IE})|| exp\{-\alpha_2(t - t_0 - T_{IE}\}, \, \forall t \geq t_0 + T_{IE} \tag{32}$$

where both α_1 and α_2 are positive constants.

Proof: Taking into account the following Lyapunov function candidate

$$V(\tilde{\Theta}) = \frac{1}{2}\tilde{\Theta}^T \Gamma_\Theta^{-1} \tilde{\Theta} \tag{33}$$

which satisfies the following inequality

$$\frac{1}{2}\underbrace{\lambda_{max}^{-1}(\Gamma_\Theta)}_{\lambda_M} ||\tilde{\Theta}||^2 \leq V \leq \frac{1}{2}\underbrace{\lambda_{min}^{-1}(\Gamma_\Theta)}_{\lambda_m} ||\tilde{\Theta}||^2 \tag{34}$$

The time derivative of (33) yields

$$\dot{V} = \tilde{\Theta}^T \Gamma_\Theta^{-1} \dot{\hat{\Theta}} \tag{35}$$

Using switched adaptive law (26) and (23) in (35) yields

$$\dot{V}=-c_1\underbrace{\tilde{\Theta}^T\zeta^T}_{\psi}\zeta\tilde{\Theta}-c_2\underbrace{\tilde{\Theta}^T\Psi_{ff}^T}_{\chi}\Psi_{ff}\tilde{\Theta}-s\,c_{IE}\tilde{\Theta}^TQ\tilde{\Theta} \tag{36}$$

When considering the time interval $t\in[t_0,t_0+T_{IE})$ and applying an upper bound to the inequality in (36), following is obtained

$$\dot{V}\leq-c_1||\psi||^2-c_2||\chi||^2\leq 0 \tag{37}$$

Therefore, the inequality (37), which is negative semi-definite (NSD), shows uniform global stability (UGS) of the parameter and initial condition estimation error $\tilde{\Theta}(t)$.

Further, if Assumption 2 is satisfied, $\dot{V}$ in (36) can be upper bounded as

$$\dot{V}\leq-c_1||\psi||^2-c_2||\chi||^2-c_{IE}\,\lambda_{min}(Q)||\tilde{\Theta}||^2,\ \forall t\in[t_0+T_{IE},\infty) \tag{38}$$

where $\lambda_{min}(Q)$ represents the minimum eigenvalue of matrix Q. By further upper bounding (38) and incorporating (34), subsequent inequality is achieved.

$$\dot{V}\leq-\beta V,\qquad \forall t\in[t_0+T_{IE},\infty) \tag{39}$$

where $\beta=\frac{2c_{IE}\lambda_{min}(Q)}{\lambda_M}>0$ is a positive scalar. The solution estimate of the above differential inequality in (39) can be written as

$$V(t)\leq V(t_0+T_{IE})exp\{-\beta(t-t_0-T_{IE})\},\ t\geq t_0+T_{IE} \tag{40}$$

Use of (34) in the inequality (40) results in

$$||\tilde{\Theta}(t)||\leq\sqrt{\frac{\lambda_m}{\lambda_M}}\,||\tilde{\Theta}(t_0+T_{IE})||\,exp\{-\frac{\beta}{2}(t-t_0-T_{IE}\} \tag{41}$$

Observing (41) and (32), it can be distinguished that α_1 and α_2 are $\sqrt{\frac{\lambda_m}{\lambda_M}}$ and $\frac{\beta}{2}$ respectively. These two entities when carefully noticed are uniform with respect to the initial conditions. As, the Lyapunov function candidate in (33) is radially unbounded, the algebraic inequality in (41) necessitates the origin of the parameter estimation error dynamics achieves uniformly global exponential stability (UGES) $\forall t\geq t_0+T_{IE}$ (in a delayed sense).

Boundedness of signals

The proof based on Theorem 1 establishes that $\tilde{\Theta}(t)\in\mathcal{L}_\infty$. Since, Θ remains constant, therefore, $\hat{\Theta}(t)\in\mathcal{L}_\infty$. From Assumption 1 and $\rho(t)$, the stable low-pass filter (14) signifies that $\Psi_f(t)\in\mathcal{L}_\infty$. The stable dynamics (16), demonstrates that $\Omega(t)\in\mathcal{L}_\infty$. Therefore, (11) proves that $\tilde{z}(t)\in\mathcal{L}_\infty$ which in turn proves that $\tilde{y}(t)\in\mathcal{L}_\infty$. Therefore, both $\hat{z}(t),\hat{y}(t)\in\mathcal{L}_\infty$. From (20) it can be verified that $\zeta(t),\,g(t)\in\mathcal{L}_\infty$. Hence, the dynamical equations (21) and (22), representing the linear and stable filters, prove that $\Psi_{ff}(t),\,u_f(t)\in\mathcal{L}_\infty$, respectively. Finally, from (26) it can be proved that $\dot{\hat{\Theta}}(t)\in\mathcal{L}_\infty$. ■

IV. SIMULATION RESULTS

The subsequent dynamics is taken into account for simulation, characterizing the linearized lateral behavior of a remotely piloted aircraft (refer [11], p. 188).

$$\begin{aligned}
\dot{z}(t)&=Az(t)+B\begin{bmatrix}\theta_1\,u_1(t)\\ \theta_2\,u_2(t)\end{bmatrix}\\
y(t)&=Cz(t)+\begin{bmatrix}\theta_3\\0\\0\end{bmatrix}\\
A&=\begin{bmatrix}-0.277&0&-32.9&9.81&0\\-0.1033&-8.525&3.750&0&0\\0.3649&0&-0.639&0&0\\0&1&0&0&0\\0&0&1&0&0\end{bmatrix}\\
B&=\begin{bmatrix}-5.432&0\\0&-28.64\\-9.49&0\\0&0\\0&0\end{bmatrix},\ C=\begin{bmatrix}0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}
\end{aligned} \tag{42}$$

where $z\triangleq[z_1,\,z_2,\,z_3,\,z_4,\,z_5]^T$ represents side slip, roll rate, yaw rate, bank angle and yaw angle, respectively, $u\triangleq[u_1,\,u_2]^T$ are rudder and aileron, respectively, $y\triangleq[y_1,\,y_2,\,y_3]^T$. The inputs $u_1(t)$ and $u_2(t)$ are affected by faults represented by scalars θ_1 and θ_2, respectively. A fault in the sensor measurement corresponding to $y_1(t)$ is also considered affected by the scalar bias θ_3, while there is no bias fault corresponding to the output components $y_2(t)$ and $y_3(t)$. The following choice of regressor matrices ϕ and φ renders (42) equivalent to (1) and leads to a common parameter vector to appear both in the system dynamics and the output relation

$$\phi=\begin{bmatrix}u_1&0&0\\0&u_2&0\end{bmatrix},\ \varphi=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$$

The actual system parameters to be determined are provided as: $\theta_1=.75$, $\theta_2=1.25$ and $\theta_3=.25$. The stabilizing feedback gain is chosen as $L=[7.5\ -7\ -1.5\ 1\ -1.6;9.8\ 0\ 0\ 1\ 0;-29.1\ 2.8\ 4.9\ 0\ 6.2]^T$. The initial states are taken as $z_0=[1\ 1\ 1\ 1\ 1]^T$, $\hat{z}_0=[1\ 1\ 1\ 1\ 1]^T$ and $\hat{\theta}(0)=[1\ 1\ 0]^T$. The selection of adaptation gain is made as $\Gamma_\theta=1000I_8$, $c_1=0.0001$, $c_2=0.0001$, $c_{IE}=1$ and $c_f=0.01$. The input signals are $u_1(t)=0.5\,exp(-0.3\,t)[2\,sin\,(5t)+2\,cos\,(9t)+2\,sin\,(10t)2+2\,sin\,(\sqrt{5}\pi t)]$ and $u_2(t)=0.5\,exp(-0.3\,t)[2\,sin\,(10t)+2\,cos\,(15t)]$, which possess finite energy and therefore are not PE signals, however, are IE.

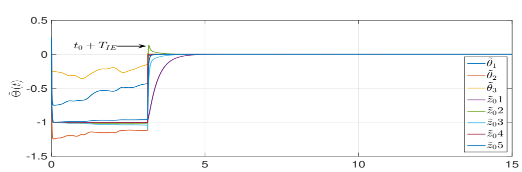

Fig. 2: Parameter estimation error $\tilde{\Theta}(t)$

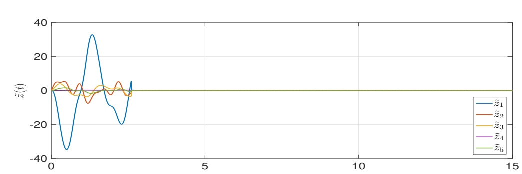

Fig. 3: State estimation error $\tilde{z}(t)$

Figure (2) depicts parameter estimation error where it is evident that the IE condition is met at approximately $t=$

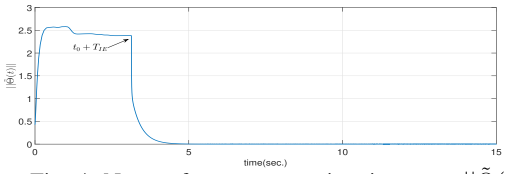

Fig. 4: Norm of parameter estimation error $||\tilde{\Theta}(t)||$

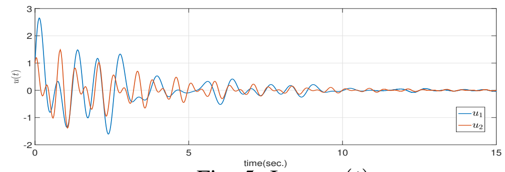

Fig. 5: Input $u(t)$

$3.1s$. As a result, the system parameter estimates converge to their actual values, demonstrating a UGES (Uniform Global Exponential Stability) result. Fig. (3) illustrates the graph representing the state estimation error $\tilde{z}(t)$. Furthermore, in figure (4), the norm of parameter estimation $||\tilde{\Theta}(t)||$ is displayed, confirming that when the IE-condition is met, the proposed adaptive observer achieves exponential convergence of parameter estimation error. Figure (5) illustrates an input signal with finite energy.

V. Conclusion

This paper proposes an IE-based adaptive observer for MIMO LTI systems, where uncertain parameters are present in both the system dynamics and the output relationship. A two-tier filter architecture is utilized where the first layer helps in achieving linear regression, where the unknown coefficient of regression is constructed by combining the unknown initial condition of the state vector with the uncertain parameter vector, creating an extended parameter estimator. While in the second layer, this approach eliminates the requirement for a restrictive PE condition. In a nutshell, the wholesome design facilitates a switched adaptive law, guaranteeing UGES of error (applicable in a delayed sense). In turn, this eases the identification of both sensor and actuator faults simultaneously without necessitating the satisfaction of strong PE condition. Further, the simulation results comply with the claims in the proposition.

References

[1] R. Carroll and D. Lindorff, "An adaptive observer for single-input single-output linear systems," *IEEE Trans. on Auto.Cont.*, vol. 18, no. 5, pp. 428–435, 1973.

[2] G. Luders and K. Narendra, "An adaptive observer and identifier for a linear system," *IEEE Trans. on Auto. Cont.*, vol. 18, no. 5, pp. 496–499, 1973.

[3] ——, "A new canonical form for an adaptive observer," *IEEE Trans. on Auto. Cont.*, vol. 19, no. 2, pp. 117–119, 1974.

[4] K. Ichikawa, "Principle of lüders-narendra's adaptive observer," *International Journal of Control*, vol. 31, no. 2, pp. 351–365, 1980.

[5] G. Kreisselmeier, "Adaptive observers with exponential rate of convergence," *IEEE Trans. on Auto. Cont.*, vol. 22, no. 1, pp. 2–8, 1977.

[6] P. Nikiforuk, H. Ohta, and M. Gupta, "Adaptive observer and identifier design for multi-input multi-output systems," *IFAC Proceedings Volumes*, vol. 10, no. 6, pp. 189–196, 1977.

[7] Q. Zhang and B. Delyon, "A new approach to adaptive observer design for MIMO systems," in *Proceedings of ACC*, vol. 2, 2001, pp. 1545–1550.

[8] Q. Zhang, "Adaptive observer for multiple-input-multiple-output (MIMO) linear time-varying systems," *IEEE Trans. on Auto. Cont.*, vol. 47, no. 3, pp. 525–529, 2002.

[9] X. Li, Q. Zhang, and H. Su, "An adaptive observer for joint estimation of states and parameters in both state and output equations," *International Journal of adaptive control and signal processing*, vol. 25, no. 9, pp. 831–842, 2011.

[10] M.-A. Massoumnia, G. C. Verghese, and A. S. Willsky, "Failure detection and identification," *IEEE transactions on automatic control*, vol. 34, no. 3, pp. 316–321, 1989.

[11] J. Chen and R. J. Patton, *Robust model-based fault diagnosis for dynamic systems*. Springer Science & Business Media, 2012, vol. 3.

[12] Q. Zhang, "An adaptive observer for sensor fault estimation in linear time varying systems," *IFAC Proceedings Volumes*, vol. 38, no. 1, pp. 137–142, 2005.

[13] Q. Zhang and G. Besancon, "An adaptive observer for sensor fault estimation in a class of uniformly observable non-linear systems," *International Journal of Modelling, Identification and Control*, vol. 4, no. 1, pp. 37–43, 2008.

[14] L. Zhao, X. Li, and P. Li, "Adaptive observer design for a class of mimo nonlinear systems," in *Proceedings of the 10th World Congress on Intelligent Control and Automation*. IEEE, 2012, pp. 2198–2203.

[15] X. Zhuang, H. Wang, S. Ahmed-Ali, and Y. Tian, "Design of a joint adaptive observer for a class of affine nonlinear sampled-output system with unknown states and parameters," *International Journal of Adaptive Control and Signal Processing*, 2021.

[16] K. S. Narendra and A. M. Annaswamy, *Stable adaptive systems*. Courier Corporation, 2012.

[17] S. B. Roy, S. Bhasin, and I. N. Kar, "Parameter convergence via a novel PI-like composite adaptive controller for uncertain Euler-Lagrange systems," in *IEEE 55th CDC*, 2016, pp. 1261–1266.

[18] ——, "A UGES switched MRAC architecture using initial excitation," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 7044–7051, 2017.

[19] ——, "Combined MRAC for unknown MIMO LTI systems with parameter convergence," *IEEE Trans. Auto. Cont.*, vol. 63, no. 1, pp. 283–290, 2018.

[20] ——, "Composite adaptive control of uncertain Euler-Lagrange systems with parameter convergence without PE condition," *Asian Journal of Control*, vol. 22, no. 1, pp. 1–10, 2020.

[21] S. K. Jha, S. B. Roy, and S. Bhasin, "Initial excitation-based iterative algorithm for approximate optimal control of completely unknown LTI systems," *IEEE Trans. Auto. Cont.*, vol. 64, no. 12, pp. 5230–5237, 2019.

[22] A. Katiyar, S. Basu Roy, and S. Bhasin, "A switched adaptive observer design without persistence of excitation," in *2019 Fifth Indian Control Conference (ICC)*. IEEE, 2019, pp. 318–323.

[23] ——, "Initial excitation based adaptive observer with multiple switching," in *2019 IEEE 58th CDC*. IEEE, 2019, pp. 2910–2915.

[24] ——, "Initial excitation based robust adaptive observer for MIMO LTI systems," *IEEE Transactions on Automatic Control*, pp. 1–8, 2022.

[25] ——, "Finite excitation based robust adaptive observer for mimo lti systems," *International Journal of Adaptive Control and Signal Processing*, vol. 36, no. 2, pp. 180–197, 2022.

[26] ——, "Initial excitation based fast adaptive observer," in *20th IEEE European Control Conference (ECC),*, 2022, pp. 01–08.

[27] R. Kamalapurkar, B. Reish, G. Chowdhary, and W. E. Dixon, "Concurrent learning for parameter estimation using dynamic state-derivative estimators," *IEEE Trans. on Auto. Cont.*, vol. 62, no. 7, pp. 3594–3601, 2017.

[28] R. Kamalapurkar, "Simultaneous state and parameter estimation for second-order nonlinear systems," in *56th CDC*. IEEE, 2017, pp. 2164–2169.

[29] G. Luders and K. Narendra, "Stable adaptive schemes for state estimation and identification of linear systems," *IEEE Trans. on Auto. Cont.*, vol. 19, no. 6, pp. 841–847, 1974.

[30] G. Bastin and M. R. Gevers, "Stable adaptive observers for nonlinear time-varying systems," *IEEE Trans. on Auto. Cont.*, vol. 33, no. 7, pp. 650–658, 1988.

[31] R. Marino and P. Tomei, "Adaptive observers with arbitrary exponential rate of convergence for nonlinear systems," *IEEE Trans. on Auto. Cont.*, vol. 40, no. 7, pp. 1300–1304, 1995.

[32] E. Panteley, A. Loria, and A. Teel, "Relaxed persistency of excitation for uniform asymptotic stability," *IEEE Trans. on Auto. Cont.*, vol. 46, no. 12, pp. 1874–1886, 2001.