

Nonparametric Modelling Based Model Predictive Control for Human Heart Rate Regulation during Treadmill Exercise

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Abstract—This paper applies a kernel-based nonparametric modelling method to estimate the heart rate response during treadmill exercise and proposes a model predictive control (MPC) method to perform heart rate control for an automated treadmill system. This kernel-based method introduces a kernel regularisation term, which brings prior information to the model estimation phase. By adding this prior information, the experimental protocol can be significantly simplified and only a small amount of model training experiments are needed. The model parameters were experimentally estimated from 12 participants for the treadmill exercise with a short and practical exercise protocol. The modelling results show that the model identified using the proposed method can accurately describe the heart rate response to the treadmill exercise. Based on the identified model, an MPC controller is designed to track a predefined reference heart rate profile. An advantage is the speed and acceleration of the treadmill can be limited to within a safe range for vulnerable exercisers. The proposed controller was experimentally validated in a self-developed automated treadmill system. The tracking results indicate that the desired automatic treadmill system can regulate the participants’ heart rate to follow the reference profile efficiently and safely.

I. INTRODUCTION

During physical exercise, as the intensity of exercise changes, the cardiovascular system adjusts to the amount of blood and oxygen delivered to the working muscles, resulting in heart rate (HR) changes and respiratory rate changes. Creating a mathematical model for the cardiovascular system might give us a better understanding of exercise physiology [1]. Comprehending the aetiology of HR behaviours throughout the course of an exercise may also help predict and reduce the mortality from cardiovascular disease [2]. This is also conducive to improving athletes’ performance and designing more effective weight loss procedures for obese people. It also helps to assess individual physical health [3].

Modelling and controlling HR response during treadmill exercise has received considerable attention in the [4]–[6]. The variance of HR response measurement can be quite large because of the limitation of HR sensor accuracy and dislocation of the sensor [7]. Also due to the complexity of the human cardiovascular system, it is hard to use a simple parametric model to describe the responses of the cardiovascular system to exercise. Accordingly, in this paper we employ a nonparametric model, called a finite impulse response (FIR) model, to describe the HR response. However, due to the fact that the size of the FIR model is relatively large, the traditional system identification method usually

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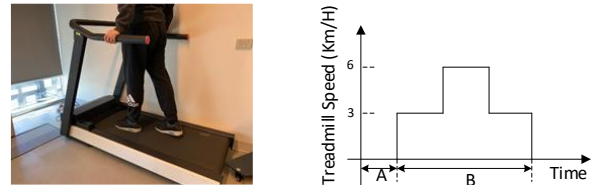


Fig. 1: The proposed automatic treadmill system and speed profile during the identification period. (A) Resting. (B) Walking.

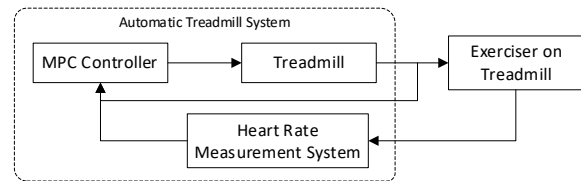


Fig. 2: Schematic of the automatic treadmill system

requires a very complicated dynamic input to provide enough information for the model establishment, which will in turn lead to a long experiment time and sharp variations during model estimation. The inherent ill-posed problem caused by sensor noise and insufficient dynamic information can be solved by adding the regularisation term in the index function. This term reforms the problem into a regularised least square estimation (ReLS) problem [8]. However, ReLS only solves the ill-condition problem and is incapable of providing any prior information to the model estimation process. To this end, we reform the FIR model estimation problem as Gaussian Process modelling [9]. By adding a kernel term in reproducing kernel Hilbert space (RKHS), the prior information is embedded in the identification process by assigning a covariance which is also called a kernel in the machine learning field [10]. The participation of this prior information means fewer experiments can provide enough information for model identification. The contributions of this study are summarised as follows: (a) An effective kernel-based nonparametric modelling method is developed for identifying the HR response model. By applying this method, we can significantly reduce the number of experiments and the complexity of the experimental protocol to reach the desired modelling accuracy. (b) An effective HR tracking controller is developed by integrating the proposed nonparametric modelling method and the MPC. This new model predictive controller can also limit the speed and acceleration ranges to ensure the safety of the exercisers. To the best of the authors’ knowledge, it is the first time that the kernel-based

nonparametric modelling approach has been integrated with MPC control for HR regulation during treadmill exercises. (c) The proposed modelling and control algorithms have been experimentally validated on 12 participants and have achieved the desired HR tracking accuracy.

II. KERNEL-BASED ESTIMATION METHOD OF HEART RATE RESPONSE MODEL

The heart rate (HR) response model can be dynamically described by its impulse response $g(k)$ as follows:

$$y(t) = \sum_{k=1}^{\infty} g(k)u(t-k) + e(t), t = 1, 2, \dots, N, \quad (1)$$

where the $e(t)$ is the noise, and N is the total number of sampling. We can compute the system output $y(t)$, here it is the HR, by knowing the corresponding impulse response $g(k)$ and input signal $u(t)$, here it is the treadmill speed. In general, $e(t)$ is supposed to be independent of $u(t)$.

For a stable system, its impulse response decays exponentially. Thus, the system can be approximately decryped by its m th order finite impulse response (FIR):

$$y(t) = \sum_{k=1}^m g(k)u(t-k) + e(t), t = 1, 2, \dots, N. \quad (2)$$

When we stack all the rows in $y(t)$ and $u(t)$ to the vectors form and define $[g(1), g(2), \dots, g(m)] = \theta \in R^m$, then equation (2) becomes

$$Y = \phi\theta + E. \quad (3)$$

Apparently, the least-squares estimator of the model (2) is

$$\hat{\theta} = \arg \min_{\theta} \|Y - \phi\theta\|^2. \quad (4)$$

In industrial applications, when the experiments are well designed (e.g., PRBS inputs) and comprehensively performed, the information matrix associated with equation Eq. (4) contains enough information to identify the parameter. As a result, even a classical least-square estimator can be applied to identify $\hat{\theta}$ which is the parameter of the nonparametric model. However, for the physiological model, in which human factors are involved, the experiments are often limited to input strength and duration. While the experimental protocol should not be too complicated, this is often the case for the modelling of the HR impulse. To ensure the safety of the treadmill exercisers, the input signal (i.e., the profile of the treadmill speed) is often confined to rectangular with moderate magnitude (treadmill speed). That is why most existing literature uses a simple parametric model, often a first-order model, to approximately descript the HR response to treadmill speed. To better accommodate the differences of various exercisers, a nonparametric model can be employed. This has the potential to develop personalised sports medicine based on accurate prediction of the cardiorespiratory response to exercise. However, using a high dimension impulse response model with limited model stimulation often leads to an ill-conditioned problem, i.e. a small error in the measurement can lead to a large estimation

error. To address this issue, a commonly used technique is that of adding a regularisation term to the estimator (4). In contrast to the regularised least square estimation (ReLS) method introduced in [8], we add a kernel regularisation term to the estimator [11]

$$\begin{aligned} \hat{\theta} &= \arg \min_{\theta} \|Y - \phi\theta\|_2 + \gamma\theta^T \beta^{-1}\theta \\ &= \beta\phi^T (\phi\beta\phi^T + \gamma I_N)^{-1} Y, \end{aligned} \quad (5)$$

where the second item is a kernel regularisation term that denotes the squared Euclidean norm in reproducing kernel Hilbert space (RKHS). β is an N -by- N kernel matrix containing the prior information of FIR. Comparing to the ReLS method, the advantage of the kernel method is that it has a stronger capacity to minimise the mean square error of FIR. The ReLS method only considers solving the ill-condition problem. More importantly, the prior information brought by the kernel allows us to build an impulse response model.

III. MPC CONTROLLER DESIGN

During treadmill exercise, the walking speed and acceleration must be limited to within a safe range to guarantee the safety of exercisers. Because model predictive control (MPC) has the inherent ability to deal with constraints, it is the most suitable choice. MPC depends on the dynamic model of the process to predict and optimise the future behaviour of the process. MPC uses current measurements, including the dynamic information of the current process, the model, process reference trajectory, and constraints, to calculate future changes in manipulated variables. MPC usually only implements the first optimal sequence to the plant and repeats the calculation when the next change is needed.

Let $y(t)$ and $\hat{y}(t+1|t)$ represent the current measurement and predicted measurement, respectively. The control output $u_t, u_{t+1}, \dots, u_{t+q-1}$ can be obtained by solving the following constrained optimisation problem [6]:

$$\begin{aligned} &\text{minimize} && \sum_{l=0}^{p-1} \|\hat{y}_{t+l+1} - r_{t+l+1}\|_{Q_y}^2 + \sum_{l=0}^{q-1} \|\Delta u_{t+l}\|_S^2 \\ &\text{subject to} && u_{min} \leq u_t \leq u_{max} \quad t = 0, \dots, N-1, \\ &&& \Delta u_{min} \leq \Delta u_t \leq \Delta u_{max} \quad t = 0, \dots, N-1. \end{aligned} \quad (6)$$

Use the kernel-based nonparametric model to predict the future output of a certain range p (called the prediction horizon) at each time t . These future outputs \hat{y} are predicted based on the given information (past inputs and outputs) up to time t and the future control output u generated by the controller up to time $t+q$, where q is called the control horizon. Q_y and S are the penalty matrix for prediction errors and control moves. Here, u and Δu are constrained speed and acceleration, respectively.

Based on the model $\theta = [g(1), g(2), \dots, g(m)]$ estimated

TABLE I: Participant information.

Col 1	Mean	SD	Range
Age(year)	28.67	0.94	27-30
Body mass(kg)	66.58	13.07	50-86
Height(cm)	174	10.35	156-187
BMI(kg/m ²)	21.75	2.22	18.37-24.59

n=12, 5 female, 7 male.

by Eq.5, the predictor can be described as:

$$\hat{y}(t+l|t) = \sum_{k=1}^l g(k)u(t+l-k) + \sum_{k=l+1}^m g(k)u(t+l-k) + y_m(t) - \sum_{k=1}^m g(k)u(t-k), \quad (7)$$

where $y_m(t)$ is the measured value at time t .

IV. EXPERIMENTS AND DISCUSSION

In this section, we introduce the procedure of the experiment and discuss the result. The experiment is divided into two phases. The first one is the model estimation phase, which builds the HR response model for each participant by using the kernel-based estimation method. The second phase is MPC control by using the proposed model. In this phase, we achieve HR tracking with constrained speed and acceleration.

The modelling and control data was obtained from 12 participants. The detail is shown in Table I. The experiment was performed under the approval of the UTS Human Research Ethics Committee (ETH17-1758). The data collection was based on voluntary participation, and the informed consent from all participants was obtained before the data collection.

Before each phase, the participants were permitted to consume a light meal 2 h before the experiment. High-intensity exercise was not allowed 3 h before the experiment. Each participant wore a wireless HR sensor and stood on the treadmill for 2 minutes before the experiment started. The room temperature was set to 22 °C.

A. Experimental Equipment

The proposed heart rate (HR) regulation treadmill system and its MPC control system are shown in Fig.1. A TRACKMASTER FVX 325 medical-grade treadmill, which is manufactured by Full Vision Inc, is used in the automated system. This system can send treadmill speed to a personal computer and manipulate the treadmill speed via the serial port. The HR is measured by a wireless wearable Zephyr HR sensor. The sensor collects the analogue electrocardiogram signal and calculates the HR by using the edge detection method. The sensor transmits HR data to the control system every second via Bluetooth. However, it is observed that the measured HR is often polluted by electromagnetic interference generated by other environmental equipment. To address this issue, we use the proposed kernel-based nonparametric MPC for HR tracking.

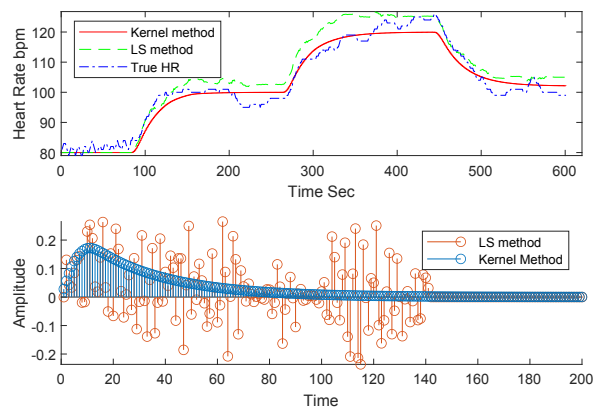


Fig. 3: (Top) Typical estimated heart rate comparison between Kernel method and LS method. (Bottom) The estimated impulse response for one participant.

B. Model Estimation

In the model estimation phase, the participants are required to walk on the treadmill according to the desired speed protocol shown in Fig.1. They first stand on the treadmill for 80 seconds. Then, they are asked to walk at 3 km/h for 3 minutes, followed by a faster walk at 6 km/h for 3 minutes. Then, another walk for 160 seconds at a speed of 3 km/h. The entire process takes 10 minutes, including 80 seconds resting period. The expertise protocol is relatively easy as it only contains two accelerations. The exerciser is informed 5 seconds before each acceleration.

For the finite impulse response model Eq.(2), the order m was selected as 250 and the sampling time was selected to 1 second. The proposed kernel-based estimator Eq. (5) was employed to identify the FIR model by using the Tuned/Correlated (TC) kernel, Diagonal kernel (DI) kernel, and Stable spline (SS) kernel. The fit error is defined as the normalised root mean square error between the estimated HR and true HR.

To select the best kernel, we estimated each participant model using three different kernels. The parameter selection method of each kernel was given in [11]. We calculated the fit error of each estimated model. As a comparison of the conventional method, we calculated the fit error of the model by using the latest square (LS) method Eq. (4). The fit error is recorded in Table II. The kernel with the lowest fitness error was selected for the model estimation. It demonstrated that the proposed kernel method outperforms the conventional LS method. Specifically, Fig.3(Top) shows a typical estimated HR comparison between the kernel-based method and LS method. The figure indicates that the proposed model response has a lower fitness error and is smoother than the LS model response in general. Fig.3(Bottom) is the estimated impulse response by using different methods. The LS method impulse response is extremely noisy. This indicates that the kernel-based method can solve the ill-posed problem.

TABLE II: Fitness error.

Subject	TC(%)	SS(%)	DI(%)	LS(%)
Participant 1	33.86	31.68	30.79	41.91
Participant 2	21.61	30.61	21.23	25.63
Participant 3	19.05	17.59	18.81	21.85
Participant 4	26.83	27.50	29.24	30.26
Participant 5	25.56	25.53	25.46	34.61
Participant 6	50.36	66.37	37.69	53.52
Participant 7	28.75	29.07	18.41	44.62
Participant 8	48.05	48.15	47.43	82.40
Participant 9	25.85	26.60	25.84	26.83
Participant 10	54.18	56.83	53.70	84.65
Participant 11	55.61	39.07	38.00	54.51
Participant 12	22.53	23.18	22.50	30.41
Average	34.35	35.18	30.76	44.27
Standard deviation	13.11	14.07	10.85	20.24

C. MPC Heart Rate Regulation

In the heart rate (HR) regulation phase, using the kernel-based nonparametric model presented in Section IV-B, we designed a model predictive controller to track a predefined reference HR. The participants were required to stand on the treadmill for 60 seconds. The reference HR was then set to be 100 beats/minutes and to last for 6 minutes. The prediction horizon was $p = 10$ and control horizon was $q = 1$. Penalty matrix for prediction errors and control moves were set to be $Q_y = 1$ and $S = 5$, respectively. To maintain the exercise safety, the maximum speed was limited to 6 km/h and the acceleration was limited to ± 0.5 km/h/sec. Fig.4 demonstrates

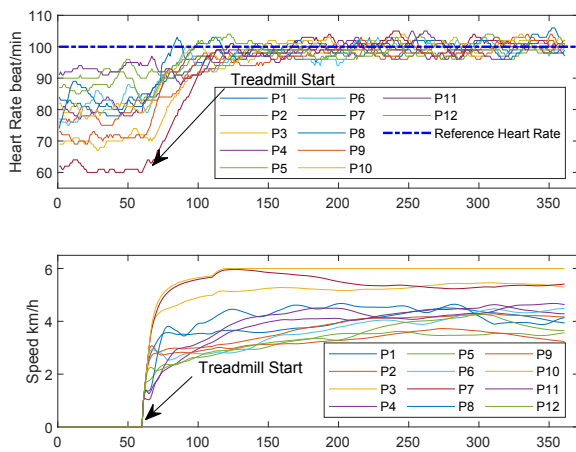


Fig. 4: Heart rate tracking results for all 12 subjects.

that the proposed kernel-based nonparametric model and MPC controller achieved the desired tracking performance. All 12 participants reached the target HR within 60 seconds after the treadmill started and without steady-state error. These performances are comparable with those discussed in previous literature [6]. However, it should be emphasised that the performance acquired by the proposed method requires easier experiments during the model estimation phase.

V. CONCLUSION

In this paper, we proposed a nonparametric model and a kernel-based estimation approach describing the HR re-

sponse during treadmill exercise. The proposed model and estimation method were applied to 12 participants. The experimental results indicated that the fit error of the proposed approach is lower than the least square method. In addition, the model estimation phase needs less time and does not contain complex exercise protocols. By using this nonparametric model, an automatic treadmill system was built and employed for HR tracking during treadmill exercise. The MPC technique was implemented, which could achieve safe exercise by constraining both exercise speed and acceleration. Experimental results demonstrated that the proposed HR regulation system achieved low HR tracking error under the predefined acceleration and speed constraints. The proposed HR response model and MPC control approach were experimentally validated and might have important implications for cardiovascular rehabilitation, the creation of effective training plans for athletes and the development of efficient weight loss plans to combat obesity.

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