2024 European Control Conference (ECC)
June 25-28, 2024. Stockholm, Sweden

Convex lifting-based path planning for overtaking maneuver on highways

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***Abstract*— This paper revisits the convex lifting method for space partition, emphasizing the generation of safe corridors in the context of navigation with obstacle avoidance guarantees. This enables the agent to navigate within the designated corridors, disregarding obstacles. The paper also emphasizes that the method can be adapted to a highway scenario, particularly in the context of overtaking maneuvers.**

I. INTRODUCTION

Automated mobility, involving the use of advanced technologies for vehicles and transportation systems to operate autonomously or with minimal human intervention, aims to enhance safety, efficiency, and accessibility in transportation. The European Union (EU) has actively focused on this concept in terms of safety, applicability, and environmental benefits, with an anticipated large-scale deployment by 2030.[1]The application of self-driving cars has great potential in the pursuit of short and mid-term goals targeted at improving efficiency and safety. According to the report of the EU Commission,[2] facilitating these vehicles and addressing the challenges originating from highway and urban mixed traffic, it is aimed that road fatalities and serious injuries are to be reduced to zero by 2050. In this context, advanced driver assistance systems (ADAS), play a crucial role in enhancing road safety eventually by taking the driver out of the loop and this is one of the priorities of Renault.[3] [4]

A spectrum of methodologies and strategies has been investigated, specifically for autonomous highway driving. Within the automated mobility framework, [1] introduce their research on platooning of connected and automated vehicles. However, given the current lack of connectivity between vehicles and technological limitations, our research will focus on decentralized approaches. In [2], the decentralized method for merging problems in highways has been studied. [3] provides a comprehensive review of motion planning algorithms for highway driving. Notably, [4] present their solutions to overtaking maneuvers, employing multi-objective optimization approaches. [5] deploys model predictive control (MPC) for trajectory control on clothoidal paths, while [6] integrates an obstacle avoidance solution into the problem.

The convex lifting in the path planning is a recent framework and it has been explored in [7]. This framework enables to decompose of the space in a cluttered environment and to establishment of safe navigation corridors for agents such as autonomous vehicles, robots, etc, which can utilize those corridors as kinematic constraints.

In ADAS, the path planning module relies on the perception module which is shown in Fig. 1. It detects and tracks obstacles and ensures full awareness of surrounding vehicles. These topics are of interest in control theory, computer science, and related fields. Thus, we focus on such an environment that the ego vehicle has complete information on its surroundings and target vehicles.

II. PATH PLANNING USING CONVEX LIFTING

The efficient design of corridors relies heavily on the effective utilization of space partitions.

Definition 1: Disjoint obstacles $\mathcal{P} = \bigcup_{i\in\mathcal{I}}\mathcal{P}_i$ in a subspace $\mathcal{X}$. The sets $\{\mathcal{X}_i\}_{i\in\mathcal{I}}$ satisfying, (i) $\mathcal{X} = \bigcup_{i\in\mathcal{I}}\mathcal{X}_i$, (ii) $\mathrm{int}(\mathcal{X}_i)\cap\mathrm{int}(\mathcal{X}_j) = \emptyset, (i,j)\in\mathcal{I}^2$ (iii) $\mathcal{P}_i\subset\mathrm{int}(\mathcal{X}_i), \forall i\in\mathcal{I}$ is called a partition of $\mathcal{X}$ induced by the obstacles $\mathcal{P}$.

The configuration space, denoted as $C_{\mathcal{X}}(\mathcal{P}) = \mathcal{X}/\mathcal{P}$, represents the complete set of potential poses that an agent can attain.

Definition 2: Given a polyhedral partition of a finite-dimensional space $\mathcal{X} = \bigcup_{i\in\mathcal{I}}\mathcal{X}_i$, a convex lifting is defined as a piecewise affine function $z:\mathcal{X}\to\mathbb{R}$ satisfying the following properties: $z(x) = a_i^T x + b_i$ for $x\in\mathcal{X}_i$ and $z(x) > a_j^T x + b_j,\ \forall x\in\mathrm{int}(\mathcal{X}_i),\ (i,j)\in\mathcal{I}^2, \forall i\neq j$

The lifting can be performed optimization problem in (1).

$$\min_{a_i,b_i} J = \sum_{i=1}^{N_0} \| \begin{bmatrix} a_i^T & b_i \end{bmatrix} \|_2^2 \tag{1a}$$

$$\text{s.t.}\quad a_i^T v + b_i \le M, \forall v\in\mathcal{V}(\mathcal{P}_i), \forall i\in\mathcal{I}, \tag{1b}$$

$$a_j^T v + b_j \ge a_i^T v + b_i + \epsilon, \forall v\in\mathcal{V}(\mathcal{P}_j), \forall i\neq j \tag{1c}$$

[1]The European Commission (2021). Sustainable and Smart Mobility Strategy.

[2]Grater, A., Harrer, M., Rosenquist, M., and Steiger, E. (2022). Connected, cooperative and automated mobility roadmap.

[3]The European Commission (2018). Advanced driver assistance systems.

[4]This thesis fits with the objectives of Renault and the work of the first author is financially supported by the company.

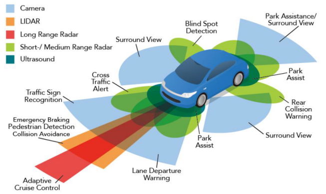

Fig. 1. ADAS perception module. [8]

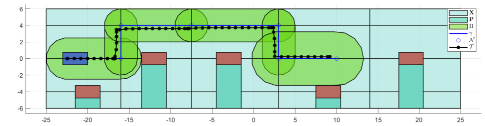

Fig. 2. MPC-based trajectory for overtaking maneuver.

After obtaining the lifting function, one can compute the lifted polyhedron by calculating the epigraph.

$$\mathcal{L} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^{d+1} : \begin{bmatrix} a_i^T & -1 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \leq -b_i, i \in \mathcal{I} \right\} \quad (2)$$

Then, one can obtain the space partition, by projecting them back into the space. $\mathcal{X}_i = \text{proj}(\mathcal{F}_i^{d-1}(\mathcal{L}), \mathcal{X})$ where $i \in \mathcal{I}$ The interconnected graph of paths is generated as $\Gamma(\mathcal{N}, \mathcal{E}, f)$ as a function of nodes, $\mathcal{N}$, edges, $\mathcal{E}$, and weight function, f. Then, by using any graph search algorithm a path for the agent can be found.

Definition 3: With given obstacles $\mathcal{P}$, a corridor between two nodes $(x_0, x_f) \in \text{int}(C_{\mathcal{X}}(\mathcal{P}))$, is defined by of two functions, $\gamma : [0,1] \rightarrow C_{\mathcal{X}}(\mathcal{P})$ and $\rho : [0,1] \rightarrow \mathbb{R}_{>0}$ satisfying,$\gamma(0) = x_0$ and $\gamma(1) = x_f$ where $\gamma(\theta) \oplus \mathbb{B}^2_{0,\rho(\theta)} \subset C_{\mathcal{X}}(\mathcal{P})$, $\forall \theta \in [0,1]$. Then, a corridor is defined as,

$$\Pi = \{\ x \in \mathbb{R}^d : \exists \theta \in [0,1]\, \text{s.t.}\ \ x \in \gamma(\theta) \oplus \mathbb{B}^2_{0,\rho(\theta)} \} \quad (3)$$

The corridors can be defined as a combination of convex sets as $\Pi = \bigcup_{i=1}^{N_c} \Pi_i$.

III. MPC-based trajectory generation

Linear time-invariant dynamics of an agent is shown as,

$$x_{k+1} = Ax_k + Bu_k, \quad (4)$$

With, $x_k \in \mathbb{R}^d$ denotes the state vector, and $u_k \in \mathbb{R}^m$ is the input vector. Using a quadratic cost in MPC:

$$\begin{aligned} \mathcal{J}(N_p, \bar{x}_i, x_k, U) &= \|x_{k+N_p|k} - \bar{x}_i\|_P^2 + \\ \sum_{l=1}^{N_p-1} \|x_{k+l|k} - \bar{x}_i\|_Q^2 &+ \sum_{l=1}^{N_p-1} \|\Delta u_{k+l|k}\|_R^2 \end{aligned} \quad (5)$$

where N_p is prediction horizon, $\bar{x}_i$ is reference point such that $\bar{x}_i \in \Pi_i$, Q is state, R is control increment, and P is the terminal cost penalty matix. The vector $U = \begin{bmatrix} u_{k|k} & \dots & u_{k+N_p-1|k} \end{bmatrix}^{\text{T}}$ is the optimization argument:

$$\mathcal{T}(\Pi_i, N_p, \mathcal{X}_f, \bar{x}_i, \mathcal{X}) : \min_U \mathcal{J}(N_p, \bar{x}_i, x_k, U) \quad (6a)$$

$$\text{s.t.} \quad x_{k+l+1|k} = Ax_{k+l|k} + Bu_{k+l|k}, \quad (6b)$$

$$u_{k+l|k} \in \mathcal{U},\ \forall l = 1 : N_p - 1, \quad (6c)$$

$$x_{k+l|k} \in \Pi_i, \quad (6d)$$

$$x_{k+N_p|k} \in \mathcal{X}_f(\bar{x}_i) \quad (6e)$$

By integrating state-space dynamics (6b), input constraints (6c), and state constraints (6d) within the corridor, the MPC problem in (6) is systematically managed at each time step. During corridor transitions, ensuring the agent's trajectory aligns with controllable dynamics and terminal constraints (denoted as $\mathcal{X}_f$) is crucial for safe navigation. This controlled invariance set ensures the system remains safe throughout transitions and navigation along subsequent corridors. An important aspect of MPC adjustment is choosing a prediction horizon to guarantee reaching $\mathcal{X}_f$ from any initial point in the corridor. By using the Backward Reachable Set construction, we can ascertain the necessary prediction steps for recursive feasibility.

IV. Results

The method is applied in a highway scenario for overtaking. Fig. 2 displays MPC-generated trajectory for a blue ego vehicle. Parameters for convex lifting optimization are set as $\epsilon = 1.0$ and $M = 0.01$. Vehicle positions $y = 0$ and $y = 4$ denote center and objective lanes. Red target vehicles are extruded to slow lanes to block possible paths. Green tubes indicate safe corridors along MPC-generated trajectory (black line).

V. Conclusion

In this paper, we revisit the convex lifting framework for space partitioning and we generate obstacle-free corridors and construct an MPC-based path for the ego vehicle. Our investigation focuses on a static highway scenario. Future work will address dynamic highway environments.

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