A Novel Approach to Proportional-Integral-Retarded Controller Tuning for Second Order Non-Minimum Phase Systems

E. Moreno-Negrete[1], I. Boussaada[2,3], S.-I. Niculescu[3], A. Ramírez[4], and C.-F. Méndez-Barrios[1]

Abstract— This paper presents a novel approach to develop a tuning rule for a Proportional-Integral-Retarded (PIR) controller when controlling a second-order system with unstable zeros. Particularly, we address the stability and performance issues that the right-half-plane zeros impose. With this aim, we use the *multiplicity-induced-dominance* properties to achieve a partial pole placement strategy guaranteeing the stability of the closed-loop system. Through this method, we derive analytical formulas for the parameters of the PIR controller that induce a predefined algebraic multiplicity for a group of real and dominant roots, ultimately enhancing system's response. Finally, numerical examples and a simulation benchmark conducted on a switched power converter show the effectiveness of the method.

I. INTRODUCTION

In recent years, there has been a growing interest in delay-based controllers, which incorporate an artificial time-delay as a design parameter. Such an interest arises from their applications in real-world control systems, where it has been shown that delay-based controllers improve performance, stability margins and robustness (see, for instance [1]–[3]). As highlighted in [4], incorporating delay into the control law facilitates the stabilization of several classes of dynamical systems. Moreover, in the context of time-delay systems, various methodologies are available for stability analysis. However, challenges arise from the need of designing low-complexity controllers capable of achieving optimal performance while considering the trade-offs between controller complexity, stability and robustness [5]. Since proportional-integral-derivative (PID) controllers are widely adopted in industrial control loops due to their simplicity and performance, the issue of parameter selection for these controllers becomes particularly relevant (e.g., [6]).

In this paper, we propose an analytical approach for designing *Proportional-Integral-Retarded (PIR)* controllers, which, similarly to PID, aims to provide a clear and easily implementable alternative for controller parameter selection. Although infinite-dimension, such a controller belongs to the class of *low complexity controllers*. For example, other controllers belonging to this class are represented by delay blocks [7] used to stabilize integrator chains or delayed PD controllers encountered in human balancing stabilization

[8]. For a more general discussion on such controllers as well as some of their applications, the reader is referred to [9]. Previous research has established tuning guidelines for PIR controllers operating on second-order systems without unstable zeros, demonstrating their effectiveness in achieving desired exponential decay rates and significantly enhancing closed-loop system performance [10]. However, our study focuses on the control of second-order non-minimum phase systems, where the presence of Right-Half Plane (RHP) zeros imposes limitations on the achievable closed-loop bandwidth, potentially leading to instability and performance deterioration [11]. To the best of the authors' knowledge, there is currently no available methodology to solve this specific problem.

Spectral methods can inform about the stability properties of linear systems by examining the distribution of the roots of their characteristic function. While for finite-dimensional linear time-invariant (LTI) systems, one of the most straightforward approaches to controlling the dynamic behavior of closed-loop solutions is through the pole placement methodology [12], this technique does not generalize to the case of LTI delay systems, which are of infinite-dimensional nature. However, a diverse array of specialized spectral techniques are available. In Ramírez et al. [10], for example, a tuning rule for PIR controllers is developed using algebraic geometry and elimination theory. However, the presence of a RHP zero represents a challenge that requires a deeper analysis to derive appropriate practical tuning rules. Hence, there is a justified need for another analysis that comprehensively addresses this specific scenario.

The *multiplicity-induced-dominancy* (MID) property [13]–[15] systematically assigns roots, allowing for *over-order* multiplicities in the characteristic roots. Building on MID principles, the Generic Multiplicity-Induced Dominancy (GMID) method reveals the stability implications of having a real root of maximal multiplicity in certain time-delay systems [16]. This property underscores the real root's dominancy, establishing it as the spectral abscissa under specified conditions. Furthermore, a *Partial Pole Placement* (PPP) method, as outlined in [17], can help determining controller parameters ensuring the designated real root emerges as the rightmost, enhancing system stability through GMID.

The main objective of the paper is to utilize the PPP strategy to develop analytical formulas for fine-tuning the parameters of the PIR controller. The analysis is grounded on a comprehensive model of a non-minimal phase second-order system, considering the prerequisites necessary to extend the MID property in this case. We validate our approach

[1] Faculty of Engineering, Universidad Autónoma de San Luis Potosí, 78210, Mexico. Email: emone@ieee.org

[2] Institut Polytechnique des Sciences Avancées (IPSA), 94200, Ivry-sur-Seine, France.

[3] Université Paris-Saclay, CNRS, CentraleSupélec, Inria, Laboratoire des Signaux et Systèmes, 91190, Gif-sur-Yvette, France.

[4] CONAHCYT-Division of Control and Dynamical Systems, IPICYT, 78216, SLP, Mexico. Email: adrian.ramirez@ipicyt.edu.mx

using a numerical example and the `TDS-Control` software package [18]. In addition, we apply this tuning criteria to a synchronous boost DC-DC converter using a power electronics simulation environment and assess its effectiveness with different pole placement options, numerical dominancy and an evaluation of different performance indices.

The rest of the paper is structured as follows: In Section II, we provide essential definitions necessary to comprehend the MID property, particularly within the framework of second-order non-minimum phase systems. Section III is dedicated to deriving the algebraic equations that compose the PIR controller tuning rule, which stands as our main contribution. In Section IV, we demonstrate the effectiveness of this tuning rule by means of a design example conducted in a power electronics simulation platform. Finally, the article concludes with a discussion of the insights and findings resulting from the proposed analysis.

II. PRELIMINARIES & PROBLEM FORMULATION

Consider a general model of a single-input/single-output (SISO) second-order non-minimum phase system whose transfer function given by

$$G(s) := c \cdot \frac{1 - zs}{s^2 + as + b}, \quad (1)$$

where $a, b, c \in \mathbb{R}_+$, and the (unstable) real zero $1/z > 0$. As mentioned in the Introduction, we consider the proportional-integral-retarded controller defined by the transfer:

$$C(s) := k_p + \frac{k_i}{s} - k_r e^{-hs}, \quad (2)$$

where $h > 0$ is the intentional delay parameter, and $(k_p, k_i, k_r) \in \mathbb{R}^3$ are the controller gains.

Considering this perspective, for systems with time delays in the feedback loop, spectral methods [19] can be employed to analyze the asymptotic behavior of the system trajectories by finding the solutions of a certain characteristic function or *quasi-polynomial*. According to definitions [20], [21], the closed-loop characteristic quasi-polynomial associated with (1)-(2) can be written as

$$Q(s; h, k_p, k_i, k_r) := P_0(s; k_p, k_i) + P_1(s; h, k_r) e^{-hs}, \quad (3)$$

where P_0, and P_1 are defined as

$$\begin{aligned} P_0 &:= s^3 + s^2(a - ck_pz) + s(b + ck_p - ck_iz) + ck_i, \\ P_1 &:= ck_r(zs^2 - s). \end{aligned} \quad (4)$$

The degree of the quasi-polynomial Q given in (3), in the sense of Pólya-Szegö Bound (see, Proposition 1 below). In this case, the degree is equal to 6, see, e.g., [22]. As proved in [23], the following result states a direct link between the quasi-polynomial's degree and the permissible multiplicity of certain spectral values confined within a horizontal strip $\alpha \leq \Im(s) \leq \beta$ defined as

Proposition 1 (Pólya-Szegö Bound, [24]). *Let Q be the quasi-polynomial given by (3) and $\alpha, \beta \in \mathbb{R}$ be such that $\alpha \leq \beta$. Denote by M the number of roots of Q counted with*

multiplicities contained in the set $\{s \in \mathbb{C} : \alpha \leq \Im(s) \leq \beta\}$, then

$$\frac{h_k(\beta - \alpha)}{2\pi} - \deg(Q) \leq M \leq \frac{h_k(\beta - \alpha)}{2\pi} + \deg(Q), \quad (5)$$

where h_k satisfy the condition $0 := h_0 < h_1 < \ldots < h_k$.

Setting $\alpha = \beta = 0$, the above theorem yields $M \leq 6$, which determines the maximal multiplicity of roots in the point spectrum of Q.

Remark 2. *In the spirit of Proposition 1, one can deduce that for a given root $s_0 \in \mathbb{C}$ of Q, its multiplicity is at most equal to 6. Furthermore, if such a maximal multiplicity is reached, then s_0 is necessarily real, and the GMID property holds, that is s_0 necessarily defines the corresponding spectral abscissa. Despite the interest of this property from a purely analytic view point, a control implementation based on the GMID lacks of robustness, see for instance [25]. As a matter of fact, for the sake of robustness with respect to the model's parametric uncertainties, it appears that it will be more appropriate to relax constraints on the choice of the closed-loop spectral abscissa. This can be carried out using the MID property by assigning a root with an over-order (intermediate) multiplicity. Specifically, when considering the PIR controller characterized by the four parameters (k_p, k_i, k_r, h), it is more appropriate that s_0 represents a root with a multiplicity of 4 within Q, which corresponds to the least over-order intermediate multiplicity. This multiplicity arises as a result of the controller's structure and design.*

In the context of dominancy, it is important to emphasize additional results to the quasi-polynomial for s_0. These definitions highlight the assignment of the dominant root within the MID property.

Definition 3. *The spectral abscissa of the quasipolynomial Q is given by the real part of the rightmost characteristic root s_0, that is:*

$$\Re(s_0) = \max\{\Re(s) : Q(s) = 0\}. \quad (6)$$

As a function of the system parameters (including the gains and the delay), the spectral abscissa is a *continuous*, *finite* and *bounded function*, see, e.g., [4]. As shown in [26], such properties are valid for general characteristic functions of systems of retarded type.

Proposition 4 ([8]). *If the quasi-polynomial (3) has a real root s_0 with multiplicity at least M, then Q can be written as*

$$Q(s) = (s - s_0)^M \left(1 + \int_0^1 e^{-h(s - s_0)t} h \frac{R_{M-1}(s_0; ht)}{(M-1)!} dt\right), \quad (7)$$

where

$$R_{M-1}(s_0; ht) = \sum_{i=0}^{M-1} \binom{M-1}{i} P_0^{(i)}(s_0) \cdot (ht)^{M-1-i}. \quad (8)$$

For a comprehensive and extended exploration of these definitions, we refer to [4], [8], [27] and the references therein.

III. Main Result

Multiple approaches have been reported for determining the numerical values of delay-based controller parameters. Recently, the $\mathscr{D}$-decomposition method has been utilized, which has the advantage of being a geometric approach that enables the visualization of the so-called γ-stability properties via a stability plot [10]. Nevertheless, this approach becomes complex as the number of available controller parameters increases. In contrast, the PPP strategy provides a systematic approach by achieving stability through algebraic analysis, eliminating complex exponential term, regardless of the controller's parameters.

Lemma 5. *Let* $\Lambda : \mathbb{C} \to \mathbb{C}$ *be the polynomial defined as*

$$\Lambda(s) := \lambda_2(s)h^2 + \lambda_1(s)h + \lambda_0, \tag{9}$$

where $\lambda_0, \ldots, \lambda_3$ *are given by*

$$
\begin{aligned}
\lambda_0 &:= 6\left(bz^2 + az + 1\right), \\
\lambda_1(s) &:= -3s^2\left(s^2z^3 - 4sz^2 + 6z\right) + \lambda_3(s), \\
\lambda_2(s) &:= s(sz-1)(s^3z^2 - 3s^2z + bz + 3s + a), \\
\lambda_3(s) &:= -6s\left(bz^2 + az + 1\right) + 3\left(bz + a\right).
\end{aligned}
\tag{10}
$$

If $\Lambda(s_0) = 0$*, then* s_0 *is a root of multiplicity four for the quasi-polynomial Q.*

Remark 6. *The polynomial* Λ *introduced in Lemma 5 is commonly referred as the Elimination-Produced Polynomial (EPP) in the literature, see, e.g., [28]. This polynomial establishes a relationship between the admissible solutions* s_0 *with respect to h. As we are interested in solutions with* $h > 0$*, throughout the remainder of the paper, we select* s_0 *accordingly while ensuring that* $\Re\{s_0\} < 0$*.*

Knowing that we can obtain the delay h if a real-root is assigned from the elimination-produced polynomial, in the context of forcing appropriate multiplicity and the dominancy, two questions appear:

- What should be the values of k_p, k_i and k_r such that (s_0, h) satisfy the conditions of the elimination-produced polynomial?
- What is the sufficient condition that ensures that s_0 is dominant?

In light of the findings previously outlined by [22], to tackle both questions we propose some appropriate sufficient conditions for the controller parameters guaranteeing the dominance of the root s_0. To simplify the presentation, we introduce the function $\widehat{R} : \mathbb{C} \times \mathbb{R}_+ \mapsto \mathbb{R}_+$, defined by the following expression:

$$\widehat{R}(s_0; h) := |P_0(s_0)|h^4 + 3|P_0'(s_0)|h^3 + |P_0''(s_0)|h^2 + 6h. \tag{11}$$

Similarly, we introduce the following real constants $f_0, \ldots, f_3$:

$$
\begin{aligned}
f_0 &:= hs_0\left(hs_0z - h + 18z\right) + 3(h+2), \\
f_1 &:= 6 + (as_0^2z - 3s_0^2 - a)h^2 + (6 + (-6z+3)a)h, \\
f_2 &:= 3h^2s_0^3z, \\
f_3 &:= bh^2s_0(zs_0 - 1) - 18hs_0^2 + 3b\left(h(2s_0z+1)+2\right) + 6a.
\end{aligned}
$$

With the notations and definitions above, we have the following result:

Proposition 7. *Consider the closed-loop characteristic quasi-polynomial (3), and let* $s_0 \in \mathbb{R}_-$*. Choose the control parameters* $(h, k_p, k_i, k_r) \in \mathbb{R}_+ \times \mathbb{R}^3$ *as follows:*

$$h := \frac{-\lambda_1 \pm \sqrt{\lambda_1^2 - 4\lambda_2\lambda_0}}{2\lambda_2}, \tag{12a}$$

$$k_p := \frac{6}{zcf_0}\left[f_1 - f_2 + \left(1 - \frac{5h}{2}\right)zs_0 + (a+h)z\right], \tag{12b}$$

$$k_i := \frac{1}{zcf_0}\left[\frac{f_1}{z} + \left(\frac{2}{z} + s_0 + 4h\right)f_2 + f_3\right], \tag{12c}$$

$$k_r := \frac{6e^{hs_0}}{chf_0}. \tag{12d}$$

Assume that there exists a non-empty interval $\mathcal{I} \subset \mathbb{R}_-$ *such that for all* $s_0 \in \mathcal{I}$ *and h given by (12a), the inequality*

$$\widehat{R}(s_0; h) < 3! \tag{13}$$

holds. Then, the following statements are true:

(a) the root s_0 *of Q has multiplicity 4;*

(b) the root s_0 *is a strictly dominant root of Q.*

Sketch of Proof. Since $h > 0$ holds for a given s_0, the proof of *(a)* follows from Lemma 5. Considering that s_0 is a root of multiplicity 4, the proof of *(b)* follows from Proposition 4 taking (11) into account.

A. Explanatory example

For a better understanding of our main result, we proceed to perform a numerical example using the values of the general model presented in (1). In this case, we have taken the following values: $a = 100, b = c = 10000$, and placed the RHS zero at $z = 1/100$. Furthermore, it is important to highlight that the open-loop system is stable since its characteristic poles are located in $-50 \pm j86.60$.

Since the EPP only requires a value of s_0 to solve in terms of the delay, we choose to assign three dominant roots at $[-200, -150, -100]$. Performing the computations of (12), the resulting numerical values are listed in Table I.

TABLE I

RESULTING PARAMETERS USING THE PPP ALGORITHM.

Dominant Root	Controller Gains			
	k_p	k_i	k_r	h
$s_0 = -100$	0.4417	22.1045	0.3748	0.0065
$s_0 = -150$	3.1141	41.2223	2.6271	0.0020
$s_0 = -200$	8.8718	60.8585	8.0624	0.0008

In Fig. 1, we present the numerical results that illustrate the characteristic curve of the EPP, spectrum of the quasi-polynomial in the complex plane, and the response of the system to a unit step input. Theplots confirm the existence of the dominant root s_0 for the assigned values, thus validating our choice of controller parameters. Furthermore, when analyzing the unit step response, we can see an improvement

in the speed of convergence of the solutions towards the reference. Assigning the root s_0 further to the left has an impact on the speed of convergence. On the other hand, a greater undershoot is observed and the appearance of an overshoot arises due to the non-minimum phase nature of the system.

In the next section, we expand these ideas by carrying out an example design of a power electronics converter with similar characteristics to those we have investigated so far by applying a systematic methodology adapted to this particular case.

Parameter	Details		
	Description	*Value*	*Units*
f_s	Switching frequency	150×10^3	Hz
E	Voltage input	48	V
D	Duty Cycle	0.6	
P	Power	100	W
L	Inductor	2.7648×10^{-3}	H
C	Capacitor	1.66×10^{-6}	F
R	Resistance	144	Ω
X_2	Voltage output	120	V
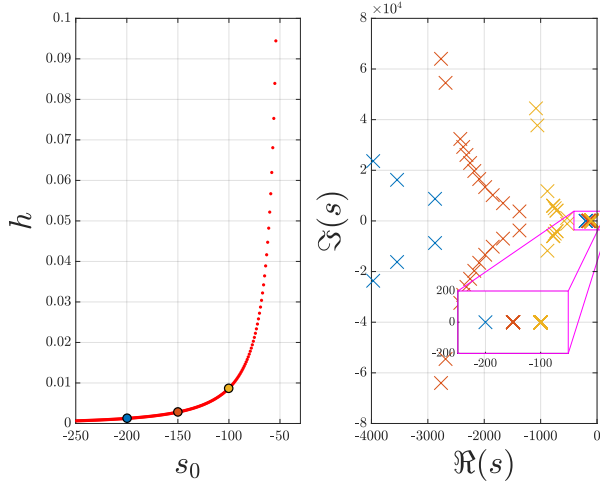
Fig. 1. EPP solvability curve, and the spectrum of $Q(s)$ using the data provided in Table I. Computation of the roots is with `TDS-Control`.

IV. DESIGN EXAMPLE

DC-DC converters play an important role in voltage regulation. Among the most studied and widely adopted converters is the synchronous boost converter due to its higher efficiency and broader operating range [29]. In it's linear analysis [30], the transfer function of the synchronous boost converter, which describes the relationship between the output voltage and the duty cycle, is given by the second-order non-minimum phase transfer:

$$G(s) := \frac{\tilde{x}_2(s)}{\tilde{d}(s)} = \frac{(1-D)ER - LEs}{LCRs^2 + Ls + R(1-D)^2}. \quad (14)$$

By keeping all system parameters fixed as in the general form of (1), we have

$$a = \frac{1}{CR}, \; b = \frac{(1-D)^2}{LC}, \; c = \frac{(1-D)E}{LC}, \; z = \frac{L}{R(1-D)}. \quad (15)$$

All the parameters of the converter during its nominal operation are detailed in Table II.

In our model configuration, all devices are considered ideal, and switches operate according to the conventional model provided by the software. To implement our proposed controller using the PPP strategy, we substitute whole numerical values on formulae (12), first analyzing the elimination-produced polynomial to locate the solvability curve from

which the sufficient dominance condition holds (see, for instance, Fig. 2) Numerical analysis reveals a vertical asymptote located at -2166.0086. In addition, we know that the open-loop poles are situated at $-2083.3 \pm j5.5$, which serves as a compelling rationale for situating the root s_0 to the left of this threshold. Since for higher values of s_0, the solvability curve passes through regions of negative delay, and previous studies [26] have demonstrated that under such conditions, the resulting equations are categorized as advanced-type delay differential equations, which leads to instability. Finally, by applying the tuning criteria above,
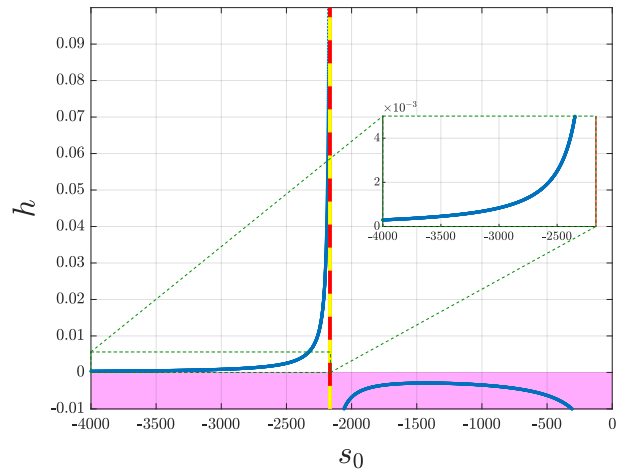

Fig. 2. EPP solvability curve of the synchronous boost converter in closed-loop with a PIR controller.

the resulting data is fed into PLECS/PLEXIM software for power electronics simulation and control design in the time-domain.

Fig. 3 presents the circuit and control schema,which illustrates the key aspects of our simulation setup. This allows us evaluating the response performance of the system in a broader environment. Here, we can also incorporate additional elements, such as input/output disturbances, or consider unknown parasites in the converter among others. In order to achieve this, we have once more utilized `TDS-Control` to compute all of the characteristic equation's roots using the controller values that the algorithm
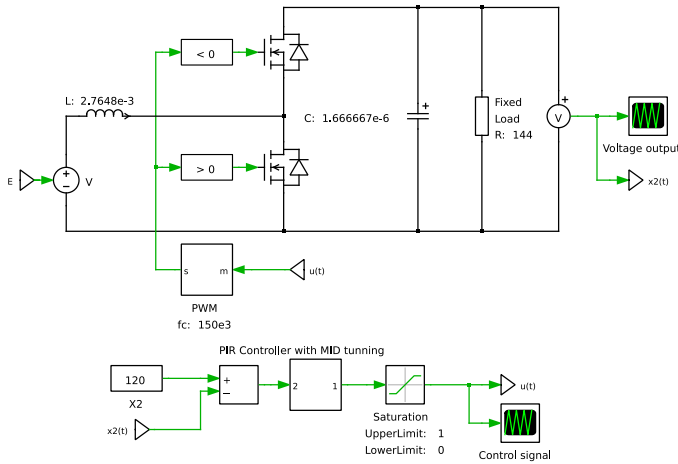
Fig. 3. Closed-loop simulation platform in PLECS/PLEXIM of a 100W synchronous boost converter with step-up conversion ratio of 48V-120V.

Dominant	Controller Gains			
Root	k_p	k_i	k_r	h
$s_0 = -5000$	1.1×10^{-4}	2.9	8.4×10^{-4}	1.5×10^{-4}
$s_0 = -10000$	0.01	10.4	0.01	2.6×10^{-5}
$s_0 = -15000$	0.07	19.4	0.06	8.7×10^{-6}

we set the system equilibrium point at $X_2 = E/(1-D) = 120$V. We also consider that E can exhibit variations, from 48V to 58V at 10ms, and back at 20ms. The results are presented in Fig. 5. As in the previous (explanatory) example, we have observed that an assignment of s_0 further to the left in the complex plane results in faster convergence rates. Additionally, we noticed a significant reduction of overshoot/undershoot associated with the considered abrupt changes of the input voltage.

On the other hand, the control signal, which has a saturation block to limit its values between 0V and 1V for safety reasons, remains within these limits at all times maintaining the system in a linear regime. However, as s_0 increases, we observe a ripple in this signal due to the substantial growth of the controller parameters at different choices of s_0. Finally, Table IV quantitatively presents the following performance indices: M_p-Maximum overshoot; t_{ss}-Settling time; t_r-Rise time; IAE-ntegral of the absolute magnitude of the error; ISE-Integral square error. These results highlight the qualitative observations from Fig. 5 and complement the understanding of the system's behavior and its response under input voltage steps.

produces. We see in Fig. 4 that s_0 is correctly assigned for each choice, as well as the matching output voltage response of the converter, by scanning s_0 in the range $[-10000, -5000]$.
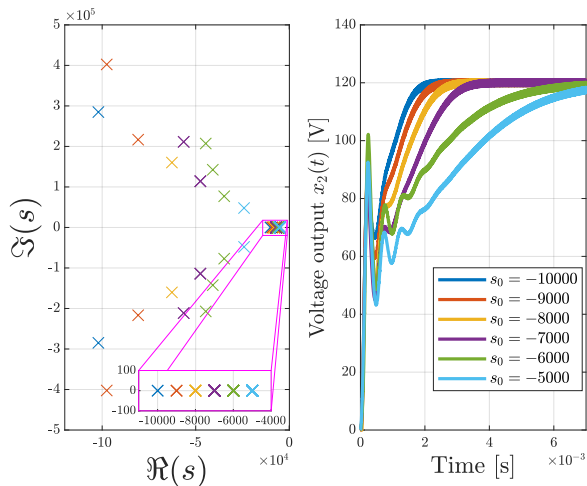

Fig. 4. Roots distribution based on PPP and system's response with various s_0 asignments. Computation of the roots is with `TDS-Control`.

A. Performance Summary

To gauge system's performance, we apply a step change in the input voltage. This example illustrates a specific scenario in which the converter might be used, such as in renewable energy applications where energy harvest is intermittent. Our primary goal is to evaluate how our controlled system responds to abrupt voltage shifts; it is important to note that we have considered an ideal case in terms of efficiency. To achieve this, we introduced three different initial values for s_0 in $[-5000, -10000, -15000]$. The resulting data for each value is summarized in Table III.

We are interested in analyzing the response times and oscillations (if any) of the output voltage. With this aim,
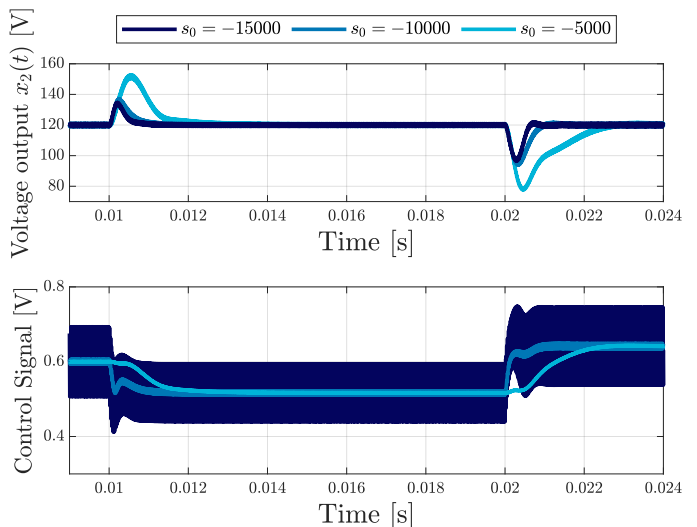

Fig. 5. Comparative of the voltage outputs $x_2(t)$ considering several assignments of s_0 and the corresponding control signals produced by the PIR controller. Simulations based on the platform shown in Fig. 3.

V. CONCLUSION

Considering a second-order non-minimum phase system, this paper presents an extended design methodology using

TABLE IV
CLOSED-LOOP SYSTEM PERFORMANCE.

Dominant Root	Performance Indices				
	M_p [V]	t_{ss} [ms]	t_r [ms]	IAE	ISE
$s_0 = -5000$	152.77	2.7	4.2	0.27	12.20
$s_0 = -10000$	137.14	0.9	1.2	0.08	3.28
$s_0 = -15000$	134.77	1.0	0.6	0.05	2.16

partial pole placement for tuning a delay-based controller known to as the PIR controller. The effectiveness of the tuning rule is illustrated through a detailed analysis and a design example conducted in a power electronics simulation platform. The performance indices, such as maximum overshoot, settling time, rise time, integral of the absolute magnitude of the error, and integral square error, further validate the benefits of these types of controllers with artificial delays. Future work can extend the proposed tuning rule to larger classes of non-minimum phase systems, such as higher order systems or systems with multiple inputs and outputs, and explore its performance in different scenarios.

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