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Dictionary Learning Strategies for Cortico-Muscular Coherence Detection and Estimation

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Abstract— **The spectral method of cortico-muscular coherence (CMC) can reveal the communication patterns between the cerebral cortex and muscle periphery, thus providing guidelines for the development of new therapies for movement disorders and insights into fundamental motor neuroscience. The method is applied to electroencephalogram (EEG) and surface electromyogram (sEMG) recorded synchronously during a motor task. However, synchronous EEG and sEMG components are typically too weak compared to additive noise and background activities making significant coherence very difficult to detect. Dictionary learning and sparse representation have been proved effective in enhancing CMC levels. In this paper, we explore the potential of a recently proposed dictionary learning algorithm in combination with an improved component selection algorithm for CMC enhancement. The effectiveness of the method was demonstrated using neurophysiological data where it achieved considerable improvements in CMC levels.**

I. INTRODUCTION

The functional connections between the cortex and associated body muscles can be examined by cortico-muscular coherence (CMC) analysis, which gives a principal measure of linear dependency between electroencephalogram (EEG) and surface electromyogram (sEMG) signals during controlled motor tasks [1], [2], [3]. Evidence shows that the level of β range $(14-36)$ Hz CMC depends on the intensity of the static force and is modulated by afferent stimuli [1], [4]. However, β range CMC is very low and frequently not detectable due to a considerable amount of noise and interference activities involved in EEG and sEMG signals during the monitored control tasks [5], [6].

One approach to solve this problem is Independent Component Analysis (ICA) [7], [8]. However, the good performance of this method requires high-channel count neurophysiological signals. Dictionary learning algorithms have been successfully used in image denoising [9], and therefore they are worthy of further exploration towards elevating CMC levels when analysing single-channel neurophysiological signals [2]. Dictionary learning is efficient in reconstructing signals with inherent structure using few waveforms. Based on this advantage, learning an underlying dictionary can make only few high amplitude dictionary vectors able to represent sensory-motor components. On the other hand, due to the lack of structure, unrelated activities and noise will spread over many low-amplitude components [10]. Hence, the coherent components could potentially be easily extracted using dictionary learning strategies. In [2] only the traditional K-SVD dictionary learning algorithm was investigated in the context of CMC enhancement leaving space for considerable improvements particularly in light of subsequent recent developments in the domain of dictionary learning [11]. Moreover, the original component selection algorithm proposed in [2] is a greedy time consuming algorithm that needs to process sequentially every coefficient in the sparse representation matrices. This paper compares three dictionary learning algorithms: K-SVD [12], Simultaneous Codeword Optimization (SimCO) [13] and Block Total Least Squares (BLOTLESS) [11], developed very recently, inspired in part by this particular application in CMC enhancement. We also simplify the component selection algorithm further to reduce the computational complexity. The experimental results demonstrate that our method achieves better performance and faster operation speed in extracting coherent EEG-EMG components.

This paper is organized as follows. Section 2 formulates the EEG/EMG communication model and the sparse representation problem under learned dictionaries and describes the proposed component selection algorithm. Section 3 presents evaluation results. Section 4 provides a summary.

II. METHODS

A. Coherence analysis

In movement control, the signal transmission between the cortex and the periphery is not instantaneous, but with a delay due to the transmission time of the neural conduction [14]. Moreover, several nerve fibers are involved in nerve conduction, introducing different attenuation and delays [15], [16]. Therefore, the EMG signal can be modelled as

$$y(t) = \sum_{i=1}^{N_x} \alpha_{x,i} x_0(t - \beta_{x,i}) + n_y(t), \tag{1}$$

where $x_0(t)$ is the cortical signal that causes muscle activities, while $\alpha_{x,i}$ and $\beta_{x,i}$ are the corresponding propagation attenuation and delays, $n_y(t)$ is the noise and N_x is the

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number of descending pathways. One can further model the corresponding EEG signal $x(t)$ as

$$x(t) = x_0(t) + n_x(t) \tag{2}$$

where $n_x(t)$ is the noise component. This model can be generalised in a straightforward manner to include bidirectional signalling.

Such synchronous linear coupling between the cerebral motor cortex and related body muscles is commonly detected and characterised using coherence analysis. The coherence $C_{x,y}(\omega)$ between two stationary processes $x(t)$ and $y(t)$ is defined as

$$C_{x,y}(\omega) = \frac{\left|S_{x,y(\omega)}\right|^2}{S_{x,x}(\omega) \cdot S_{y,y}(\omega)}, \tag{3}$$

where $S_{x,x}(\omega), S_{y,y}(\omega)$ are the power spectral densities of $x(t)$ and $y(t)$, respectively, and $S_{x,y}(\omega)$ is their cross spectral density [17]. In this work, we estimated the coherence between EEG and sEMG in the short-time Fourier transform domain [18] to keep their statistical properties fairly constant over analysis windows. In the case of a linear coupling between the cortex and the periphery as modelled by (1) and (2), the coherence between EEG and EMG signals has the form

$$C_{X,Y}(\omega) = \frac{|A(\omega)|^2 S_{x_0,x_0(\omega)}{}^2}{[S_{x_0,x_0}(\omega) + S_{n_x,n_x}(\omega)][|A(\omega)|^2 S_{x_0,x_0}(\omega) + S_{n_y,n_y}(\omega)]}. \tag{4}$$

where $S_{x_0,x_0}(\omega)$ is the power spectral density of the control component, $S_{n_x,n_x}(\omega)$ and $S_{n_y,n_y}(\omega)$ are the power spectral densities of the noise components and $A(\omega)$ is the frequency response of the propagation channel. Significant coherence can be defined by setting the confidence limit (CL) [19] to 95% which is estimated as

$$CL = 1 - (1 - 0.95)^{\frac{1}{L-1}}, \tag{5}$$

where L is the number of trials in the signal set.

The value of the coherence is a real number between zero to one, with zero indicating that the two processes are unrelated, while one indicating that one is driving the other. According to (4), it is evident that the coherence value is one in the noiseless condition. However, in the presence of noise, the coherence will drop to a relatively low value, potentially below the significance threshold. This paper explores using dictionary learning and sparse expansions to remove the noise from EEG and sEMG signals, thereby enhancing CMC levels.

B. Learning sparse representation dictionaries

The goal of dictionary learning is to learn an overcomplete dictionary of k vectors $\mathbf{d}_i \in \mathbb{R}^n, i = 1, \ldots, k$, where $k \geq n$, in which a given set of m signals $\mathbf{r}_i \in \mathbb{R}^n, i = 1, \ldots, m$ can be represented in a sparse manner. This sparse representation can be formulated as an optimisation problem

$$\arg\min_{\mathbf{D},\mathbf{S}} \|\mathbf{R} - \mathbf{DS}\|_2^2 \quad s.t. \quad \|\mathbf{s}_i\|_0 \leq T, \ \forall i, \tag{6}$$

where $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_k]$ is the matrix of the dictionary vectors, $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_m]$, is the matrix of given signals, $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_m]$, $\mathbf{s}_i \in \mathbb{R}^k$, is the coefficient matrix, that is, its i-th column is the vector of expansion coefficients of $\mathbf{r}_i$ with respect to the dictionary, and T is a constant known as the sparsity of sparse coding.

The neurophysiological signals are first used to form data matrices of EEG signals, $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_L], \mathbf{x}_i \in \mathbb{R}^n$, and EMG signals, $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_L], \mathbf{y}_i \in \mathbb{R}^n$, where L is the number of trials in the experiment. Next, a common dictionary in which $\mathbf{X}$ and $\mathbf{Y}$ have sparse representations should be learned. Specifically, the matrix $\mathbf{R} = [\mathbf{X}, \mathbf{Y}]$ is formed and the dictionary $\mathbf{D} \in \mathbb{R}^{n \times k}$ is generated by solving the problem in (3) for the data matrix $\mathbf{R}$. Sparse representations of EEG and sEMG signals in this dictionary are then given by

$$\hat{\mathbf{X}} = \mathbf{DS}_{\mathbf{X}} \qquad \hat{\mathbf{Y}} = \mathbf{DS}_{\mathbf{Y}}, \tag{7}$$

where $\mathbf{S}_{\mathbf{X}}$ and $\mathbf{S}_{\mathbf{Y}}$ are the sparse coefficient matrices corresponding to $\mathbf{X}$ and $\mathbf{Y}$.

C. Finding sparse expansions via ADMM

For a given dictionary $\mathbf{D}$, alternating direction method of multipliers (ADMM) is applied for sparse coding. The optimization objective is formulated as:

$$\begin{aligned} \min\nolimits_{S_X,S_Y}\{\frac{1}{2}\|\mathbf{X} - \mathbf{DS}_{\mathbf{X}}\|_F^2 + \frac{1}{2}\|\mathbf{Y} - \mathbf{DS}_{\mathbf{Y}}\|_F^2 \\ + \lambda_1\|\mathbf{S}_{\mathbf{X}}\|_1 + \lambda_2\|\mathbf{S}_{\mathbf{Y}}\|_1\}, \end{aligned} \tag{8}$$

where the regularization parameters $\lambda_1 > 0$ and $\lambda_2 > 0$ determine the trade-off between data fidelity and model simplicity: larger λ_1 and λ_2 typically lead to *sparser* coefficients $\mathbf{S}_{\mathbf{X}}$ and $\mathbf{S}_{\mathbf{Y}}$ (i.e., simpler models) but larger representation errors $\mathbf{X} - \mathbf{DS}_{\mathbf{X}}$ and $\mathbf{Y} - \mathbf{DS}_{\mathbf{Y}}$. As the ℓ_1 terms in the objective function are not differentiable, auxiliary variables $\mathbf{Z}_{\mathbf{X}}$ and $\mathbf{Z}_{\mathbf{Y}}$ are introduced in the ADMM formulation:

$$\begin{aligned} \min_{S_X,S_Y}\{\frac{1}{2}\|\mathbf{X} - \mathbf{DS}_{\mathbf{X}}\|_F^2 + \frac{1}{2}\|\mathbf{Y} - \mathbf{DS}_{\mathbf{Y}}\|_F^2 \\ + \lambda_1\|\mathbf{Z}_{\mathbf{X}}\|_1 + \lambda_2\|\mathbf{Z}_{\mathbf{Y}}\|_1\} \text{ s.t. } \mathbf{Z}_{\mathbf{X}} = \mathbf{S}_{\mathbf{X}}, \mathbf{Z}_{\mathbf{Y}} = \mathbf{S}_{\mathbf{Y}}. \end{aligned} \tag{9}$$

The corresponding augmented Lagrangian is given by:

$$\begin{aligned} L = \frac{1}{2}\|\mathbf{X} - \mathbf{DS}_{\mathbf{X}}\|_F^2 + \frac{1}{2}\|\mathbf{Y} - \mathbf{DS}_{\mathbf{Y}}\|_F^2 \\ + \lambda_1\|\mathbf{Z}_{\mathbf{X}}\|_1 + \lambda_2\|\mathbf{Z}_{\mathbf{Y}}\|_1 \\ + \frac{\rho_3}{2}\|\mathbf{S}_{\mathbf{X}} - \mathbf{Z}_{\mathbf{X}}\|_F^2 + \langle \boldsymbol{\beta}_3, \mathbf{S}_{\mathbf{X}} - \mathbf{Z}_{\mathbf{X}} \rangle \\ + \frac{\rho_4}{2}\|\mathbf{S}_{\mathbf{Y}} - \mathbf{Z}_{\mathbf{Y}}\|_F^2 + \langle \boldsymbol{\beta}_4, \mathbf{S}_{\mathbf{Y}} - \mathbf{Z}_{\mathbf{Y}} \rangle, \end{aligned} \tag{10}$$

where $\rho_3 > 0$ and $\rho_4 > 0$ are ADMM penalty constants, $\boldsymbol{\beta}_3$ and $\boldsymbol{\beta}_4$ are Lagrange multipliers. This formulation allows ADMM updates of variables $\mathbf{S}_{\mathbf{X}}$, $\mathbf{S}_{\mathbf{Y}}$, $\mathbf{Z}_{\mathbf{X}}$, and $\mathbf{Z}_{\mathbf{Y}}$ to admit closed-form solutions: $\mathbf{S}_{\mathbf{X}}$ and $\mathbf{S}_{\mathbf{Y}}$ are solutions of the corresponding least squares problems, and the closed form solutions of $\mathbf{Z}_{\mathbf{X}}$ and $\mathbf{Z}_{\mathbf{Y}}$ involve soft-thresholding function.

D. Component selection

Finding sparse representations of EEG and EMG signals is not sufficient for increasing the level of coherent components relative to noise and background activity. There still are remaining interference activities affecting the level of CMC. Therefore, in order to enhance the relative level of coherent components, we employ a component selection algorithm to remove the irrelevant coefficient from sparse coefficient matrices $\mathbf{S_X}$ and $\mathbf{S_Y}$.

To that end, dictionary vectors are successively tested for their relevance for representing coherent EEG and sEMG processes. This is done by replacing corresponding rows in the coefficient matrices $\mathbf{S_X}$ and $\mathbf{S_Y}$ by zero vectors, and if that reduces the coherence, the vector is considered relevant and the corresponding row is restored, otherwise it is kept equal to zero. This procedure is repeated until every row of the coefficient matrix has been processed, first for EEG and then for sEMG signals. The corresponding pseudo-code is described in Algorithm 1. The component selection algorithm in [2] was treating every signal in the data set individually, as a consequence, its numerical complexity was L times higher, where L is the number of trials, *i.e.* signals in the data set.

The coherence between EEG and EMG signal is finally computed by (4) using the EEG and EMG matrices obtained after the component selection.

Algorithm 1 Component Selection Algorithm

Input: $\mathbf{S_X}$: the coefficient matrix of EEG signals; $g \leftarrow 1$; N: the number of rows of matrix $\mathbf{S_X}$; L: the number of columns of matrix $\mathbf{S_X}$; $\vec{\lambda}$: a row vector of length L; CMC_0: the original CMC value between $\mathbf{S_X}$ and $\mathbf{S_Y}$.
Output: reconstructed $\mathbf{S_X}$.

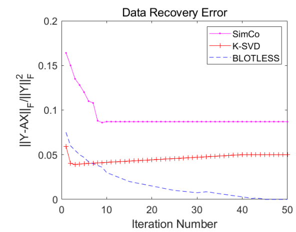
repeat
  $\vec{\lambda} \leftarrow \mathbf{S_X}_{g,:}$
  $\mathbf{S_X}_{g,:} \leftarrow \mathbf{0}$;
  calculate CMC_g using (1);
  if $CMC_g > CMC_{g-1}$ **then**
    remain $\mathbf{S_X}_{g,:}$
  else
    $\mathbf{S_X}_{g,:} \leftarrow \vec{\lambda}$
  end if
  $g \leftarrow g + 1$;
until g >N

III. EVALUATIONS

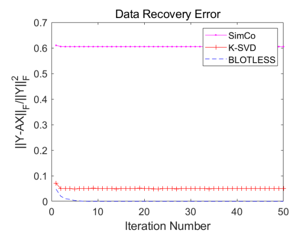
A. Performance of dictionary learning algorithms

In this study we consider three dictionary learning algorithms: K-SVD, SimCO and BLOTLESS. The main difference between these algorithms lies in the dictionary update stage. K-SVD updates dictionary items sequentially, one by one. SimCO updates all dictionary items simultaneously, whilst BLOTLESS updates dictionary items block by block [11]. In the tests with SimCO, we used regularised SimCO [13] with the block size of 100, and the regularization parameter of 0.05.

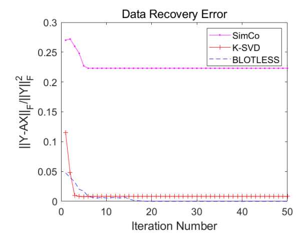
We first compared the learning algorithms according to their ability to learn dictionaries under which the simulated data can be represented accurately using only few dictionary atoms. Fig. 1 shows how the error of sparse expansions of the simulated data depends on the number of iterations of the considered dictionary learning algorithms. From the plots in Fig. 1, it can be seen that the convergence rate of BLOTLESS is the fastest. All three plots reflect the phenomenon that SimCO falls into local optimum easily.



(a) $-20\ dB$



(b) $-10\ dB$



(c) $-5\ dB$

Fig. 1. Performance of the three dictionary learning algorithms under different SNRs ($-20dB$, $-10dB$ and $-5dB$) in function on the number of iterations. The curves represent relative errors of representing considered data sets using 10 elements of learned dictionaries.

We then compare the three considered dictionary learning algorithms based on their performance in enhancing CMC values using synthetic signals with signal-noise ratios (SNRs) of $-20dB$, $-10dB$ and $-5dB$, adding synchrony only in β range. For each signal-to-noise ratio, we simulated neurophysiological signals of 200 trials according to the model introduced in section II.A. We went through 50 cycles to simulate the signals and then use the dictionary learning algorithms to improve CMC. In the dictionary learning stage, the size of dictionary was chosen as 400. The peak values of CMC are recorded each epoch, and the mean values of 50 outcomes obtained by different dictionary learning algorithms are summarised in Table 1. It can be observed that CMC levels obtained using BLOTLESS are higher than the levels obtained using K-SVD and SimCO.

TABLE I
CMC BETWEEN SYNTHETIC SIGNALS BEFORE AND AFTER SPARSE REPRESENTATION. K-SVD, SIMCO AND BLOTLESS WERE CONSIDERED.

SNR(dB)	CMC			
	Original	K-SVD	SimCO	BLOTLESS
-20	0.0325	0.0503	0.0507	0.0528
-10	0.0724	0.0787	0.0811	0.0862
-5	0.2218	0.2232	0.2266	0.2425

In the above evaluations in terms of convergence rate and the ability to enhance coherence, BLOTLESS is the best-performing algorithm. Thus, when processing the physiological signals, BLOTLESS is adopted at the dictionary learning stage.

B. Neurophysiological signals

The neurophysiological signals used in this evaluation were recorded from seven healthy subjects aged 24-62 years [20], during a motor control task. All subjects were right-hand dominant by self-report. Subjects were holding a 15 cm plastic ruler, grasping the end 2 cm of the ruler in a key grip, and keeping the ruler 2 cm above and parallel to the table surface. They were asked to hold the ruler gently against an electromechanical tapper, which provided pulses of lateral displacement, so to maintain the position of the ruler, but not to resist the perturbation. The length of a single trial was 5 s, with the stimulus given 1.1 s after the start of the trial. The stimuli were generated at pseudorandom intervals varying between 5.6 and 8.4 s (mean 7 s) so that subjects would not predict the arrival of the next stimulus. The entire experiment consisted of up to 8 blocks of 25 trials each. Thus, up to 200 trials of data were collected from each subject. Bipolar surface EMG was recorded over the first dorsal interosseous (FDI) muscle of the dominant hand, using a Nicolet Viking IIIP EMG machine[20]. Bipolar EEG was recorded from the scalp overlying the contralateral hand area of the motor cortex as described in [16]. EEG and EMG signals were sampled at 1024 Hz, amplified and band-pass filtered ($0.5 - 100$ Hz for EEG; $5 - 500$ Hz for EMG). Raw data were reviewed offline by visual inspection and epochs of data containing movement or blink artifacts were rejected.

The CMC was estimated via short time Fourier transform (STFT), which was computed at $M = 512$ frequencies, using $125ms$ Hanning window with $9.8ms$ shifts between consecutive analysis windows, as this choice of parameters was found to provide the most suitable time-frequency resolution for the analysis of the considered physiological signals [21].

In this study we focus on the second prominent peak of the cortico-muscular coherence (Peak 2), which occurs between 1.5 and $4.5s$, has less bidirectional signaling, represents more stable interactions between EEG and sEMG signals, and is typically difficult to detect [21]. To extract the coherent components, 256-sample segments are first extracted around Peak 2. BLOTLESS is chosen as the algorithm for dictionary learning, followed by ADMM to get more accurate representation. The regularisation parameters λ_1 and λ_2 are chosen as 0.05 via cross-validation.

Fig. 2 compares the CMC values of subject B before and after processing. Fig. 2(a) shows the original CMC between EEG and FDI signals. Fig. 2(b) shows the CMC between extracted segments around Peak 2. Then the extracted segments were processed using BLOTLESS and ADMM. The coherence plot obtained at this stage is displayed in Fig. 2(c). Fig. 2(d) gives the final CMC plot with noise and irrelevant background activities removed using the component selection algorithm. From the plots, one can observe that the coherent EEG/sEMG components in frequency band other than β are also enhanced.

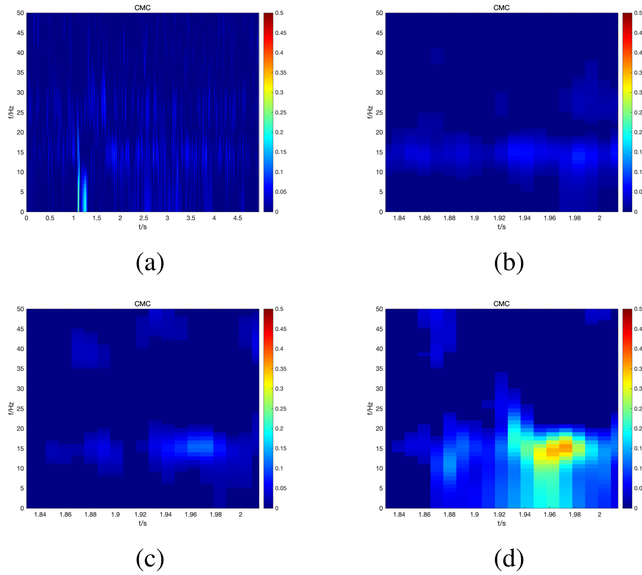


Fig. 2. CMC plots of experimental signals of subject B. CMC values below the 95% confidence limit are set to zero. (a) Original CMC plot, (b) CMC of 256-sample segment, (c) CMC plot before selection, (d) CMC plot after selection.

TABLE II
COMPARISON OF CMC VALUES

Subject	CMC		
	Original	Before selection	After selection
B	0.0787	0.1087	0.3672
G	0.1275	0.1432	0.3371
J	0.0947	0.1039	0.1900
K	0.1660	0.1867	0.2798
L	0.0771	0.0774	0.2732
N	0.0578	0.1326	0.4274
Q	0.0997	0.1169	0.2678

Table 2 summarizes the CMC values at the second peak for all subjects. In general, the CMC of all subjects has been increased to a considerable extent. Except for subject K, the CMC alues increased by more than 100%. We applied analysis of variance (ANOVA) to the original CMC values and the values which are obtained after component selection. When the significance level α is 0.05, the increase of CMC is significant with p-value of $3.62026e - 05$.

TABLE III
COMPARISON OF CMC INCREASE BY [2] AND OUR IMPROVED METHOD

Subject	CMC increase of Peak 2 $(\%)$	
	Xu's method	Improved method
B	313.57	**366.58**
J	64.96	**100.63**
K	**71.48**	68.55
L	118.42	**254.35**
N	244.98	**639.45**

Finally we compared the effectiveness of our proposed algorithm to the effectiveness of the algorithm proposed in [2]. Table 3 shows results for the 5 subjects considered in [2]. It can be observed that, except for subject K, where a minor performance degradation is observed, the new algorithm exhibits more improvement in increasing CMC levels.

IV. Conclusion

Sparse signal representations based on dictionary learning have proved effective in enhancing levels of cortico-muscular coherence estimated using EEG and surface-EMG signals collected synchronously during motor control tasks. However, previous work in this domain has been restricted to dictionary learning using the K-SVD algorithm in combination with a numerically inefficient greedy algorithm for selection of relevant components [2], hence the full potential of this technique in the context of motor neuroscience remains unexplored. In this paper, we propose using BLOTLESS, a recently developed dictionary learning algorithm, for learning underlying EEG-EMG dictionaries in combination with a simplified component selection algorithm, and achieve remarkable improvements in enhancing EEG-EMG coherence levels.

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