Abstract—MicroRNA-based gene therapy for cancer treatment via nanoparticles (NPs) requires navigation of multiple physical and physiological barriers in order to efficiently deliver the miRNAs to the cancer cell cytoplasm. We here present a mathematical model to investigate the variability associated with tumor, NP, and miRNA characteristics, and identify the limiting factors in miRNA delivery to tumors. Through global parameter analysis, the miRNA release rate from NPs and NP degradability were found to have the most significant impact on cytosolic accumulation of miRNAs. These NP properties can be fine-tuned in order to optimize the delivery system for enhancing the efficacy of miRNA-based therapy.

Clinical Relevance—Understanding the effect of nanoparticle, tumor, and miRNA characteristics in governing the efficacy of miRNA-based cancer therapy will support its clinical translation.

I. INTRODUCTION

MicroRNAs (miRNAs) are small non-protein-coding RNAs (~22 nucleotides) that regulate cell signaling by binding to genes that code for proteins, thereby inhibiting protein translation. It is increasingly recognized that miRNA dysregulation causes aberrant cell signaling associated with cancer. miRNAs are either found to be downregulated [1] or upregulated [2] in tumors, which manifests as downregulation of tumor suppressor effects and upregulation of oncogenic effects, respectively. Therefore, exogenous administration of miRNAs or their antagonists to normalize aberrant signaling pathways is currently an attractive approach to inhibit tumor cell proliferation. However, given the short half-life of naked miRNAs due to degradation by plasma ribonucleases, limited tumor penetration and cellular uptake due to their negative charges, and off-target effects due to non-specific delivery, nanoparticle (NP)-based delivery systems have been used to ensure targeted and efficient delivery to tumor cells [3].

Whereas the application of nanomaterials in cancer has shown potential to improve targeted drug delivery to tumors [4, 5], challenges with low tumor deliverability due to insufficient understanding of the collective effects of NP properties on NP pharmacokinetics and tumor penetration still exist. To this end, mechanistic mathematical modeling can be used as a valuable in-silico tool [6-8] to improve the understanding of NP-mediated miRNA delivery to cancer cells in vivo, and can provide practical guidelines to optimize NP design for enhanced miRNA therapy efficacy.

II. METHODS

Figure 1. Model schematic. Model consists of a plasma and tumor compartment, with the latter sub-compartmentalized into vascular, interstitial, cell membrane, and cell cytosolic space. Yellow circles represent nanoparticles (NPs) loaded with miRNAs (green stars). Red arrows indicate transport processes. Curved orange arrow indicates tumor growth. Black hammerhead arrows indicate inhibition of tumor proliferation. Magenta arrows represent degradation and yellow arrow indicates clearance of NPs.

A. Model development

We have developed a multiscale mechanistic model, calibrated to mice, to study the pharmacokinetics and pharmacodynamics of NP-delivered, miRNA-based cancer therapy. As shown in Fig. 1, the model consists of a plasma (systemic circulation) and a tumor compartment, where the latter is further divided into vasculature, interstitium, and cytosolic sub-compartments. Following intravenous (i.v.) injection of miRNA-loaded NPs into the plasma compartment, NPs are cleared from the systemic circulation via renal and/or hepatobiliary excretion mechanisms (based on their physicochemical characteristics) [9, 10]. In the current model, clearance is parameterized by $k_{Cl}$ (units, $\text{wk}^{-1}$), which empirically varies with NP diameter ($\phi_{NP}$; units, cm) as: $k_{Cl} = \frac{0.11e^{-1.33\cdot \phi_{NP} - 0.001e^{-9.7\cdot \phi_{NP}}}}{\ln(2)}$, obtained by fitting the plasma half-life data of quantum dots of varying sizes from the literature [10, 11].

NPs circulating in the plasma compartment are transported between the plasma and tumor vasculature sub-compartment via perfusion, defined by the plasma flow rate $Q$ (units, $\text{mL} \cdot \text{g}^{-1} \cdot \text{wk}^{-1}$), which is determined by the following empirical relationship with tumor volume $B(t)$ (units, $\text{cm}^{3}$): $Q = 2843 \cdot e^{-0.65 \cdot B(t)}$, obtained by fitting a monoexponential function to data reported in [12]. From the tumor vascular sub-compartment, NPs can unidirectionally permeate and accumulate in the tumor extravascular regions to deliver the cargo to the cancer cells (a phenomenon referred to as the enhanced permeability and retention (EPR) effect). As shown in Fig. 1, we model this serial process in three steps, characterized by the following rate constants: i) The

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extravasation rate of NPs from the vascular to interstitial sub-
 compartment is characterized by the permeability (P)-surface
area (S) product (P · S), where P (units, cm · wk⁻¹) is a
function of tumor vascular porosity and the ratio of NP size
(φNP) to tumor vascular pore size (φpore; units, cm) [13], and
S (units, cm²/cm³) relates to tumor volume B(t) as
S = 0.26 · e⁻⁴.5(B(t)) + 138 · e⁻⁰.⁰⁴(B(t)) obtained empirically from
data in the literature [14]. ii) The diffusion rate of NPs across
the tumor interstitium to reach the cancer cells is characterized by
the ratio D/ L², where D is NP diffusivity (units, cm² ·
wk⁻¹) and L is the intercapillary distance (units, cm),
suggestive of the characteristic length of the interstitium
between tumor vessels and cancer cells. iii) Once in the
vicinity of cancer cells, NPs undergo endocytosis
(characterized by the rate of endocytosis (kendo; units, wk⁻¹))
to enter the cell cytosolic region.

The endocytosed NPs then release miRNAs into the
cytoplasm at a rate characterized by krel (units, wk⁻¹), while
undergoing degradation inside the endosome at a rate δNP
(units, wk⁻¹). The released miRNAs act on their target site
(messenger RNA molecules) to suppress protein translation and
exhibit tumor suppressor effects, so the tumor growth rate
constant σ (units, wk⁻¹) is scaled by the dimensionless factor
1/1 + (D/ L²)φpore, where Cm(t) is the cytosolic concentration of
miRNA and EC50 is the concentration of miRNA inducing half
of the maximum possible tumor inhibition. Inside the cytosol,
miRNA also undergoes degradation characterized by the rate
constant δM (units, wk⁻¹). The various model parameters
were either known a priori or obtained through data fitting
from the literature (Table I).

### TABLE I. LIST OF MODEL PARAMETERS STUDIED IN GSA

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Units</th>
<th>Value (Range)*</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>Tumor growth rate constant</td>
<td>wk⁻¹</td>
<td>2.99 (1.3-6.5)</td>
<td>[15]</td>
</tr>
<tr>
<td>φpore</td>
<td>Dia. of tumor vessel pore</td>
<td>nm</td>
<td>1700 (300-4700)</td>
<td>[16]</td>
</tr>
<tr>
<td>L</td>
<td>Tumor intercapillary length</td>
<td>μm</td>
<td>100 (10-250)</td>
<td>[17]</td>
</tr>
<tr>
<td>ηT,B</td>
<td>Dynamic viscosity of tumor blood</td>
<td>cP</td>
<td>7.42 (4.5-10)</td>
<td>[14]</td>
</tr>
<tr>
<td>ηT,I</td>
<td>Dynamic viscosity of tumor interstitium</td>
<td>cP</td>
<td>3.5 (2-5)</td>
<td>[18]</td>
</tr>
<tr>
<td>φNP</td>
<td>Dia. of NP</td>
<td>nm</td>
<td>70 (25-150)</td>
<td>—</td>
</tr>
<tr>
<td>δNP</td>
<td>NP degradation rate</td>
<td>wk⁻¹</td>
<td>0.1 (0.01-1)</td>
<td>arb.</td>
</tr>
<tr>
<td>N₀</td>
<td>No. of NPs per injection (represents dose of miRNA)</td>
<td>—</td>
<td>2.5e+10 calc. (6.4e+9 - 1e+12) ≡ 4μg miRNA (1-156 μg)</td>
<td></td>
</tr>
<tr>
<td>EC₅₀</td>
<td>EC₅₀ of miRNA</td>
<td>nM</td>
<td>(0.1-100)</td>
<td>[19]</td>
</tr>
<tr>
<td>krel</td>
<td>Release rate of miRNA from NPs</td>
<td>wk⁻¹</td>
<td>0.42 (0.16-9.7)</td>
<td>arb.</td>
</tr>
<tr>
<td>δM</td>
<td>Decay rate of miRNA</td>
<td>wk⁻¹</td>
<td>4.8 (2.4-29.1)</td>
<td>[20]</td>
</tr>
</tbody>
</table>

*Parameter value range used for GSA

### B. Model equations

The various transport and pharmacodynamic processes
described above have been formulated into a system of
ordinary differential equations (ODEs; Eqs. 1–7) to
numerically simulate a reference treatment protocol. We then
used the model to further understand the role of different
model parameters in governing NP and miRNA delivery, and
treatment outcome.

**Equation for NP mass kinetics in plasma (NP(t)):**

\[
\frac{dNP(t)}{dt} = \left( \frac{NP(t)}{V_{TP}} \right) \cdot Q \cdot B(t) - \frac{NP(t)}{V_{TP}} \cdot \left( 1 + \frac{2}{N_0} \right) B(t) - k_{CI} \cdot NP(t),
\]

\[
NP(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(1)

where, \(V_{TP} (= f_e \cdot B(t))\) and \(V_P (1 \text{ mL})\) [21] are volumes of
tumor vascular and plasma compartments, respectively; \(f_e (= 0.17)\) is the vascular fraction of the tumor [14]; \(N_0\) is the
injected dose of NPs and \(i\) represents the injection times (in
weeks) post inoculation of tumor in mice \(i = 2, 3, 4, 5\).

**Equation for NP mass kinetics in tumor vasculature (NP(t)):**

\[
\frac{dNP(t)}{dt} = \left( \frac{NP(t)}{V_{TP}} - \frac{NP(t)}{V_{TP}} \right) \cdot Q \cdot B(t) - P \cdot S \cdot NP(t),
\]

\[
NP(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(2)

**Equation for NP mass kinetics in tumor interstitium (NP(t)):**

\[
\frac{dNP(t)}{dt} = \left( 1 + \frac{2}{N_0} \right) B(t) - \frac{NP(t)}{V_{TP}} \cdot \left( 1 + \frac{2}{N_0} \right) B(t) - k_{endo} \cdot NP(t),
\]

\[
NP(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(3)

**Equation for NP mass kinetics in cancer cell membrane (NC(t)):**

\[
\frac{dNC(t)}{dt} = \frac{D}{L^2} \cdot NC(t) - k_{endo} \cdot NC(t),
\]

\[
NC(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(4)

**Equation for NP mass kinetics in cancer cell cytosol (NC(t)):**

\[
\frac{dNC(t)}{dt} = \frac{D}{L^2} \cdot NC(t) - \frac{NP(t)}{V_{TP}} \cdot \left( 1 + \frac{2}{N_0} \right) B(t) - k_{endo} \cdot NC(t),
\]

\[
NC(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(5)

**Equation for miRNA concentration kinetics in cancer cell cytosol (CM(t)):**

\[
\frac{dCM(t)}{dt} = k_{rel} \cdot \frac{NC(t) \cdot M_{miRNA} \cdot e^{-x_{rel} \cdot t}}{V_{TC}} - \delta_M \cdot CM(t),
\]

\[
CM(t) = \begin{cases} 0, & t = 0 \\ N_0, & t = i \\ \end{cases}
\]

(6)

where \(M_{miRNA}\) is the mass of miRNAs loaded in a single NP; \(V_{TC}\)
is the cytosolic volume of tumor \((= f_e \cdot f_{cyt} \cdot B(t))\), where
\(f_{cyt} (= 0.4)\) is the cancer cell volume fraction of a tumor [22]
and \(f_{cyt} (= 0.4)\) is the cytoplasmic volume fraction of a cancer
cell [23]. Note that the concentration of tumor suppressive
miRNAs is downregulated in cancer; therefore, it is reasonable
to assume its initial condition to be zero.

**Equation for tumor volume kinetics (B(t)):**

\[
\frac{dB(t)}{dt} = \frac{σ \cdot \left( 1 - \frac{B(t)}{B^*} \right) \cdot B(t)}{V_{T,C} - 1 + \frac{2}{N_0}} \cdot (1 + \frac{2}{N_0}) \cdot B(t),
\]

\[
B(0) = B_0
\]

(7)

where \(B_0\) is the inoculated size of tumor \((0.001 \text{ cm}³ =
10^5 \text{ cells})\) [24] and \(B^*\) is the tumor carrying capacity \((1 \text{ cm}³)\).

The model is solved numerically as an initial value problem in
MATLAB R2018a using the built-in function ode45.

**C. Parametric and statistical analyses**

To understand the effect of various delivery barriers, and
physicochemical and physiological processes on NP-
mediated miRNA delivery to the tumor cytosolic space,
which in turn affects tumor inhibition, we performed global
sensitivity analysis (GSA). In our analysis, a total of 11 model parameters were simultaneously perturbed from their baseline value (within a predefined physiological range, Table I). We focused on the following six model outputs of interest: \(\text{AUC}_{P}^{NP}, \text{AUC}_{V,T}^{NP}, \text{AUC}_{T}^{NP}, \text{AUC}_{T,C}^{NP}, \text{AUC}_{M}^{NP}, \) and \(t_{90}\), which represent area under the curve (AUC) of NP mass kinetics in -plasma, -tumor vasculature, -tumor interstitium, -tumor cytosol, miRNA concentration kinetics in tumor cytosol, and time to reach 90% of carrying capacity, respectively. While AUCs are indicative of bioavailability of NPs or miRNA in a given compartment, \(t_{90}\) indicates inhibition of tumor proliferation. Note that AUC is calculated through numerical integration using the built-in MATLAB function \(\text{trapz}\). To investigate the vast multidimensional parameter space, Latin hypercube sampling was used to obtain 11,000 sets of parameters, ten times. Each of the ten parameter batches and their corresponding model outputs was subject to multivariate linear regression analysis to obtain a distribution of regression coefficients for each parameter, which served as the sensitivity index (SI) to quantify the significance of the parameters in governing the model outputs. Finally, one-way ANOVA and Tukey’s test were conducted to rank the parameters in order of their SI value (higher the SI, the more sensitive the measured output is to changes in the parameter) [25-28]. All analyses were performed in MATLAB R2018a.

III. RESULTS AND DISCUSSION

Treatment simulation and global sensitivity analysis

We used the model to simulate a literature-based treatment regimen of miRNA therapy [24], which involves a weekly IV injection of 0.15 mg·kg\(^{-1}\) miRNA (equivalent to 4 μg/mouse of miRNA) for 4 weeks, starting 2 weeks post tumor inoculation. Of note, the number of NPs (reference size: 70 nm in diameter) required to load the reference dose of 4 μg miRNA is \(\approx 2.5 \times 10^8\); this number was estimated based on NP volume, molecular weight of miRNA (\(\approx 7\)kDa [29]), and the assumptions that the miRNA is a sphere of dia. \(\approx 2.5\) nm [30] and packed randomly into a hollow NP with a packing efficiency of \(\approx 64\%\) [31]. In our GSA, to simulate a different NP size (or miRNA dose) other than the reference value, we had to recalculate the number of NPs required to load a given dose. Also, to be consistent with other reports in the field, NP mass was represented by percent of injected dose (%ID), instead of actual number of NPs in a given compartment.

As shown in Fig. 2, the numerical solution of the model, based on the reference parameter values (Table I and Methods) and the implemented treatment regimen, appropriately predicted the temporal evolution of NP mass in various model compartments (Fig. 2a-2d), miRNA concentration in cancer cell cytosol (Fig. 2e), and tumor size (Fig. 2f). Following each injection, NP kinetics in the plasma was primarily governed by clearance mechanisms, represented by the lumped \(k_{C1}\) parameter, leading to almost complete elimination of the injected dose within a week (Fig. 2a). Further substantiated by GSA (Fig. 3a), NP size (\(\phi_{NP}\)), which exerts its impact by modulating \(k_{C1}\), was found to be the most significant parameter in affecting NP bioavailability in plasma (\(\text{AUC}_{P}^{NP}\)).

From the systemic circulation, the blood flow-dependent delivery to tumor causes an incremental change in NP mass in tumor vasculature with every injection (Fig. 2b), which is attributable to incremental rise of NP mass with every injection in plasma compartment (Fig. 2a), due to the remaining residual of the previous injection (change conspicuous in logscale, not shown). Additionally, as shown in Fig. 3b, tumor growth rate constant \(\sigma\) is the most significant parameter in affecting NP bioavailability in tumor vasculature (\(\text{AUC}_{V,T}^{NP}\)), which suggests that due to the growing tumor the rate of influx of NPs into the tumor vasculature increases with time because the tumor vasculature volume increases in parallel with the tumor volume (first term of Eq. 2), thereby causing the increments.

Further, NP mass kinetics in the tumor interstitium is coupled to tumor vascular kinetics (Fig. 2c), suggesting that permeation into or diffusion through the interstitium is not a rate limiting factor for the given baseline conditions of NP and tumor characteristics. However, as we identified through GSA (Fig.3c-e), \(\eta_{T,B}\) (tumor blood viscosity), \(\eta_{T,J}\), and \(\phi_{pore}\) (tumor vessel wall pore size) are significant in affecting the NP bioavailability in tumor interstitium (\(\text{AUC}_{T,C}^{NP}\)), which suggests that under a different set of parameter values (e.g., a hypovascular, or densely stromal, or tumor with smaller vessel wall pores, or larger NPs), we may observe decoupling from vascular kinetics that can make extravasation or interstitial diffusion a limiting factor in delivery to the cytosol.

Similarly, endocytosis through the membrane, which is a relatively fast process depending upon NP size (\(\phi_{NP}\)), does not decouple from vascular kinetics in limiting delivery to the inside of the cell for the studied NP size (Fig. 2c). However, as shown in Fig. 3d, \(\phi_{NP}\) is one of the most significant factors that affect the NP bioavailability in tumor cytosol (\(\text{AUC}_{T}^{NP}\)), in addition to the degradation rate of the NP (\(\delta_{NP}\)), suggesting that cytosolic accumulation of NPs is sensitive to their size and degradability. Moreover, as shown in Fig. 2d, the characteristics of the hypothetical NPs used in the simulations allow incremental accumulation of NPs in the cytosol with every injection, eventually surpassing the well-accepted
median value (0.7 %ID) of NP accumulation in the tumor following a single injection [32]. The NPs accumulated in the cytosol release miRNA into the cytosol, such that as shown in Fig. 3e, miRNA release rate ($k_{rel}$) is the most significant factor, followed by degradation rate of miRNA ($\delta_{m}$), in governing miRNA bioavailability in tumor cytosol ($A_{UMC, FC}$). The reference parameter values of the hypothetical miRNA used in the study allow attainment of miRNA concentration in the cytosol greater than the reference EC$_{50}$ value required for therapeutic efficacy, throughout the duration of treatment (Fig. 2f). As a result, as shown in Fig. 2f, following weekly injections of miRNA-loaded NPs, the tumor shows growth inhibition, resulting in a smaller size during the treatment timeframe in comparison to the control tumor. As observed, the treated tumor takes ~16 days longer to reach 90% carrying capacity ($t_{90} = 5.35$ wks), with respect to the no-treatment scenario ($t_{90} = 3$ wks). Note that $t_{90}$ is only being used for the purpose of quantitative comparison and has no clinical relevance. However, post treatment termination, as the miRNA concentration falls below the effective threshold the treated tumor manages to reach the carrying capacity, equaling the untreated tumor in size. This can be attributed to the lack of a death mechanism in the logistic tumor growth model (Eq. 7). For simplicity, the model only contains a single mechanism of action via which the miRNA exhibits its effect, i.e., miRNA concentration-dependent suppression of tumor growth rate constant $\sigma$, which is the most significant factor in affecting tumor growth (Fig. 3f). However, to allow the possibility of miRNA-induced or co-administered chemotherapeutic-induced death, future updates to the pharmacodynamic component of the model will be necessary.

IV. Conclusion

Using a simplified, multicompartment mechanistic model, we studied the pharmacokinetic and pharmacodynamic aspects of NP-mediated miRNA therapy for tumor inhibition, and also investigated the effects of simultaneous parameter perturbations (through GSA) on key model outputs, including the bioavailability of NPs in various compartments, the bioavailability of miRNAs in tumor cytosol, and the resulting therapeutic outcome of miRNA-induced tumor growth inhibition. In addition to the importance of NP size, through the analysis we were also able to identify the importance of miRNA release rate from NPs and degradability of NPs in governing cytosolic accumulation of miRNAs, which directly impact tumor growth based on the EC$_{50}$ of a given miRNA. These tunable NP properties thus provide opportunities to optimize the drug delivery system for enhancing the efficacy of miRNA therapy. This model represents a first step towards a more complete mechanistic model that can account for additional NP characteristics (e.g., targeting to improve endocytosis) and also include specific molecular signaling pathways in the cytosolic compartment to better represent the mechanisms of action of miRNAs.

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