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Neural Dissociations between Magnitude Processing of Fractions and Decimals

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Abstract—**Fraction and decimal magnitude processing are crucial for mathematic achievement. Previous neuroimaging results showed that fraction and decimal processing activated both overlapping and distinct neural substrates, but temporal dissociations between fraction and decimal processing remained unknown. This event-related potential (ERP) study explored differences in neural activities between magnitude processing of fractions and decimals, by examining the notation effect (fraction vs. decimal) and distance effect (far vs. close) on early components of P1/N1, P2 and N2. Results showed that decimals elicited larger N1 and smaller P1 than fractions at the parietal region. Fractions demonstrated the significant distance effect on fronto-central P2 while decimals showed the distance effect on left anterior N2. ERP results reflect distinct processing of identification and semantic access stages between fractions and decimals. Identification is located at the visual-related region with enhanced perception acuity and identification efficiency for decimals. Semantic access activates the fronto-central region associated with elaborative magnitude manipulation for fractions, while semantic access reflects automatic phonological retrieval for decimals. Our findings disintegrate the magnitude processing of fractions and decimals from identification to magnitude processing. It reveals that temporal discrepancies between fraction and decimal magnitude processing appear as early as post-stimulus 100 ms.**

I. INTRODUCTION

Fractions and decimals are the most pervasive notations of rational number encountered in everyday life. Magnitude processing is the foundation of rational number ability. However, many international studies have documented the poor understanding of fractions and decimals by students [1]. Magnitude processing of fractions and decimals has become a significant challenge in learning mathematics.

Fractions and decimals are ratios written in two different notations. In the ratio-integrated view [2], fractions are a subset of ratios and decimals are ten-based formats of ratios. Fractions consist of hierarchically arranged integers (i.e. a/b) that inherently refer to part-whole relationship. The equivalent decimals lose this bidimensional information, instead provide a static unit [3]. Moreover, fraction dilemma is more salient than decimals among rational number notations, due to inaccurate, error-prone conceptual and procedural knowledge [4]. Although previous functional magnetic resonance imaging (fMRI) studies reported that a common network centered by the intraparietal sulcus (IPS) was activated during processing of fractions and decimals [5], recent fMRI research found distinct activation patterns of fractions and decimals. Particularly, fractions were associated with activation in the right IPS and frontal gyrus, and decimals with activation in the bilateral IPS, left occipito-temporal and frontal areas [6, 7].

Magnitude processing bridges the act of seeing a nonsymbolic entity or symbolic fraction to accessing its numerosity. According to serial-stage model [8], magnitude processing involves serially organized identification and semantic access stages. For identification, the behavioral study suggested that recognition and discrimination mainly depend on notations [3]. For sematic access, numerous studies with comparison tasks involving entity or symbolic fraction have observed a distance effect. Distance effect refers to that comparisons of numerical pairs closer in magnitude lead to the longer response time (RT) and lower accuracy (ACC) than those are farther apart (e.g., 0.7 0.8 vs. 0.2 0.9) [7, 9, 10]. In behavioral studies, a more dramatic distance effect was observed for fractions than for the equivalence of decimals, suggesting that fractions elicit less precise magnitude [3, 11].

Although previous studies indicated that fraction processing was inconsistent with decimal processing, temporal neural mechanisms underlying magnitude processing of fraction and decimal are still unclear. How to disintegrate the magnitude processing in temporal dimension and at which stage the dissociation between fraction and decimal appears have not been answered. The serial-stage model might provide an assumption of discrepant patterns underlying magnitude processing of fraction and decimal, while evidence from neurophysiology is still needed.

In our study, Event-related potentials (ERPs) with high temporal resolution were applied to dissociate the underlying neural activities between fraction and decimal magnitude processing. Based on conceptual, procedural difficulties within fractions and serial-stage hypothesis, we expected to identify distinct identification and semantic access stages by examining the notation effect (fraction vs. decimal) and distance effect (far vs. close) in the comparison tasks [12]. According to previous ERP studies [9, 13, 14], the notation factor would evoke P1/N1 components which are associated with visual processing. The distance factor would influence P2 and N2 components distributed at fronto-parietal regions.

II. METHODS

A. Participants

Sixty-seven right handed college students (33 males, age: M=21.83, SD=1.84) without known dyscalculia were

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recruited for this study. Nine participants were excluded from further data analysis due to excessive artifacts during EEG recording, resulting in less than 30 trials for each condition. All subjects were asked to read and sign an informed consent form before the experiment. The study was approved by the Academic Committee of the School of Biological Science and Medical Engineering, Southeast University, China.

B. Stimulus and Procedure

Participants performed a comparison task that consisted of two kinds of symbolic fraction notations (fraction and decimal). A total of 320 comparison pairs were designed by manipulating the magnitudes of symbolic fraction and entity. For each block, the participants were firstly presented with an entity in bar or pie diagram with or without lines that divided the entity into equal parts. They were required to remember the entity magnitude. It was followed by the presentation of a single-digit fraction (2/3, 7/8...) for the fraction block or a two-digit decimal (0.67, 0.88...) for the decimal block. The symbolic fraction was presented in the screen until the participant responded by pressing the "F" key with the forefinger of the left hand if the entity magnitude was larger and the "J" key with the forefinger of the right hand if the symbolic fraction magnitude was larger. Different block orders were counterbalanced across subjects. The instructions emphasized both speed and accuracy. To control for the number of digits on the screen and prompt participants to use holistic representation, the moderately difficult comparison pairs were designed with magnitude-matched symbolic fractions. Two types of distances were used, with an average for the close distance of 0.11 and an average for the far distance of 0.45. The whole procedure was controlled by E-prime 2.0 (Psychology Software Tools, Pittsburgh, PA).

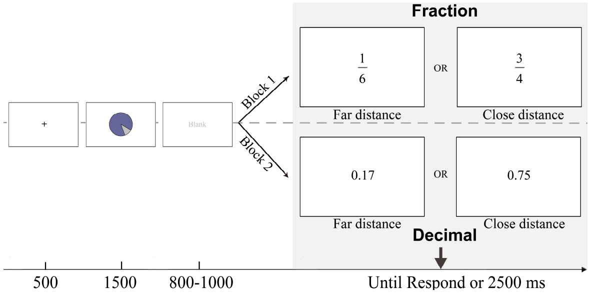

Figure 1. Experimental design for processing fractions and decimals.

C. Data Analysis

Continuous EEG data were recorded by a 64-channel Neuroscan Inc. (Herndon, Virginia, and USA) using the international 10-20 system, with the average reference on the bilateral mastoids. The sampling rate was 1000 Hz with an analog passband filter of 0.1-40 Hz. All inter-electrode impedances were maintained below 5 kΩ. Electrooculography (EOG) signals were simultaneously recorded by horizontal and vertical electro-oculograms. Preprocessing of EEG data was carried out in Scan 4.5, including rejecting paroxysmal time windows, reducing ocular artifacts using signals recorded by EOG electrodes, and low-pass filtering below 30 Hz (24 dB/oct). EEG was segmented into 1.2 s time-locked to the onset of a symbolic fraction with the baseline of 200ms. Artifact rejection was set at a threshold of ±80 μV. Only epochs with correct response were kept for calculation.

After a visual inspection of grand average ERP waveforms with consideration of results from relevant ERP studies [5, 13, 15], the P1/N1, P2 and N2 components on nine electrodes (F3, Fz, F4, C3, Cz, C4, P3, Pz, P4) were selected for further analysis. Time windows for each ERP component was as follows: P1/N1 (75-125 ms), P2 (170-250 ms) and N2 (250-350 ms). A significance level of 0.05 was used to apply the Greenhouse-Geisser correction for nonsphericity and the Bonferroni correction for multiple comparisons.

III. RESULTS

A. Behavioral Results

Mean RT and ACC for fraction and decimal notations were presented in Fig 2. Repeated measures ANOVA was conducted with notation (fraction vs. decimal) and distance (far vs. close). For the RT, main effect of notation was significant ($F(1,57)=46.45, \eta_p^2=0.45$, $p<0.001$), with 93 ms faster for decimals compared to fractions. Main effect of distance was significant ($F(1,57)=438.39$, $\eta_p^2=0.89$, $p<0.001$), with 238 ms faster for the far distance than the close distance. For the ACC, the main effect of notation was significant ($F(1,57)=47.02$, $\eta_p^2=0.45$, $p<0.001$), with 3.35% higher accuracy for decimals. Main effect of distance was also significant ($F(1,57)=598.09$, $\eta_p^2=0.91$, $p<0.001$), with 15.08% higher accuracy for the far distance. Additionally, notation by distance interactions of RT and ACC were both significant (RT: $F(1,49)=23.74$, $\eta_p^2=0.29$, $p<0.001$; ACC: $F(1,49)=34.64$, $\eta_p^2=0.38$, $p<0.001$). Follow-up simple effect analyses showed that for fractions, the distance effect was more profound, with 18.1% higher accuracy ($p<0.001$) and 271 ms faster for the far distance than the close distance ($p<0.001$). For decimals, the far distance had 12% higher accuracy ($p<0.001$) and was 206 ms faster than the close distance ($p<0.001$).

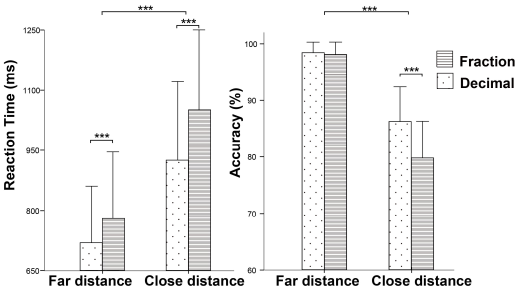

Figure 2. Mean RT and ACC in the fraction and the decimal notations as a function of distance. *** p <0.001

B. ERP Results

Repeated measure ANOVA was conducted on mean amplitudes of P1/N1, P2 and N2 with notation × distance ×electrode. Grand average ERP waveforms on nine electrodes were shown in Fig. 3 and topographies of mean difference potentials during the N1/P1, P2 and N2 time windows were illustrated in Fig. 4 and Fig. 5.

For the P1/N1, at frontal to parietal regions, the main effect of notation was detected ($F(1,57)=21.33$, $p<0.001$, $\eta_p^2=0.27$), with fractions having smaller N1 and larger P1 component (-0.46 vs. -1.13 μV). There was a significant notation by electrode interaction ($F(8, 456)=2.89$, $p<0.05$, $\eta_p^2=0.05$). Follow-up simple effect analysis indicated a pronounced notation effect (*ps*<0.001) in centro-pareital electrodes (C3:-0.66 vs. -1.43 μV, Cz:-1.08 vs. -1.93 μV, C4:-0.69 vs.

-1.35 μV, P3:1.08 vs. 0.44 μV, Pz:0.02 vs. -0.96 μV, P4:1.23 vs. 0.57 μV) and this effect was also significant (*ps*<0.05) in frontal electrodes (F3:-1.15 vs. -1.69 μV, Fz:-1.52 vs. -2.08 μV, F4:-1.35 vs. -1.77 μV).

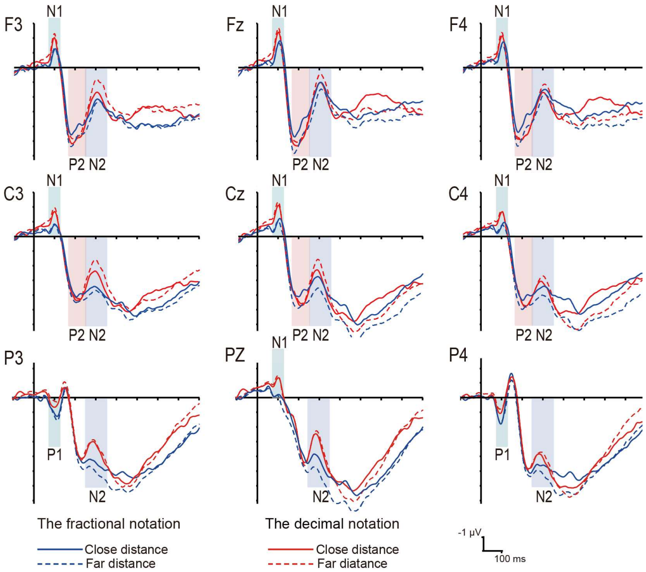

Figure 3. Grand average ERP waveforms on nine electrodes for fraction and decimal notations.

Fig. 4 demonstrates notation effect topographies at P1/N1 time window by subtracting the ERP of the fractions from that of the decimals. The notation effect was characterized by widely distributed N1 component over the fronto-central scalps and P1 component over the lateral parietal scalps.

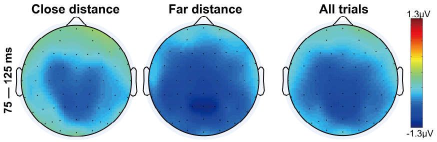

Figure 4. Topographical maps of notation effect for close-distance, far-distance and all trials during post-stimulus 75-125 ms.

For the P2, the interaction between distance, notation and electrode was significant ($F(8,456)=2.91$, $p<0.05$, $\eta_p^2=0.05$). Follow-up simple effect analysis revealed larger amplitude for the far distance than the close distance for fractions at the frontal to central electrodes (*ps*<0.05) (F3:4.50 vs. 3.79 μV, Fz:4.62 vs. 3.61 μV, F4:4.26 vs. 3.45 μV, C3:4.18 vs. 3.72 μV, Cz:4.34 vs. 3.51 μV, C4:4.58 vs. 3.93 μV).

For the N2, the notation and electrode interaction was detected ($F(8, 456)=9.44$, $p<0.001$, $\eta_p^2=0.14$), with fractions having smaller N2 component at F3, C3 and Cz (*ps*<0.01) (F3:2.49 vs. 1.61 μV, C3:3.70 vs. 2.44 μV, Cz:3.31 vs. 2.46 μV). Notably, there was a significant notation by distance interaction ($F(1, 57)=8.75$, $p<0.01$, $\eta_p^2=0.13$). For decimals, the distance and electrode interaction was shown ($F(8,456)=3.60$, $p<0.01$, $\eta_p^2=0.06$), due to the far distance elicited larger N2 component at F3, C3 and Cz (*ps*<0.05) (F3:1.23 vs. 1.99 μV, C3:2.10 vs. 2.78μV, Cz:2.17 vs. 2.76 μV). For fractions, the distance and electrode interaction was revealed ($F(8,456)=2.97$, $p<0.05$, $\eta_p^2=0.05$), with reversed distance effect for the close distance having larger N2 component at Cz, C4, P3, Pz and P4 (*ps*<0.01) (Cz:2.92 vs. 3.71 μV, C4:3.56 vs. 4.19 μV P3:4.51 vs. 5.10 μV, Pz:4.47 vs. 5.37 μV, P4:4.73 vs. 5.34 μV,).

Fig. 5 shows the topographic maps of distance effect by subtracting the ERP of the close distance from that of the far distance. At P2 time window, the significant distance effect was characterized by more profound focal positivity over the fronto-central electrodes for fractions, but not for decimals. Moreover, at N2 time window, the distance effect for fractions was widely distributed and centered at the parietal area, while the distance effect of reverse polarity for decimals was obvious at left lateralized fronto-central region.

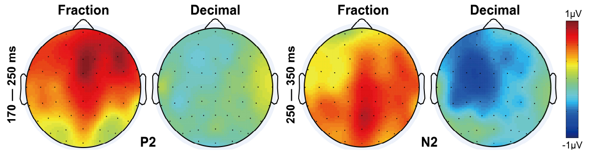

Figure 5. Topographical maps of distance effect for fraction and decimal notations during post-stimulus 170-250 ms and 250-350 ms.

IV. DISCUSSION

This study investigated the underlying neural activities between fraction and decimal magnitude processing. Identification and semantic stages during magnitude processing between two notations were identified and compared based on relevant ERP components.

Identification depends on notations [3]. Consistent with our prediction, the notation effect was revealed with more negative visual evoked potentials (VEP) N1/P1 for decimals, suggesting distinct notation differences occurred during symbolic fraction identification. Larger midline and anterior N1 and bilateral posterior P1 with same amplitude order replicated previous studies [12, 14, 16]. N1 modulation reflects access to automatic attention [15] and arousal state [17] for exogenous stimulus based on visual pathway. Furthermore, N1 is dependent on cognitive processing. Anterior N1 showed a dipole pair in the IPS, which correlated with number sense. It reflected that obligatory magnitude extraction appeared as early as 75 ms [18]. Identification of decimals was earlier primed and more intensive indexed by higher P1/N1 negativity. It indicated the advantage of decimal notation in enhanced perception acuity and higher magnitude retrieval efficiency. Moreover, the amplitude of N1 varied inversely with task difficulty [19] and had a positive relation with performance on numerosity comparison [20]. Difficult conceptual meanings of fractions, featured by part-whole relationship, might result in reduced N1. The notation effect in our behavioral data confirmed the same way as previous behavioral studies with more accurate and faster RT for decimals [3, 11]. Different level of expertise also profoundly modulates the P1/N1 component. In previous research, the P1 evoked by novel and unfamiliar stimuli significantly deviates from the processing of skilled ones [21]. When task or stimulus were in participants' field or participants were familiar with them, N1 is larger and sometimes left lateralized [22]. Exposure frequency and proficiency of symbolic fractions might contribute to timing and intensity differences between fast retrieval of highly overlearned decimal and slow extraction of less known fractions. Notably, the fact that the equivalent magnitude was strictly matched between two notations suggests that asymmetry in P1/N1 between fractions and decimals was not due to magnitude differences but due to different identification complexity. Lastly, the notation effect without any distance effect detected in early P1/N1 component was new evidence of the serial-stage hypothesis in rational number field.

Distance effect is related to semantic access stage [7, 9, 10]. Consistent with what is seen for number comparison tasks, we detected similar distance effects on the P2 and N2 components [13, 15]. As expected, significant interactions of notation and distance were first observed for fractions during the P2 time window, indicating notation-dependent semantic access. The distance effect for fraction was in line with the result of Temple and Posner's experiment, with larger P2 for far distance [23]. Luo et al. reported larger P220 components for shortcut calculation (rounding) in fronto-central sites [24]. We suggested that participants were more likely to apply approximate estimate strategy for fractions, resulting in increased anterior P2. Notably, the semantic processing of fractions involved an earlier and more intensive fronto-central activation which was usually found in complex task. This finding replicated two previous studies, which revealed frontal activations in the fraction processing [6, 8]. At N2 time window, left anterior distance effect for decimals indicated the phonological processing for the retrieval of decimal-related facts and arithmetic procedures [24, 25]. More complex fraction calculation might not be stored in rote memory and failed to be retrieved. Therefore, parietal area which related to elaborative processing of numbers was activated instead. Accordingly, semantic access activated magnitude manipulation with more profound distance effect for fractions compared to automatic retrieval of semantic distance for decimals by phonological processing. In accord with the P2 and N2 interaction, our behavior data also revealed more dramatic distance effect for fractions. These results indicated that harder procedural manipulation was conducted for fractions during semantic access stage since numerators and denominators changed synchronously.

Overall, our results supported the serial-stage model of magnitude processing in rational numbers domain. We found that identification depended on visual perception and was less intensive, less efficient for fractions than decimals. Semantic access relied on high-level cognition of magnitude manipulation and was earlier, more effortful for fractions, while semantic access indicated the automatic phonological retrieval for decimals. Understanding neural mechanism underlying more complex conceptual identification and procedural semantic access for fractions relative to decimals can help explain the fraction dilemma, which has theoretical and instructional significance.

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