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# A Laplacian-Gaussian Mixture Model for Surface EMG Signals from Upper Limbs*

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***Abstract*— Existing literature suggests that the probability density function (pdf) of surface Electromyography (sEMG) signals follows either a Gaussian or Laplacian model. In this paper, a Laplacian-Gaussian mixture model is proposed for the EMG signals extracted from the upper limbs. The model is validated using both quantitative and qualitative perspectives. Specifically, for a benchmark dataset, the Kullback–Leibler (KL) divergence is computed between the proposed model and the histogram based empirical probability density function (mpdf). For a sample signal, a goodness of fit plot with $R$ squared value and a visual comparison between the histogram based mpdf and the estimated pdf from the proposed model are presented. Moreover, the Expectation-Maximization (EM) algorithm is derived for the estimation of the parameters of the proposed mixture model. The weight of the Laplacian component is computed for each of the signals from a benchmark dataset. It has been empirically determined that the Laplacian component has a major contribution to the mixture.**

## I. INTRODUCTION

The knowledge of the probability density function (pdf) of the sEMG signal can significantly contribute toward improving applications of EMG signals for example in exoskeleton control [1]. The nature of the pdf is affected by many factors, such as type of the muscle, the level of muscle contraction force, and the disturbance [2]. The muscle contraction force has a causal role in the nature of the pdf of the sEMG signals. In the existing literature, the sEMG signals are modelled by either the Laplacian or Gaussian pdfs. Specifically in the earlier work, the Gaussian density was proposed through experiments on the sEMG signals at different muscle contraction levels [3], lower and medium force levels [4], and higher muscle contraction levels [5]. Hunter et al. [6] visually compared the EMG density to a Gaussian model and observed the deviation from the Gaussian density with a sharper peak at zero. Recently, Bilodeau et al. [7] reported, under constant force and slowly varying contractions, the sEMG signal density is non-Gaussian and at higher force levels, the density of EMG tends to be Gaussian. Clancy et al. [8], found that at lower muscle force levels the sEMG tends to follow the Laplacian distribution. Notably, the sEMG signal from a given channel is traditionally modelled using a single probability distribution. In many cases, this distribution may not accurately describe the nature of the data. Based on this evidence, it may be reasonable to assume that the distribution of EMG signals may have both the Gaussian and Laplacian components. In this work, a Laplacian-Gaussian mixture (LGM) model is proposed for the sEMG signals. The proposed LGM model is validated on an sEMG dataset using (1) the Kullback–Leibler divergence (KLD) with the empirical probability density function (mpdf) based on the data histogram (2) qualitative analyses such as visual comparison with the mpdf and (3) goodness of fit plot, comparison of coefficient of determination (CFD) - $R$-squared, the confidence interval for $R$-squared and the Akaike Information Criteria (AIC). Finally, a heat map of the key model parameter i.e., mixing coefficient corresponding to the Laplacian component is presented.

## II. METHODOLOGY

In this work, a Laplacian-Gaussian Mixture model is proposed for the sEMG signal from an individual channel. The key objective is to determine the nature of the mixing coefficients and thus validate the suitability of the LGM for the sEMG signals. In this exercise, the proposed model has a few unknown parameters which have to be estimated to determine the contributions of the Gaussian and Laplacian components. The unknown parameters including the mixing coefficients can be estimated from the corresponding sEMG data. Here we use the Expectation-Maximization (EM) algorithm [9] for estimation of the unknown parameters.

### *A. Laplacian-Gaussian Mixture (LGM) Model*

Consider a random variable $X(n)$ that denotes the value of the sEMG signal at the discrete index $n$. The proposed LGM model can be written as

$$f(x;\Theta) = \lambda_1 f_1(x;\theta_1) + \lambda_2 f_2(x;\theta_2) \quad (1)$$

here $x$ is a realization of the random variable $X(n)$ and $\Theta = [\lambda_1, \lambda_2, \theta_1, \theta_2]$ is the vector of unknown parameters in the mixture model. $\lambda_1$ and $\lambda_2$ are the responsibility coefficients or mixing probabilities that add to unity. $\theta_1$ and $\theta_2$ are the parameters of component densities $f_1(x;\theta_1)$, a Laplacian density and $f_2(x;\theta_2)$ a Gaussian density defined as follows

$$f_1(x;\theta_1) = \frac{1}{2\sigma_1} \exp\left( - \frac{|x-\mu_1|}{\sigma_1} \right) \quad -\infty < x < \infty \quad (2)$$

$$f_2(x;\theta_2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left( \frac{(x-\mu_2)^2}{2\sigma_2^2} \right) -\infty < x < \infty \quad (3)$$

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where $\theta_1 = [\mu_1, \sigma_1]$ and $\theta_2 = [\mu_2, \sigma_2^2]$ are the statistics of the Laplacian and Gaussian densities. As shown in the LGM model (1), the mixing coefficients $\lambda_1$ and $\lambda_2$ are hidden in the observations. It is well known that the EM algorithm provides an efficient solution to the Gaussian mixture case and with this motivation, we derive a similar EM algorithm for the proposed LGM model.

### *B. EM-based Parameter Estimation*

Consider a surface EMG signal represented as an array $\mathbf{X} = \{x_n\}_{n=0}^{N-1}$. Inspired by the indicator variable used in the Gaussian mixture model in [9], we propose a discrete random vector $\mathbf{Z} = \{Z_n\}_{n=1}^{N}$ and $Z_n = [Z_{n,1}, Z_{n,2}]$ which has only two possible states $\{Z_{n,1} = 1, Z_{n,2} = 0\}$ & $\{Z_{n,1} = 0, Z_{n,2} = 1\}$. The relation with the mixing coefficients is emphasized by the probabilities $p(Z_1 = 1, Z_2 = 0) = \lambda_1$ and $p(Z_1 = 0, Z_2 = 1) = \lambda_2$. The marginal probability of these hidden variables is given by

$$p(Z_n) = \lambda_1^{z_{n,1}} \lambda_2^{z_{n,2}} \tag{4}$$

Note that the variables $Z_n$ are assumed to be i.i.d. The conditional density of $x_n$ given $Z_n$ and the parameters $\Theta$ is

$$f(x_n|Z_n;\Theta) = \prod_{j=1}^{2} (f_j(x_n;\theta_j))^{z_{n,j}} \tag{5}$$

here, $x_n$ are conditionally i.i.d. The joint density of data, hidden parameters and the parameters is

$$f(\mathbf{X},\mathbf{Z};\Theta) = \prod_{n=0}^{N-1} \prod_{j=1}^{2} (\lambda_j f_j(x_n|\theta_j))^{z_{n,j}} \tag{6}$$

Thus the complete data log likelihood is

$$L(\mathbf{X},\mathbf{Z};\Theta) = \sum_{n=0}^{N-1} \sum_{j=1}^{2} z_{n,j} \ln(\lambda_j f_j(x_n;\theta_j)) \tag{7}$$

*1) E-step:* $\Lambda(\mathbf{X},\Theta,\Theta^{(k)})$ is the expectation of the complete data log-likelihood with respect to the conditional distribution of hidden variables given data $\mathbf{x}$ and current value of $\Theta$ as $\Theta^{(k)}$.

$$\Lambda(\mathbf{X},\Theta,\Theta^{(k)}) = E_{\mathbf{Z}|\mathbf{X},\Theta^{(k)}}\{L(\mathbf{X},\mathbf{Z};\Theta)\} \tag{8}$$

Using the Bayes theorem we can obtain the posterior probability of $Z_n$ at index $n$ as

$$P(Z_{n,j} = 1|x_n;\Theta^{(k)}) = \frac{f(x_n|Z_{n,j} = 1;\theta_j^{(k)})P(Z_{n,j} = 1)}{\sum_{l=1}^{2} f(x_n|Z_{n,l} = 1;\theta_l^{(k)})P(Z_{n,l} = 1)}$$

since the Bayesian estimate of $Z_n$ is

$$E(Z_{n,j}|x_n,\Theta^{(k)}) = P(Z_{n,j} = 1|x_n,\theta_j^{(k)}) \tag{9}$$

let $\gamma_{n,j}^{(k)}$ denote the above estimate and it is obtained as

$$\gamma_{n,j}^{(k)} = \frac{\lambda_j f_j(x_n;\theta_j^{(k)})}{\sum_{i=1}^{2} \lambda_i f_i(x_n;\theta_i^{(k)})} \tag{10}$$

To estimate the parameter updates $\Theta^{(k+1)}$, the corresponding expectation on the complete data log likelihood becomes

$$\Lambda(\mathbf{X},\Theta,\gamma^{(k)}) = \sum_{i=1}^{n} \sum_{j=1}^{2} \gamma_{n,j}^{(k)} \ln(\lambda_j f_j(x_n;\theta_j)) \tag{11}$$

*2) M-step:* Substituting the Laplacian and the Gaussian pdfs from (3) and (2) in (11) leads to

$$\begin{aligned}\Lambda(\mathbf{X},\Theta,\gamma^k) \quad = \quad & \sum_{n=0}^{N-1} \gamma_{n,j}^{(k)} \Big\{ \ln\lambda_1 - \ln\sigma_1 - \frac{|x_n - \mu_1|}{\sigma_1} \\ & \ln\lambda_2 - \frac{1}{2}\ln\sigma_2^2 - \frac{(x_n - \mu_2)^2}{2\sigma_2^2} \Big\} \end{aligned} \tag{12}$$

The parameters are estimated recursively by solving the optimization problem below

$$\Theta^{(k+1)} = \max_{\Theta}\ \Lambda(\mathbf{x},\Theta,\gamma^{(k)}) \tag{13}$$

by replacing the $\Lambda(\mathbf{x},\Theta,\gamma^{(k)})$ with the data likelihood in (12). The updated values of the parameters are

$$\begin{aligned}\lambda_1^{(k+1)} &= \frac{N_1}{N} \\ \lambda_2^{(k+1)} &= \frac{N_2}{N} \\ \mu_1^{k+1} &= Median\left[\left\{\frac{\gamma_{n,1}^{(k)}}{N_1}, x_n\right\}_{n=0}^{N-1}\right] \\ (\sigma_1)^{(k+1)} &= \frac{1}{N_1}\sum_{n=0}^{N-1} \gamma_{n,1}^{(k)} \left|(x_n - (\mu_1^k))\right| \\ \mu_2^{(k+1)} &= \frac{1}{N_2}\sum_{n=0}^{N-1} \gamma_{n,2}^{(k)} x_n \\ (\sigma_2^2)^{(k+1)} &= \frac{1}{N_2}\sum_{n=1}^{n} \gamma_{n,2}^{(k)} (x_n - (\mu_2^k))^2 \end{aligned} \tag{14}$$

where $N_1 = \sum_{n=0}^{N-1} \gamma_{n,1}^{(k)}$ and $N_1 + N_2 = N$. The E & M steps are repeated until convergence of the sum of the squared error between two consecutive estimates $\Theta^{(k)}$ and $\Theta^{(k+1)}$ for each of the parameters.

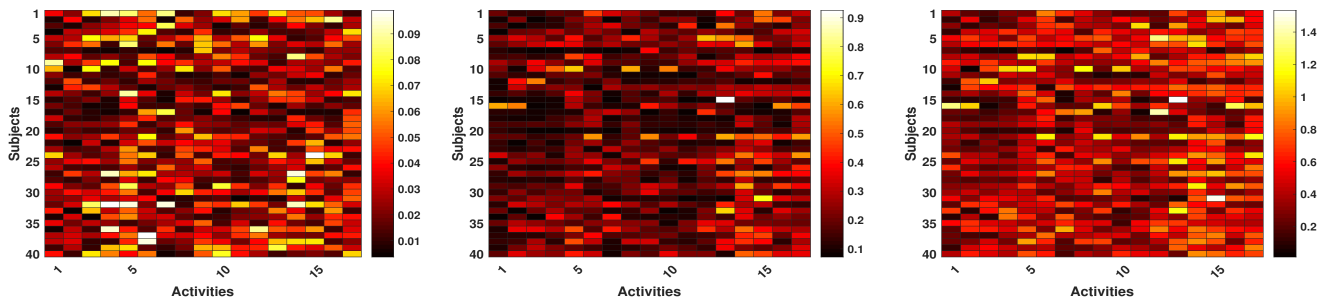
### *C. Methods for Validation*

The proposed LGM model is validated using the following quantitative and qualitative analyses.

- KL-divergence between the LGM and the mpdf: By definition, the KLD measures the similarity between two probability distributions. Let $f_1$ and $f_2$ be the probability distributions then their KL-divergence is given by

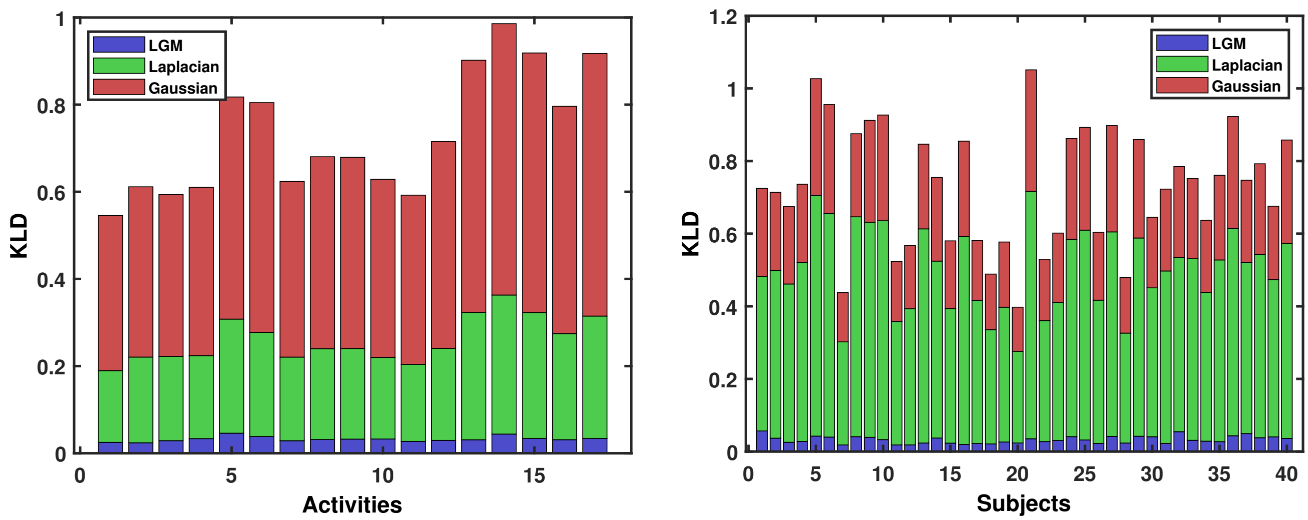
$$KL(f_1, f_2) = \sum_{x} f_1(x) \ln\left(\frac{f_1(x)}{f_2(x)}\right) \tag{15}$$

- Visual comparison: The visual comparison between the mpdf and the estimated pdf helps to understand the agreement between them. Specifically, the mpdf is fitted from the histogram and the proposed model pdf is




(a) (b) (c)

Fig. 1: KL-divergence for each of the subjects and activities (a) LGM, (b) Laplacian and (c) Gaussian models




(a) (b)

Fig. 2: Average KL-divergence (a) over the subjects for different activities (b) over the activities for different subjects

reconstructed from the estimates of model parameters from the EMG signal.

- A goodness of fit plot with $R$-squared: A goodness of fit plot is used to show the relationship between the actual EMG values and the model-based values. The closer the data points to the $45^{\circ}$ line, the better the model fit. Whereas, coefficient of determination (R-squared) gives the variation of one variable that is directly related to the change of the other variable. If this value approaches $1$, then the correlation between the two variables is stronger.
- Akaike Information Criteria (AIC): The AIC [10] is a statistical metric used to select the best model among available models. It is calculated using

$$\text{AIC} = n \log\left(\frac{SSE}{n}\right) + 2p \tag{16}$$

where $n$ is the number of observations, $SSE$ is the sum square of errors and $p$ is the number of parameters. A model with a lower AIC value is preferred.

## III. Validation and Results

### A. Data description

In this work, the proposed model is tested on the EMG Ninapro dataset DB2 [11] which consists of 3 different exercises collected from 40 subjects. Out of these 3 exercises, we consider exercise-1 which consists of 17 basic activities of the wrist and the fingers. Each of these EMG signals are collected from twelve electrodes (channels) placed at strategic muscles sites on the arm. For a given trial, the EMG signal usually corresponds to 5 sec of recording. In this study, the proposed model is constructed and analyzed for a channel with the maximum energy among the available channels for a given trial. The average value of this maximum energy among the selected channels over the $40$ subjects and $17$ activities is $3.74 \times 10^{-4}$. The proposed model parameters are estimated by EM algorithm described in section II-B. However, only the estimates of the mixing coefficients are discussed here.

### B. Model Validation

*1) KL-divergence:* The KLD is computed between the proposed model pdf and the mpdf for the data corresponding to the $40$ subjects performing $17$ different limb activities (Fig

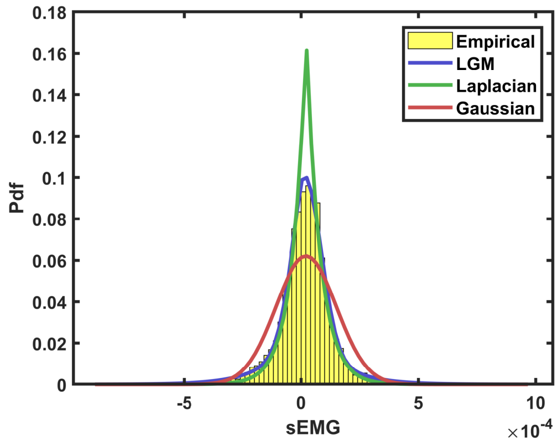



Fig. 3: Comparison of the mpdf with models LGM (blue), Laplacian (green) and Gaussian (red) for subject-15 and activity-17

1(a)). For comparison purposes, we also present similar KLD maps for the Laplacian (Fig 1(b)) and the Gaussian models (Fig 1(c)). As shown in Figs. 1(a) to (c) the KL-divergence for the LGM model is apparently the lowest, followed by the Laplacian and the Gaussian. Specifically, KLD for the LGM model, the upper bound is $0.1$ and it is the lower bound for the KLD from the Laplacian and Gaussian models. To emphasize this trend, the KLD averaged over the subjects for each of the activities is shown in Fig 2(a) and KLD averaged over the activities for each of the subjects is shown in Fig 2(b). Clearly, the average KLD for the LGM model is very low compared to the average KLD computed for the Laplacian and the Gaussian models.

In the following analyses, i.e. visual comparisons, Goodness of fit plots and the AIC, the results are presented only for the EMG signals from the data corresponding to the subject $15$ and activity $17$.

*2) Visual Comparisons:* The visual comparison as described in section II-C is given in the Fig. 3. It is evident that the area covered by the LGM pdf with empirical data is better when compared to the Laplacian or Gaussian pdfs. For this EMG signal, the LGM pdf is a better match compared to the standalone Gaussian or Laplacian pdfs.

*3) Goodness of fit plots:* Fig. 4 depicts the goodness of fit plots between different models and the actual data. It is clear that for the proposed LGM model, the data points in the scatter plot are distributed along the $45°$ line indicating better correlation. In contrast, the data points which are away from this line correspond to the Laplacian and Gaussian models (Fig. 4). Similarly from the values of R-squared, it is observed that the proposed model is better than the competing standalone models. The confidence interval for R-squared for subject-15 and activity-17 is $[0.9945,\ 0.9969]$ for LGM, $[0.816389,\ 0.829337]$ for Laplacian and $[0.8617,\ 0.8733]$ for Gaussian. For the EMG signal under consideration, the AIC values for the 3 models are LGM: $1.3905$, Laplacian: $1.5634$ and Gaussian: $1.5187$.

Similar qualitative comparative analyses are carried out for the rest of the data from the $40$ subjects and $17$ activities and the LGM has been apparently a good fit compared to the other pdfs. These results can be found at https://bit.ly/3ecyH2O. The average R-squared values for the three models over the $17$ activities of $40$ subjects are as follows, LGM: $0.9476$, Laplacian: $0.9303$ and Gaussian: $0.5199$. The confidence intervals and the AIC values for rest of the data can be found in the same link. Clearly, the LGM model is suitable for the sEMG signals from the Ninapro-DB2.

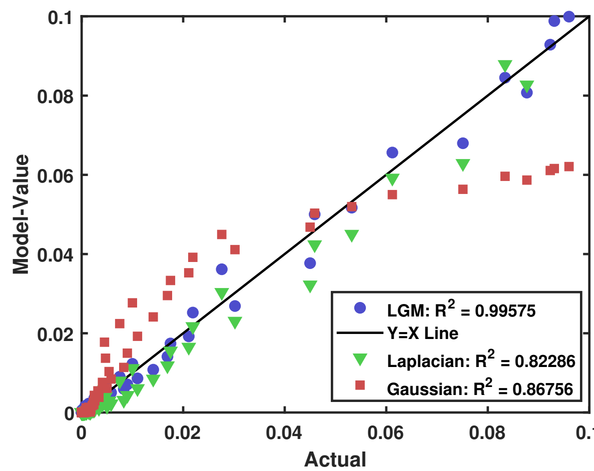



Fig. 4: Goodness of fit plots for LGM model (blue), Laplacian model (green) and Gaussian model (red) for subject-15 and activity-17

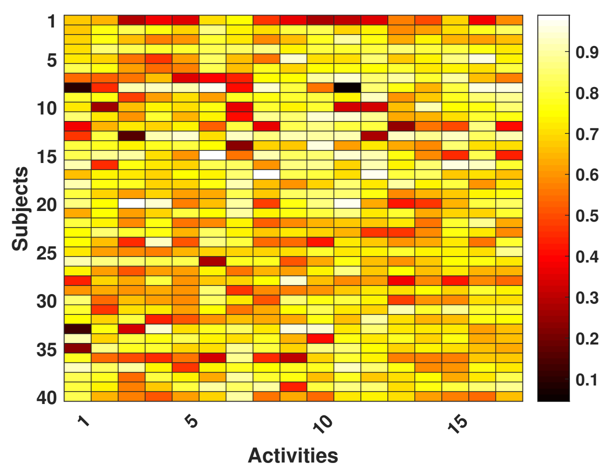



Fig. 5: Heat Map of weight coefficients of Laplacian component in LGM model for $40$ subjects and $17$ activities

*4) Analysis of weight map:* Based on the analysis of the DB2 data using the proposed LGM model, the mixing coefficient corresponding to the Laplacian component $\lambda_1$ is shown through a heat map in the Fig. 5. It is observed that in most of the cases i.e for different subjects and different limb activities, $\lambda_1$ has a higher value compared to $\lambda_2$ of the Gaussian component. As evident in Fig. 5, a few exceptions do exist to this observation, for e.g. the weights $\lambda_1$ corresponding to a few activities of subjects $7, 8, 12, 22, 25, 28, 29$ and $36$ are found to be lower. This means $\lambda_2$ is larger i.e., Gaussian component has a higher contribution for the mentioned subjects. Moreover, for most of the signals, as shown in Fig. 5, the weight $\lambda_1$ is between $0$ and $1$ which implies a positive contribution from both the distributions.

## IV. Conclusion

A Laplacian-Gaussian mixture model is proposed for the surface EMG signals. The model is tested on the sEMG signals of the benchmark Ninapro DB2 dataset. From the KL-divergence, the R-squared, confidence intervals for R-squared, the AIC and the visual comparisons with the mpdf, it is validated that the proposed model is better compared to the standalone Laplacian and Gaussian models. Also for most of the signals, both the mixture components contribute, however, the Laplacian component has a stronger weighting. In future work, we plan to extend our proposed model for the multi-channel EMG signals and demonstrate its practical applicability.

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