An Optimization Approach for Transcranial Direct Current Stimulation Using Nondominated Sorting Genetic Algorithm II

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Abstract— Transcranial direct current stimulation (tDCS) delivers weak current into the brain to modulate neural activities. Many methods have been proposed to determine electrode positions and stimulation intensities. Due to the trade-off between intensity and focality, it is actually a multi-objective optimization problem that has a set of optimal solutions. However, traditional methods can produce only one solution at each time, and many parameters need to be determined by experience. In this study, we proposed the nondominated sorting genetic algorithm II (NSGA-II) to solve the current optimization problem of multi-electrode tDCS. We also compared the representative solutions with LCMV solutions. The result shows that a group of solutions close to the optimal front can be obtained just in only one run without any prior knowledge.

I. INTRODUCTION

Transcranial direct current stimulation (tDCS) is a non-invasive brain stimulation method, which induces excitatory or inhibitory effects on specific cortical areas by injecting weak currents through electrodes attached to the scalp [1]. Due to its advantages in safety, portability, and low side effects, tDCS has become a valuable tool in many fields of neuroscience research and clinical application, including cognitive neuroscience [2], stroke rehabilitation [3], and psychiatric disorders [4]. Besides, it has been reported that tDCS could improve cognitive function in healthy subjects.

The conventional setup of tDCS is to apply a weak current (usually ≤ 2mA) through two rectangular patch electrodes (25-35 cm²). For example, the electrode montage (anode electrode over the primary motor cortex (M1) and the cathode electrode over the contralateral supraorbital forehead) is widely used when facilitating motor rehabilitation. However, the high-resolution finite element model (FEM) simulation results show that the electric field induced by tDCS is relatively diffuse by this montage. It means that the limited stimulation intensities required for security reasons could not precisely focus on the target brain region of interest (ROI). For solving this problem, high-definition tDCS (HD-tDCS) has been proposed to generate a more focal electric field with multiple smaller electrodes (1-2cm²). In order to induce the electric field as similar as possible to the desired one, the optimization of electrode montage and stimulation intensity at each electrode remains a key procedure.

Various methods have been proposed to optimize electrode montage. For simplifying the optimization problem, the international 10/10 electrode placement system is employed as a candidate to place electrodes rather than place electrodes arbitrarily on the scalp. Previous studies found a trade-off between achievable intensity (at the target) and focality. The linearly constrained minimum variance (LCMV) method was proposed, which enforces a hard linear constraint of the electric field at a single node while minimizing the electric field elsewhere. LCMV with different safety constraints were also compared with weighted least squares (WLS) and max intensity (MI) [5]. A genetic algorithm was collaborated with weighted least squares, while the former searched different montages among candidates and the latter calculated the best current values as the fitness of a given montage [6]. Some studies used genetic algorithms to allocate a limited number of sources to the anode electrode array [7]. For the sake of convenience, many studies also considered the number of active electrodes [6, 8, 9].

In general, stimulation optimization has to face a trade-off between intensity and focality, which means that stimulation optimization is a multi-objective optimization problem (MOP). However, all methods proposed above treat stimulation optimization as a single-objective optimization problem, which maximizes one of them on the premise that the other is determined or ignored. For example, in the LCMV procedure, the electric field at ROI is determined at every single run, which means the intensity is settled when maximizing accuracy [8]. In contrast, the accuracy is ignored when the max intensity procedure maximizes the intensity at ROI [10]. Because only one solution will be found at each run in these methods, it has to run many times with different parameters to get different solutions to the multi-object optimization problem. It would be difficult to find the best parameters to simplify the MOP in a clinical application. To the best of our knowledge, no research has defined stimulation optimization as a multi-objective optimization problem.

Many multi-objective evolutionary algorithms (MOEAs) have been developed over the past decades, utilizing the characteristic of evolutionary algorithms (EAs) work with a population of solutions. A set of various solutions can be achieved just in a single run. The nondominated sorting genetic algorithm II (NSGA-II) proposed in [11] is one of the most excellent and widely used multi-objective optimization algorithms so far. The existence of multiple objectives in a problem will produce a set of optimal solutions (usually called
Pareto-optimal solutions) rather than a single optimal solution. One of these Pareto-optimal solutions cannot be said to be better than the other in the absence of any further information. Based on the idea of Pareto nondominated sorting, NSGA-II classifies the solution individuals into multiple levels, and the solutions obtained are uniformly distributed.

In this study, we tried to apply the NSGA-II algorithm to optimize the current on each electrode and obtained a set of diverse solutions weighing the intensity against focality in a single run. Besides, the solutions that generate the highest intensity or most focal electric field were compared with the LCMV solutions with approximately equal intensity at the target region.

II. METHODS

A. Head Model and Electric Field Simulation

The numerical simulations were performed using a realistic head model, and the electric field generated by injected currents was solved using the finite element method. The ICBM152 template was used with finer detailed segmentation of six tissue types at 1.0 mm\(^3\) resolution. Conductivities of all tissues were assumed to be isotropic, and the values were assigned to 0.126 S/m (WM), 0.276 S/m (GM), 1.65 S/m (CSF), 0.01 S/m (skull), 0.465 S/m (scalp), 2.5 e-14 S/m (air cavities), 5.9 e-7 S/m (electrodes), 0.3 S/m (gel). We employed the international 10/10 electrode placement system with 72 electrodes as candidates to place physical electrodes.

We used the open-source software package ROAST to compute the electric field [12, 13] and optimize with the LCMV algorithm [5, 14, 15]. Consider the head model as a homogeneous volume with a scalar conductivity field \(\sigma\) and the potential distribution \(V\) in the volume was governed by Laplace's equation when applying currents through the electrode to the boundary of the volume:

\[
\nabla \cdot (\sigma \nabla V) = 0
\]

The boundary conditions consisted of current density perpendicular to the scalp at anode electrode, ground applied to the cathode electrode and insulated at all the other boundary locations, including other electrodes. Using a fixed reference electrode \(Z\) as cathode, the electric field of the remaining 71 electrodes was solved with unit current density injected into each of them to obtain 71 solutions of the electric field distribution, representing the lead field \(A\) which would be used for optimization below.

B. Objective Functions of NSGA-II

In the present study, \(C\) was the lead field that maps currents to target intensity at the target location. \(r\) represented the injected current at each electrode (indexed by \(m\), \(m = 1, ..., 71\)) except reference electrode \(Z\). \(e\) was the desired orientation of the electric field at the target. \(e_0\) accounted for the desired field at the target node.

As mentioned above, we may have to trade off intensity and focality during optimization, so the objective functions obviously represent intensity and focality. On account of the later comparison with LCMV solutions, the objective function representing field intensity was set to the electric field intensity in the desired direction at target, and it could be replaced with electric field strength if necessary:

\[
\text{maximize } f_1(i) = \sigma \cdot Ci
\]

In order to assess the focality of a given field, we defined \(F(r)\) to quantify the proportion of the electric field magnitude contained within a sphere of increasing radius around the target [5]. Half-max radius was another objective function representing the accuracy which was defined as the radius that contains half of the total electric field:

\[
r_{0.5} = r_0 | F(r) = 0.5
\]

Furthermore, a phenomenon was found in practice that evolution may fall into the local optimal Pareto-optimal solutions rather than a single optimal solution. One of these Pareto-optimal solutions cannot be said to be better than the other in the absence of any further information. Based on the idea of Pareto nondominated sorting, NSGA-II classifies the solution individuals into multiple levels, and the solutions obtained are uniformly distributed.

C. Constraints of NSGA-II

For security reasons, the most common constraint is to limit the total injected current to the maximum total current \(I_{tot}\). According to the Kirchhoff’s current law, the total inflowing current is equal to the total outflowing current, so we have the constraint as:

\[
g_1(i) = \sum_m |i_m| + |\sum_m i_m| \leq 2I_{tot}
\]

The maximum current at each individual electrode \(I_{ind}\) is also necessary when we use small electrodes. Otherwise, relatively high current density would have the risk of side effects such as skin irritation and burn. Taking all electrodes, including reference electrode into consideration, we have the constraint as:

\[
g_2(i) = |i_m| \leq I_{ind}
\]

To handle constraints in a population approach, we used the constrained-domination principle that any feasible solution has a better nondomination rank than any infeasible one [16, 17]. The definition of constrained-domination between two solutions \(i\) and \(j\) is:

**Definition 1:** A solution \(i\) is said to constrained-dominate a solution \(j\), if any of the following conditions is true.
- Solution \(i\) is feasible and solution \(j\) is infeasible.
- Solution \(i\) and \(j\) are both infeasible, but solution \(i\) has a smaller overall constraint violation.
- Solution \(i\) and \(j\) are feasible and solution \(i\) dominates solution \(j\) with the usual domination principle[18].

The constraint violation value \((CV(i))\) of a solution \(i\) was measured using the following formula:

\[
CV(i) = \sum_{j=1}^{f}(g_j(i)) + \sum_{k=1}^{K} |h_k(i)|
\]

where the bracket operator \((a)\) returned the negative of \(a\), if \(a < 0\) and returns zero, otherwise. \(g_j(i)\) was the normalized constraint function that had divided by the constant in the constraint, which was, for \(g_j(i) \geq b_j\), the normalized constraint function became \(g_j(i) = g_j(i)/b_j - 1 \geq 0\) and similarly \(h_k(i)\) can also be normalized equality constraint that could be added in other applications.

Notably, the constraint of injected current at electrodes except reference electrode (6) was implemented in the population initialization and genetic operator in practice. As a result, only inequality constraint (5) and (7) was considered when calculating \(CV(i)\). Furthermore, a phenomenon was found in practice that evolution may fall into the local optimal
solutions with opposite electric field direction. Therefore, the third condition of constrained-domination is further divided into two conditions according to the direction of the electric field $f_1$:

- Solution $i$ generates a positive electric field, and solution $j$ generates a negative electric field.
- Solution $i$ and $j$ both generate a positive or negative electric field, and solution $i$ dominates solution $j$ with the usual domination principle [18].

D. The Selected Parameters

To provide an intuitive understanding of the prospects and limitations of NSGA-II and LCMV, we compared the solutions obtained by NSGA-II in a single run versus the solutions obtained by LCMV in multiple runs. The two approaches adopted the same safety constraints ($I_{tot} = 2 \text{ mA}, I_{ind} = 1 \text{ mA}$). We chose the primary motor cortex (M1) as the target region, and the desired direction of the electric field at the target was set to radial-in, which were widely used in the test of optimization methods. The population size and generation of NSGA-II were set to 100 and 300, respectively. We set probabilities of crossover and mutation as 0.8 and 0.1, respectively. The distribution index of simulated binary crossover started at 4 and increased to 7 with generation linearly. The distribution index of the polynomial mutation operator also started at 5 and increased to 10 with generation linearly. The desired electric field in the LCMV ranges from 0.13V/m to 0.28V/m in 0.1 steps. The above parameters were set according to the pre-experiment, which can achieve the best results.

III. RESULTS

A. Comparison of Optimal Results between NSGA-II and LCMV

Fig. 1 illustrated the solutions obtained by NSGA-II and LCMV, respectively. The population obtained at the end of 300 generations ranges from 0.135V/m to 0.277V/m for electric field intensity, and ranges from 53.2mm to 74.5mm for half-max radius. Compared with the solutions of LCMV, which can also be regarded as the Pareto optimal front of this problem, the solutions of NSGA-II were closer on the side with a larger electric field. Besides, diversity in solutions was well maintained and the population is evenly distributed, while only one solution can be got in a single run of LCMV. Compared with the solutions of LCMV, the ability of NSGA-II to get a set of solutions in a single run and the excellent distribution of solutions have been demonstrated. We chose two solutions with the highest intensity and most focal electric field for further comparison.

B. Specific Solutions for Focality and Intensity

We further compared NSGA-II scheme and the $\ell_1$ norm constrained LCMV in terms of electric field simulation. We chose the four solutions selected by the red circle in Fig. 1 for further study. Fig. 2(a) showed the electric field of NSGA-II solution that generate highest intensity, and Fig. 2(b) illustrated the electric field of LCMV solution that have approximate intensity at target. The distribution range of these two solutions were both wide, while LCMV produced a large electric field in frontal lobe and NSGA-II influenced the back side of brain more. On account of the intensity-focality trade-off, it seems to be inevitable to deliver large energy outside the ROI. Moreover, due to the objective function representing intensity in NSGA-II was set to electric field in the desired direction, NSGA-II actually produces greater intensity at target according to the electric field direction in sagittal plane of Fig. 2(a). It is meaningful to apply NSGA-II to search the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Comparison between the solutions obtained by NSGA-II and LCMV. Solutions of NSGA-II got in one run are widely distributed. Solutions are closer when the electric field is larger.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Electric field simulation of (a) the solution with strongest intensity obtained by NSGA-II and (b) the solution obtained by LCMV with approximate intensity.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Electric field simulation of (a) the solution with most focal distribution obtained by NSGA-II and (b) the solution obtained by LCMV with approximate intensity.}
\end{figure}
montage that produces the max intensity at the target regardless of direction. As shown in Fig. 3, the most focal electric field generated by NSGA-II and the approximate value of intensity produced by LCMV were of little difference. Both solutions produced a very focused electric field around the ROI, while the electric field of LCMV was slightly more focal. Although the half-max radius seemed to have a wide gap in Fig. 1, the simulation results showed that the disparity is too small to directly impact practical application.

Fig. 4 showed the distribution of current intensity among electrodes, and current with intensity less than 0.01 mA were not considered for the sake of simplicity. There was no significant difference caused by different methods in the current intensity greater than 0.1 mA. Similarly, solutions that generated stronger electric fields had more current with an intensity close to 1 mA. However, the solutions obtained by NSGA-II both had more electrodes injected current greater than 0.01 mA, which would become a barrier to practical application. To deal with the problem, current that is too small to have an influence on the distribution of electric field can be discarded in the application, or constraining the number of active electrodes can be introduced to the future improved methods.

IV. DISCUSSION AND CONCLUSION

In this study, we proposed a new approach to optimizing tDCS-induced electric fields at the target brain region by considering the trade-off between intensity and focality and compared the results with the LCMV method. NSGA-II was adopted to get a set of acceptable solutions at one time assigning different weights on intensity and accuracy as alternative solutions. The multi-objective algorithm can search the Pareto optimal front without any prior knowledge. Meanwhile, the single-objective algorithm has to set many parameters and can get only one point of Pareto optimal front corresponding to the parameters in every single run. The flexible setting of objective functions makes NSGA-II available to different problems aiming to maximize electric field regardless of direction or in the special direction.

NSGA-II provides more potential choices for researchers and physicians. Especially for some subjects who have skin trauma in some areas, stimulation montages that avoid injecting currents at these areas and have a close effect can be employed instead. Other multi-objective algorithms should be undertaken on the tDCS optimization in further research.

References


