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Finite element method modeling to confirm the results of comprehensive optimization of the tripolar concentric ring electrode based on its finite dimensions model *

Oleksandr Makeyev, *Senior Member, IEEE*, Yiyao Ye-Lin, Gema Prats-Boluda, *Member, IEEE*, and Javier Garcia-Casado, *Member, IEEE*

***Abstract*— Concentric ring electrodes are noninvasive and wearable sensors for electrophysiological measurement capable of estimating the surface Laplacian (second spatial derivative of surface potential) at each electrode. Significant progress has been made toward optimization of inter-ring distances (distances between the recording surfaces of the electrode), maximizing the accuracy of the surface Laplacian estimate based on the negligible dimensions model of the electrode. However, novel finite dimensions model offers comprehensive optimization including all of the electrode parameters simultaneously by including the radius of the central disc and the widths of the concentric rings into the model. Recently, such comprehensive optimization problem has been solved analytically for the tripolar electrode configuration. This study, for the first time, introduces a finite dimensions model based finite element method model (as opposed to the negligible dimensions model based one used in the past) to confirm the analytic results. Specifically, finite element method modeling results confirmed that previously proposed linearly increasing inter-ring distances and constant inter-ring distances configurations of tripolar concentric ring electrodes correspond to an almost two-fold and more than three-fold increases in relative and normalized maximum errors of Laplacian estimation when directly compared to the optimal tripolar concentric ring electrode configuration of the same size.**

***Clinical Relevance*— This study assesses and confirms the electrode configuration that maximizes the accuracy of the estimated Laplacian recorded via concentric ring electrodes. Therefore, it is potentially useful for designing future concentric ring electrodes for diagnostic purposes such as localization of epileptic foci.**

I. Introduction

Concentric ring electrodes (CREs; tripolar configuration shown in Fig. 1A) are noninvasive and wearable sensors for electrophysiological measurement capable of estimating the surface Laplacian (second spatial derivative of surface potential) at each CRE which is not feasible with conventional disc electrodes (Fig. 1B) [1]–[10]. Significant progress has been made toward optimization of inter-ring distances (distances between the recording surfaces of a CRE), maximizing the accuracy of the surface Laplacian estimate based on the negligible dimensions model (NDM) of the CRE [11]. Namely, in [11] the inter-ring distances optimization problem has been solved for tripolar (number of concentric rings n equal to 2) and quadripolar ($n = 3$) CRE configurations and 5th and 10th percentiles of absolute value of the Taylor series truncation term coefficient for the lowest remaining term order that has been shown to be a predictor of the Laplacian estimation error [11], [12]. Obtained results have been validated using finite element method (FEM) modeling [11]. However, a significant drawback of the simplistic NDM is that a single point of negligible diameter represents the central disc of the CRE surrounded by concentric circles of negligible width that represent the concentric rings which is inconsistent with the design of currently used CREs (Fig.1A).

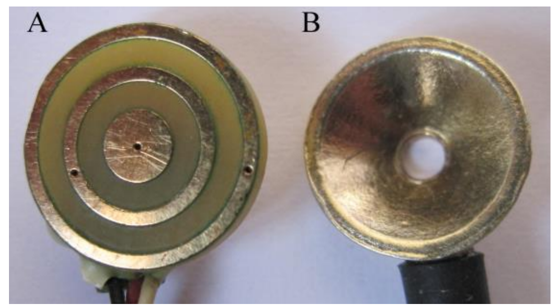

Figure 1. Tripolar concentric ring electrode (A) and conventional disc electrode (B).

Simultaneously, the comparison framework for the novel finite dimensions model (FDM) of a CRE was developed and validated on human electrocardiogram data [6] following the original proof of concept proposed in [13]. FDM allows adding the radius of the central disc and the widths of the concentric rings into the optimization problem. Such comprehensive problem permits the optimization of all of the CRE parameters simultaneously and has recently been solved analytically in [14]. Derived principles defining optimal CRE configurations have been illustrated on tripolar CREs but are likely to hold for any larger value of n. Optimal tripolar CRE has been directly compared to the previously proposed (in [6]) FDM based

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O. Makeyev is with the School of STEM, Diné College, Tsaile, AZ 86556 USA (e-mail: omakeyev@dinecollege.edu; phone: 928-724-6960).

Y. Ye-Lin, G. Prats-Boluda and J. Garcia-Casado are with the Centro de Investigación e Innovación en Bioingeniería (CI2B), Universitat Politècnica de València (UPV), Camino de la Vera s/n Ed. 8B, 46022 Valencia, Spain (emails: {yiye, gprats, jgarciac}@ci2b.upv.es).

linearly increasing inter-ring distances (LIIRD) and constant inter-ring distances (CIRD) tripolar CRE configurations of the same size. Obtained results suggested that previously proposed configurations correspond to an almost two-fold (99.33%) and more than three-fold (213.01%) increase in the Laplacian estimation error respectively compared to the optimal tripolar CRE configuration [14]. However, these analytic results have not been confirmed via FEM modeling.

This study, for the first time, adapts the NDM based FEM model from [11], [12], [15]–[17] to FDM to confirm the analytic results of the direct comparison between the three tripolar CRE configurations from [14]: relative and normalized maximum error ratios of Laplacian estimation (mean ± standard deviation for 10 CRE sizes) computed using the FEM model were equal to 1.97 ± 0.02 and 1.96 ± 0.02 respectively (LIIRD over optimal) as well as 3.07 ± 0.05 and 3.05 ± 0.07 respectively (CIRD over optimal).

II. Methods

FEM model from [11], [12], [15]–[17] was adapted from NDM to FDM to directly compare the surface Laplacian estimates for LIIRD and CIRD tripolar CRE configurations from [6] to the optimal (with respect to the accuracy of Laplacian estimation) tripolar CRE configuration of the same size from [14]. Matlab (Mathworks, Natick, MA, USA) was used for all the FEM modeling. FDMs of the three tripolar CREs compared are presented in Fig. 2. An evenly spaced square mesh of 700 x 700 points corresponding to roughly 20 x 20 cm was located in the first quadrant of the *X*-*Y* plane over a unit charge dipole oriented towards the positive direction of the *Z* axis and projected to the center of the mesh (Fig. 3). Electric potential v was generated at each point of the mesh for a dipole depth equal to 5 cm [18]:

$$v = \frac{1}{4\pi\sigma}\frac{(\bar{r}_p - \bar{r})\cdot\bar{p}}{\left|\bar{r}_p - \bar{r}\right|^3} \quad (1)$$

where $\bar{r} = (x, y, z)$ is the location of the dipole, $\bar{p} = (p_x, p_y, p_z)$ is the moment of the dipole, and $\bar{r}_p = (x_p, y_p, z_p)$ is the observation point.

This dipole depth was selected since out of the range of dipole depths (1 cm to 5 cm) that was assessed in our previous work [17] it corresponded to the lowest standard deviation of relative and maximum errors for 10 CRE sizes considered (CRE diameter ranging from 0.5 cm to 5 cm) thus making reported mean values more representative [12], [19]. The medium was assumed to be homogeneous with a conductivity σ equal to 7.14 mS/cm to emulate biological tissue [20].

The analytical Laplacian was calculated at each point of the mesh, by taking the second spatial derivative of the electric potential v [18]:

$$\Delta v = \frac{3}{4\pi\sigma}\left[5(z_p - z)^2\frac{(\bar{r}_p - \bar{r})\cdot\bar{p}}{\left|\bar{r}_p - \bar{r}\right|^7} - \frac{(\bar{r}_p - \bar{r})\cdot\bar{p} + 2(z_p - z)p_z}{\left|\bar{r}_p - \bar{r}\right|^5}\right] \quad (2)$$

In order to obtain Laplacian estimates for the three tripolar CRE configurations from Fig. 2, potentials were calculated first for all nine concentric circles as means of potentials at four points on each circle. Next, these circle potentials were

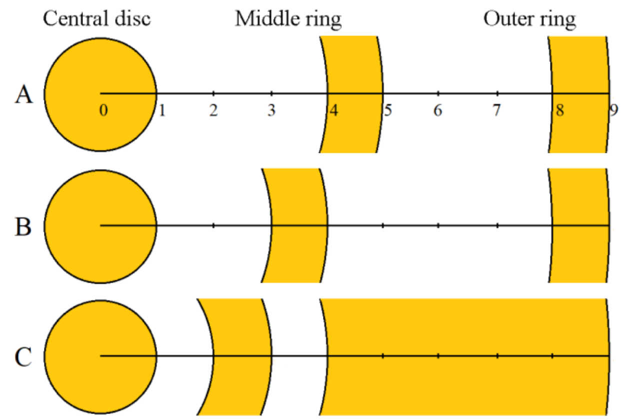

Figure 2. Finite dimensions models of three tripolar concentric ring electrode configurations including: constant inter-ring distances configuration (A), linearly increasing inter-ring distances configuration (B), and optimal configuration with respect to the accuracy of Laplacian estimation (C).

used to calculate the potentials on the three recording surfaces of each CRE configuration. For example, the potential on the central disc for all three CRE configurations in Fig. 2 is equal to the mean of the potential at the center of the central disc and potential on the smallest of the concentric circles. Finally, for each CRE configuration, two bipolar differences for each of the ring potentials minus the central disc potential were linearly combined using respective set of coefficients and divided by the square of the distance between the concentric circles [6] to produce the respective Laplacian estimate. These Laplacian estimates were computed at each point of the mesh where appropriate boundary conditions could be applied for respective CRE diameter (the total number of points ranging from 520 x 520 for the largest CRE diameter to 682 x 682 for the smallest one). Laplacian estimate coefficients for the CIRD and LIIRD configurations (Fig. 2A and 2B) were adopted from [6]: (37/130, –11/468) for CIRD and (37/90, –7/540) for LIIRD respectively. Derivation of Laplacian estimate coefficients for the optimal configuration was performed using the analytic approach from [6] applied to the FDM from [14] (Fig. 2C) and resulting in coefficients (952/1227, –6/409). These three Laplacian estimates were compared with the calculated analytical Laplacian for each point of the mesh, where corresponding Laplacian estimates were computed, using relative error and normalized maximum error measures:

$$\text{Relative error}^i = \sqrt{\frac{\sum(\Delta v - \Delta^i v)^2}{\sum(\Delta v)^2}} \quad (3)$$

$$\text{Normalized maximum error}^i = \frac{\max\left|\Delta v - \Delta^i v\right|}{\max\left|\Delta v\right|} \quad (4)$$

where i represents CRE configuration, $\Delta^i v$ represents the corresponding Laplacian estimate, and Δv represents the analytical Laplacian. While (3) is borrowed verbatim from [11], [12], [15]–[17], (4) is a slight modification of the maximum error measure used in the aforementioned previous studies:

$$\text{Maximum error}^i = \max\left|\Delta v - \Delta^i v\right| \quad (5)$$

The reason why the maximum error (5) from [11], [12], [15]–[17] was normalized in this study (4) was to make visualization

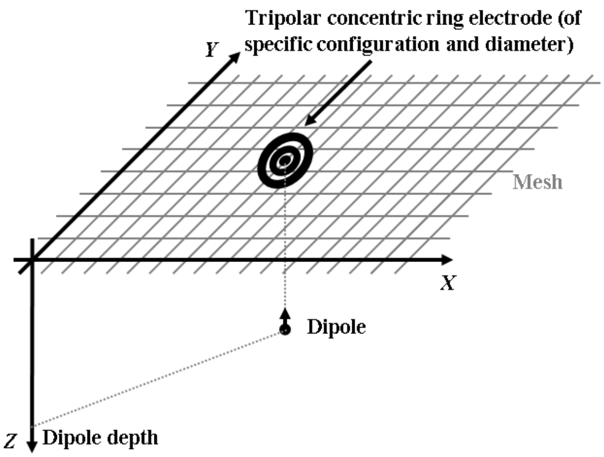

Figure 3. Schematic of the finite element method model used to compare Laplacian estimates.

of the improvement in Laplacian estimation accuracy easier by representing the error as a percentage of the maximum absolute value of the analytical Laplacian.

III. RESULTS

Relative and normalized maximum errors computed via the FEM modeling using (3) and (4) are presented in Fig. 4 for CRE diameters ranging from 0.5 cm to 5 cm. As it can be observed, the greater the electrode diameter the greater the error (both relative and normalized maximum) of the Laplacian estimation for all the CRE configurations. Relative error of up to 1.05% and normalized maximum error of up to 1.67% were obtained for CIRD configuration of 5 cm diameter which could be significant in real life noninvasive electrophysiological measurement applications. Optimal tripolar CRE configuration allows decreasing those errors to 0.35% and 0.57% respectively for the same electrode size. This decrease in Laplacian estimation error is even more meaningful for smaller dipole depths (figures not shown). For example, for dipole depth of 3 cm as considered in [11] and electrode diameter of 5 cm relative and normalized maximum errors corresponding to CIRD configuration are equal to 5.65% and 8.31% respectively while optimal tripolar CRE configuration allows decreasing them to 2.03% and 3.1%.

Overall, for every electrode diameter, optimal tripolar CRE configuration (Fig. 2C) provided a smaller error in Laplacian estimation than previously proposed CIRD and LIIRD configurations (Fig. 2A and 2B). Such improvement can be further quantified by computing the error ratios (mean ± standard deviation for 10 CRE sizes) corresponding to LIIRD over optimal and CIRD over optimal. Compared to the optimal tripolar CRE configuration relative and normalized maximum errors corresponding to its LIIRD and CIRD counterparts are larger by 1.97 ± 0.02 (relative error) and 1.96 ± 0.02 (normalized maximum error) times as well as by 3.07 ± 0.05 (relative error) and 3.05 ± 0.07 (normalized maximum error) times respectively (Fig. 4). This improvement is consistent across all the CRE diameters ranging from 0.5 cm to 5 cm as evidenced by low standard deviation values for the error ratios.

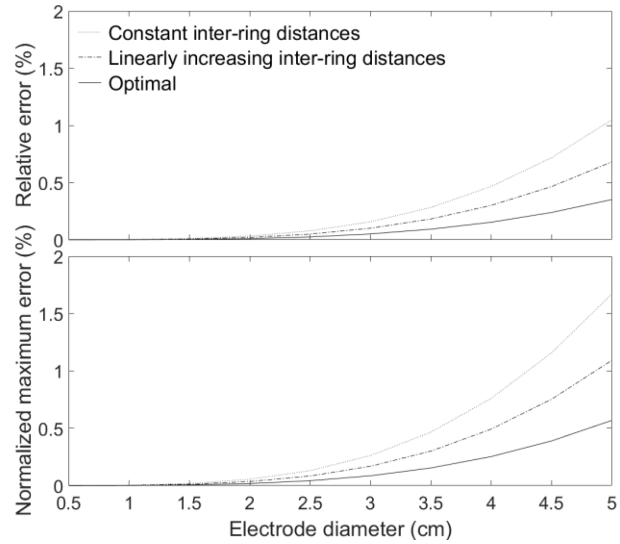

Figure 4. Relative (top) and normalized maximum (bottom) errors of surface Laplacian estimation corresponding to constant inter-ring distances, linearly increasing inter-ring distances, and optimal tripolar concentric ring electrode configurations.

IV. DISCUSSION

The FEM model from [11], [12], [15]–[17] has been adapted NDM to FDM in this work to confirm the analytic results obtained in [14]. General increase in the surface Laplacian estimation errors due to increase in the electrode size (Fig. 4) is consistent with the previously obtained results via NDM based FEM modeling [11], [12], [15]–[17] and demonstrated for the first time in this study via FDM based FEM modeling.

Analytic and FEM based increases in Laplacian estimation error corresponding to LIIRD and CIRD tripolar CRE configurations from [6] being compared to the optimal configuration of the same size from [14] are shown to be consistent (difference of less than 5%): FEM modeling based mean error ratios correspond to increases in Laplacian estimation error of 96-97% and 205-207% respectively which is comparable to increases of 99.33% and 213.01% obtained analytically in [14]. This further suggests the potential of the optimal tripolar CRE configuration from [14] in particular as well as the potential of the FDM based comprehensive optimization of the CRE design targeting maximizing the accuracy of the surface Laplacian estimation in general.

Future work will concentrate on building prototypes of optimal tripolar CREs and comparing them against LIIRD and CIRD configurations as well as against conventional single pole electrodes on real life data recordings including phantom, animal model and human for further proof. The main concern that can be addressed via the prototypes is the possibility of shorting due to salt bridges affecting the accuracy of the surface Laplacian estimation. Optimal tripolar CRE configuration from this study aims to minimize the distances between the recording surfaces and real life data can provide conclusive insight into how small these distances can get without adversely affecting the estimation accuracy. Another direction of future work is a thorough investigation of the effects of the dipole depth and orientation in the proposed FDM based FEM model as well as comparison of sensitivity and spatial resolution for three tripolar CRE configurations considered. Moreover, amplitude of the resulting Laplacian estimate signal merits additional study since amplitudes of the signals recorded via CREs have been shown to be smaller than amplitudes of the signals recorded via conventional disc electrodes [21], [22]. This makes the signal-to-noise ratios of those Laplacian estimates more important. Finally, moving from a single-layer FEM model used in this study to a more comprehensive one such as, for example, a five-layer planar model of the abdomen [23] or a four-layer concentric inhomogeneous spherical head model used recently in [9] would further validate the obtained results.

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