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Improving Nonlinear Interpolation of K-Space Data Using Semi-Supervised Learning and Autoregressive Model

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Abstract—**Parallel magnetic resonance imaging (pMRI) accelerates data acquisition by undersampling k-space through an array of receiver coils. Finding accurate relationships between acquired and missing k-space data determines the interpolation performance and reconstruction quality. Autocalibration signals (ACS) are generally used to learn the interpolation coefficients for reconstructing the missing k-space data. Based on the estimation-approximation error analysis in machine learning, increasing training data size can reduce estimation error and therefore enhance generalization ability of the interpolator, but scanning time will be longer if more ACS data are acquired. We propose to augment training data using unacquired and acquired data outside of ACS region through semi-supervised learning idea and autoregressive model. Local neighbor unacquired k-space data can be used for training tasks and reducing the generalization error. Experimental results show that the proposed method outperforms the conventional methods by suppressing noise and aliasing artifacts.**

I. Introduction

Magnetic resonance imaging (MRI) has revolutionized radiology over past four decades. It has some advantages in compared to other imaging modalities such as non-radiation and superior soft tissue contrast. However, MRI suffers the low speed problem that causes high cost. In addition, patient has to stay in the closed-bore of MRI scanner for a long time and may feel uncomfortable with tight spaces. For this reason, many techniques have been proposed to accelerate MR imaging speed. Among those techniques, parallel MRI (pMRI) [1-3] speeds up data acquisition by undersampling data along the phase-encoding direction on k-space through an array of receiver coils. Aliasing artifacts caused by the undersampling acquisition below the Nyquist sampling rate are removed by separating aliased pixels in image domain or interpolate missing k-space data using neighboring k-space points [4].

Parallel MRI reconstruction techniques can be classified as image-based [2], k-space-based [1, 3], and combinations of previous two kinds of methods [5]. Estimation of coil sensitivities is needed to perform image-based reconstruction, which can be acquired from a separate calibration scan or fully sampled data at the center of k-space. On the other hand, k-space-based methods, formulated as an interpolation procedure, directly reconstruct missing k-space signals without requiring prior knowledge of coil sensitivities. Finding accurate relationships between acquired and missing k-space data determines the interpolation performance and reconstruction quality. For example, generalized auto-calibrating partially parallel acquisitions (GRAPPA) [3], widely used in routine clinical practice, has been improved by enhancing interpolation accuracy from multiple approaches like enlarging interpolation kernel size [6], cross-validated kernel selection [7], iterative reconstruction [8], image support reduction [9], virtual coil concept [10], infinite pulse response [11], discrepancy-based adaptive regularization [12], nonlinear interpolation [13], coefficient penalized regularization [14], sparsity-promoting calibration [15], and properly selected coefficients [16].

Autocalibration signals (ACS) data are used to learn the interpolation coefficients for reconstructing the missing k-space data in GRAPPA. However, the acquired k-space lines outside of the ACS region and unacquired data are not used in calibration process, so that the learned regression model may be not accurate due to the limited training data, provided by the limited number of the acquired ACS lines. In machine learning theory, increasing the training data is helpful to enhance the generalization capability [17], which enables the k-space interpolation more accurate. However, the increased ACS lines as the training data cause the longer scan time and therefore delay imaging speed. To increase training data and improve reconstruction quality, transfer learning [20] has been used for MRI reconstruction in recent years [18, 19]. However, in medical imaging, particularly in MRI, training data and testing data may have different distributions in source and target domains under transfer learning framework. Moreover, MR coil architectures and images acquired from different MR scanner vendors may have different characteristics and features making transfer learning inefficient [18]. In addition, deep learning yields unstable reconstruction with several insatiability forms with respect to 1) certain tiny perturbations, 2) small structural changes, and 3) changes in the number of samples [24]. Scan-specific reconstruction was proposed to improve k-space interpolation accuracy with database-free deep learning [21]. Since training ACS data is limited in scan-specific manner, the 3-layers convolutional neural networks were trained for avoiding too many features and reducing the over complexity. The optimal model capacity should be selected with the suitable size of the training data for achieving the lowest possible generalization error [26].

Besides the ACS lines as training data, the unacquired and acquired k-space data outside of ACS region may be also helpful for training regression model and fitting interpolation coefficients in GRAPPA. Motivated by the semi-supervised learning (SSL) [22], both ACS data (similar to labeled data in SSL) and the unacquired and acquired data (similar to unlabeled data in SSL) outside of ACS region could enhance the training performance and improve generalization capability, rather than only ACS data used for training. On the

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other hand, since linear-predictive relationships rely on local Fourier information [23], smoothness assumption of SSL on Fourier domain may be applicable and local unacquired k-space data could be used for training tasks. Based on this analysis, we propose a nonlinear interpolation method for improving GRAPPA reconstruction using SSL and autoregressive (AR) model. In this paper, motivation and background introduction are presented in the sections I and II. The section III provides the proposed method. Experimental results and conclusion are given in the sections IV and V.

II. Related Background

A. GRAPPA Reconstruction

GRAPPA reconstruction [3] is generalized as an interpolation process to estimate missing k-space data as the following equation:

$$S_j(k_y + r \cdot \Delta k_y, k_x) = \sum_{l=1}^{L} \sum_{b=-N_b}^{N_a} \sum_{h=-H_l}^{H_r} w_{j,r}(l,b,h) \times S_l(k_y + b \cdot R \cdot \Delta k_y, k_x + h \cdot \Delta k_x) \quad (1)$$

, where S represents k-space signals, w denotes the weight coefficients estimated by using ACS data, R is acceleration factor, j is the target coil interpolated by all other coils counted by l, and b and h construct the interpolation kernel. The indices k_x and k_y represent data positions along frequency encoding and phase encoding directions, respectively. The interpolation coefficients are calculated at first by using ACS data acquired in k-space. Then, those estimated coefficients are used to interpolate missing k-space data. It can be considered as a linear regression model with complex values of k-space data for solving the linear inverse problem. This calibration and interpolation process can be simplified as a matrix equation:

$$b = Ax \quad (2)$$

, where A represents the matrix comprised of the undersampled points, b denotes the vector for the missing points, and x represents the coefficients to be estimated.

B. Autoregressive Model in MRI Reconstruction

The autoregression moving average (ARMA) model has been used to improve GRAPPA reconstruction [11]. The correlation of k-space data points is better characterized with the infinite impulse response model for improving interpolation accuracy and therefore reconstruction quality. Based on the linear predictability, missing k-space data may be accurately imputed by as a linear combination of measured samples [23]. Within a finite spatial support, Fourier samples can be interpolated by a linear combination of past and future samples with autoregressive structures. It is possible to reuse unacquired Fourier samples for estimating interpolation coefficients during the training phase.

III. Proposed Method

During calibration of GRAPPA reconstruction, generalization error can be decomposed into the estimation error and the approximation error. The error decomposition is demonstrated in Figure 1. The estimation error is caused by some factors such as noise and limited kernel size, and the approximation error is generated since ACS data and testing data on outer k-space have different distributions with different mean values and variances.

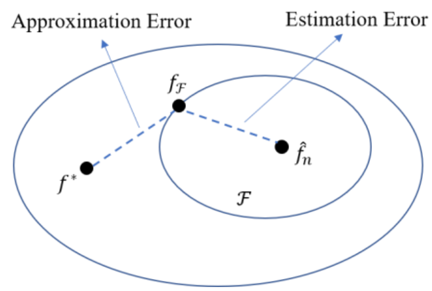


Figure 1. Estimation-approximation error decomposition with two terms at odds with each other.

The learning bias $\mathcal{F}$ is limited with smaller estimation error, and the approximation error becomes larger. The larger sample size n enables smaller estimation errors and has no effect on approximation error, but it requires more acquired k-space data with longer scanning time. Therefore, there is a tradeoff between sizes of $\mathcal{F}$ and n. The total generalization error is represented as

$$L(\hat{f}_n) - L(f^*) = \left[L(\hat{f}_n) - \inf_{f \in \mathcal{F}} L(f)\right] + \left[\inf_{f \in \mathcal{F}} L(f) - L(f^*)\right] \quad (3)$$

, and

$$f^* = \operatorname*{argmin}_f L(f) \quad (4)$$

, where the first term of the right side of the Eq. (3) is the estimation error, and the second term is approximation error, L denotes the loss function, $L(f^*)$ represents the lowest expected loss. If the sample size n is infinite, $L(\hat{f}_n) = L(f^*)$.

A. Semi-Supervised Learning

If the sample size of ACS n is increased, the estimation error can be reduced. However, increasing sampled ACS data will delay scanning time. It may be possible to using unacquired k-space data for estimating interpolation coefficients. The ACS data acquired at the central k-space is considered as labeled data, which is limited in compared to unlabeled data such as unacquired and unsampled data in outer ACS region. The unacquired k-space data near the ACS region could have similar mean values and variance of magnitude on Fourier domain, in accordance with linear predictive relationships relying on local Fourier information [23]. Unacquired k-space data near the ACS lines may be belonged to the same clusters with ACS data and they distribute on the same manifold. Neighbor unacquired data near ACS lines may meet both clustering assumption and manifold assumption of semi-supervised learning [22]. Through an iterative process, the estimated k-space data near ACS lines predicted from the previous iteration are also used for calculating and updating interpolation coefficients, so that sample size n is gradually increased.

B. Recursive Estimation of Interpolation Coefficients in Autoregressive Mode

To increase data for estimating interpolation coefficients, local neighbor unacquired k-space data are also iteratively used to augment training data, as shown in Figure 2. The initially estimated k-space data are generated using the conventional GRAPPA reconstruction. In the first iteration, two neighbor unacquired lines are added to the previous ACS regions for estimating interpolation coefficients, and then interpolated unacquired k-space data replace the predicted data in the previous iteration. The interpolation kernel is different from the conventional GRAPPA, and it covers the locations of missing k-space lines, and the interpolation is similar to the equation (6) in the reference [11]. Furthermore, nonlinear terms along both of phase-encoding and frequency-encoding directions are also added to enhance SNR, as nonlinear GRAPPA [13] does.

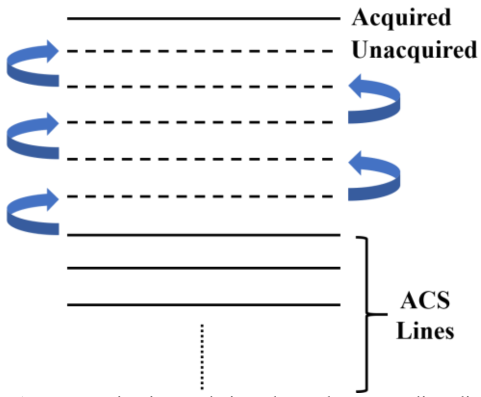


Figure 2. Autoregressive interpolation along phase encoding direction in k-space. Unacquired k-space lines are re-generated by existing reconstructed data following the order from inner k-space to outer k-space. The upward-and-downward phase-encoding directions are autoregressively interpolated. Only one direction (upward) along is demonstrated here.

IV. EXPERIMENTAL RESULTS

A. Datasets

Two datasets are used to evaluate the proposed method. The first dataset is an 8-coil phantom data and the second one is a 4-coil axial brain data. For phantom dataset, an outer reduction factor 6 and 28 ACS lines are used to undersample k-space. A convolutional kernel size 2 x 33 is used for interpolating the conventional GRAPPA to produce the initially estimated k-space. For brain dataset, the outer reduction factor is 4 with 22 ACS lines. The kernel size is 2 x 15 for producing initial k-space estimation. Then, the neighbor 2 unacquired lines along phase-encoding direction and 11 neighbors along frequency-encoding data points are used for iteratively interpolating k-space with augmented training data. The experimental procedures involving human subjects described in this paper were approved by the Institutional Review Board.

B. Reconstruction Quality Evaluation

Reconstruction results of the phantom dataset are shown in Figure 3. The conventional GRAPPA [3] and ARMA-model [11] based GRAPPA reconstructions show serious noise and aliasing artifacts at high reduction factor 6. Nonlinear GRAPPA [13] show artifacts due to low number of ACS lines used in reconstruction. The proposed method is able to reduce noise at the 1st iteration and the 10th iteration reconstructions. The latter one can also reduce tiny aliasing artifact in compared to the 1st iteration reconstruction.

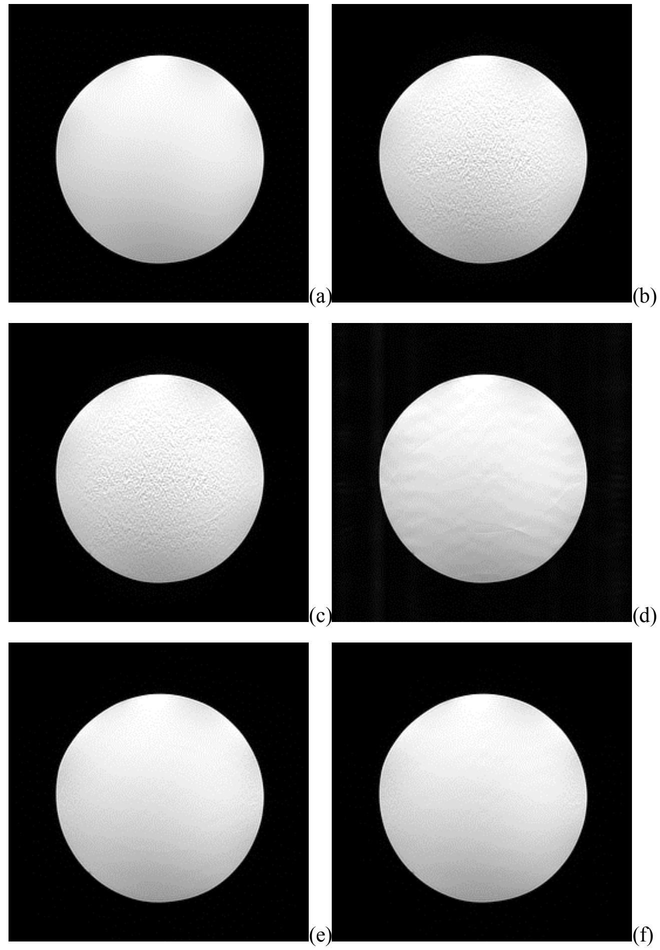
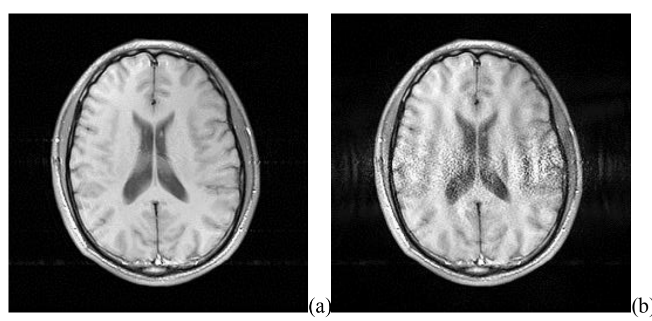


Figure 3. Phantom reconstruction with the fully-sampled reference image (a), the traditional GRAPPA (b), the ARMA model-based reconstruction (c), nonlinear GRAPPA (d), and the proposed reconstruction with the 1st iteration (e) and the 10th iterations (f).

Similar to the reconstruction results of phantom. The proposed method outperforms the conventional GRAPPA and ARMA-model based GRAPPA with suppressing noise and aliasing artifacts of brain image reconstruction, as shown in Figure 4. The 1st iteration of the proposed method shows some aliasing artifacts, and they gradually disappear in the 10th and the 30th iterations. The augmented training data by gradually enlarging neighbor unacquired lines may improve the interpolation accuracy in low-frequency area of Fourier domain.



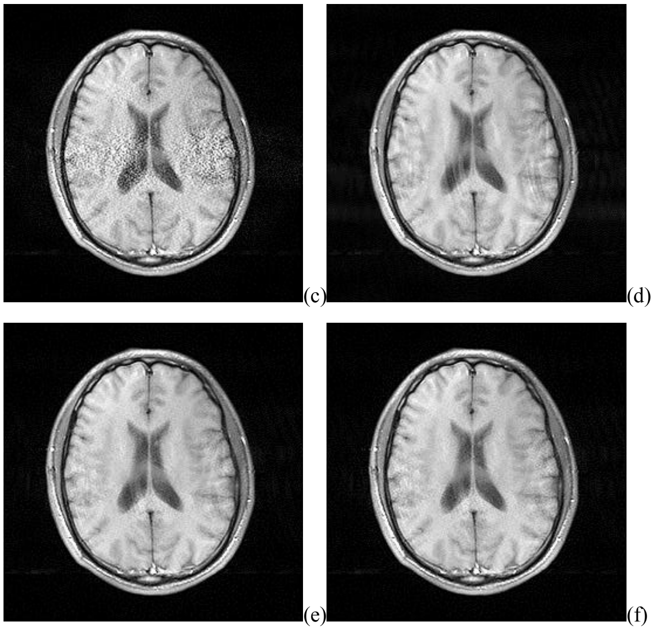


Figure 4. Fully-sampled k-space data is reconstructed as the reference image (a). Brain MRI reconstructions with the traditional GRAPPA (b), and ARMA model (c). The proposed reconstructions with the 1st, 10th, and 30th iterations are presented in (d), (e), and (f), respectively. Nonlinear GRAPPA reconstruction shows serious aliasing artifacts using the low number of ACS lines, which is not shown here.

V. Conclusion

In conclusion, an iterative reconstruction method is proposed to improve nonlinear interpolation accuracy of k-space data for enhancing reconstruction quality. Semi-supervised learning idea is applied to local neighbor k-space data with cluster assumption and manifold assumption, and training data are augmented through autoregressively adding local unacquired data near ACS region to interpolation coefficient learner. Experimental results show that the proposed method is able to suppress noise and aliasing artifacts in compared to the conventional methods. Due to learning-based MRI reconstruction suffers instabilities with perturbations [24, 25], stabilizing semi-supervised learning-based reconstruction quality will be studied in the future work.

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