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Focused ultrasound simulation through cortical bone by finite element method*

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Abstract— **Bone tissue is constantly changed adapting to its mechanical environment and capable of repairing itself. Ultrasound has recently been used as a diagnostic technique to assess bone conditions. To optimize the experimental model as best as possible computational simulation techniques have been focused on clinical applications in bone. This study aims to analyze by finite element method the propagation of ultrasound waves along the cortical bone. The wave propagation phenomenon is well studied and described by the Helmholtz equation. The first part of the work analytically solves the Helmholtz equation, and later the COMSOL Multiphysics software is used. It was established a cylindrical geometry as the bone sample. The software analyzes with "Pressure Acoustic, Frequency Domain" module. An extremely fine mesh is used for the solution in order not to lose information. According to the analytical solution, the results show the behavior of the acoustic pressure waves throughout the samples. In addition, attenuation coefficients are calculated for biological materials such as bone and muscle. Simulation methods allow to analyze adjustable parameters in the development of new devices. Thus, optimizing resources and allowing the researcher to better understanding the problem to be solved.**

I. INTRODUCTION

Bone tissue is classified by its porosity in: i) cortical, the dense part on the bones outside, and ii) trabecular or spongious, the porous tissue filling the bones [1]. Cortical bone composes the external envelope of all bones (long, short and flat bones). Cortical bone presents a dense structure of low porosity (typical porosity approximately 15%) that seems compact at the macroscopic level [2]. In recent years, cortical bone has been recognized as playing a role in bone resistance, in particular at fracture sites such as the proximal femur and the distal radius [1]. Cortical bone porosity has been increasingly recognized as a fracture risk factor [3].

To evaluate cortical bone, there exist several techniques based on the attenuation of non-ionizing radiation to quantify bone density, including single photon absorptiometry (SPA), single X-ray absorptiometry (SXA), dual X-ray absorptiometry (DXA) and quantitative computed tomography (QCT) [4]. However, they provide only limited information on bone structure and bone material properties. Researchers and clinicians are looking for new methods that should be inexpensive, more comfortable for the patient, should detect fragility in addition to decreased bone density [5].

Favorable results have been reported in the use of ultrasound methods to analyze the state of bone tissue [6], [7]. Ultrasound corresponds to a mechanical wave propagating at frequencies above the range of human hearing (20 kHz). In particular, the wave propagation depends on the intrinsic elastic properties of the medium as well as on its mass density [2]. An evaluation of the ultrasound parameters should allow the mechanical properties of the bone to be deduced.

In contrast to bone density measurements, which are based on X-ray attenuation, ultrasound parameters reflect the structural anisotropy of the bone [8]. Therefore, ultrasound methods could have greater potential to be developed into a tool for the comprehensive noninvasive assessment of three-dimensional structure and strength [9]. However, this would require ultrasonic in several directions and at different frequencies [5], which can be complex at experimental level.

The physics laws description for space and time-dependent problems is usually expressed in partial differential equations (PDE). For the vast majority of geometries and problems, PDE not always can be solved with analytical methods.

The finite element method (FEM) is considered a well-established technique for computing solutions on complex problems in different fields of science and engineering [10]. It is used mainly for problems in which no exact solution, expressible in some mathematical form, is available [11]. This technique can be used as a powerful tool that approximates the solution of differential equations describing physical processes.

Computational simulation methods offer many advantages in analyzing the interaction of ultrasound waves with bone. Having a correct analysis of sound propagation through the tissue will allow us to understand its behaviour better. The acoustic simulation in bone has had a significant impact in recent years [12]. It has been used in the analysis of construction materials and biomedical applications. This lack of analytical solutions means that the only recourse (other than computational) is to perform experiments. Because these experiments are expensive and time consuming to perform, the potential utility of computational methods is considerable. In particular, in bone applications, simulations allow the creation or manipulation of bone samples. It can be specified values of any property to explore variables associated with the biomechanical integrity of the bone. The degree to which simulation can provide greater understanding is as broad as the number of problems and questions one can ask.

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In this study, the finite element simulation software COMSOL Multiphysics is used to analyzed the propagation of sound waves along the cortical bone. The simulation uses the real parameters of the Bone Radar transducer. The Bone Radar is a device designed and built in the Medical Physics Laboratory of the University of Guanajuato. It has been used for the diagnosis of hip dysplasia in neonates [13] and evaluation of osteoporosis in women [14].

II. METHODOLOGY

A. Governing equations

The following equation describes the propagation of an acoustic wave along one dimension in space and time:

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial p}{\partial x^2} = 0 \qquad (1)$$

where p is the acoustic pressure (Pa) and c is the speed of sound (m/s) through the medium.

The equivalent of the wave equation formulated in the frequency domain is the Helmholtz equation. The Helmholtz equation forms very often the basis for the numerical analysis of acoustic problems. A deriving method is assuming that the pressure is a harmonic time signal of the type

$$p(x,t) = p(x)e^{i\omega t} \qquad (2)$$

By inserting this expression into the wave equation (eq.1) and rearranging it, the Helmholtz equation is obtained, with constant material parameters, for a harmonic time signal

$$\frac{\partial^2 p}{\partial x^2} + k^2 p = 0 \qquad (3)$$

where k is defined as the wave number $\left(k = \frac{\omega}{c}\right)$, ω is angular frequency (rad/s). To solve the Helmholtz equation it must be combined with the material parameters, boundary conditions and initial conditions that describe the physical problem in question.

The geometry proposed in the problem to be solved is a cylinder, therefore $p = p(r,\theta,z)$ written in cylindrical coordinates,

$$\nabla^2 p = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \qquad (4)$$

substituting in Helmholtz equation we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = 0 \qquad (5)$$

Solving equation 5 by the separation of variables method, the following general solution is obtained

$$\begin{aligned} p(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} & \Big[A_{mn}J_m\left(r\sqrt{k^2+n^2}\right) \\ & + B_{mn}Y_m\left(r\sqrt{k^2+n^2}\right)\Big] \\ & \times [C_m\cos(m\theta) + D_m\sin(m\theta)](E_n e^{-nz} + F_n e^{nz}) \end{aligned} \qquad (6)$$

where J_m and Y_n are Bessel functions of the first and second type, respectively.

However, since Y_n functions diverge at origin and this is part of domain they should be neglected ($B_{mn} = 0$), leaving the general solution as

$$\begin{aligned} p(r,\theta,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} & A_{mn}J_m\left(r\sqrt{k^2+n^2}\right) \\ & \times [C_m\cos(m\theta) + D_m\sin(m\theta)]\,(E_n e^{-nz} + F_n e^{nz}) \end{aligned} \qquad (7)$$

Several assumptions were made to follow the equation above in medium propagation, which are: 1) linear propagation of a sound wave through a medium, 2) density is constant and 3) pressure is time harmonic.

The following boundary conditions were applied during the simulation of the geometries: 1) all sound energy enters the transducer surface, where p=p and p_0 is the initial amplitude of the harmonic source, and 2) the pressure amplitude disappears near the cylinder walls.

B. Simulation with COMSOL Multiphysics software

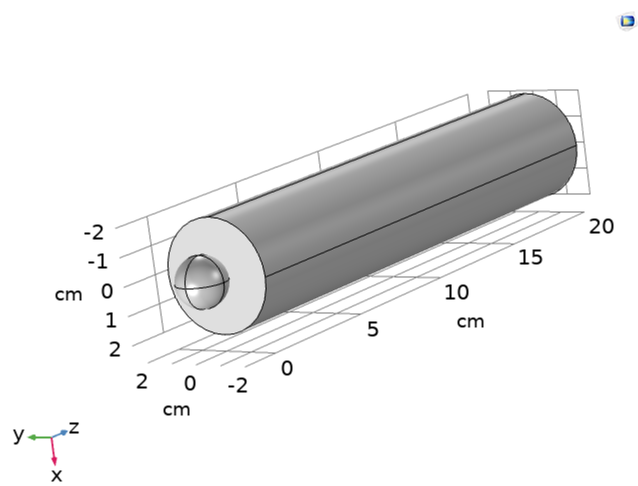

Fig. 1. Cylindrical sample of cortical bone with transducer tip

As mentioned above, cylindrical geometry was established for the bone sample. It has a radius of 2 cm and a height of 20 cm, as shown in figure 1. The tip of the ultrasound transducer, corresponding to the Bone Radar, was placed on one side of the cylinder. For the geometry of the transducer, a sphere with a radius of 0.9 cm was used. A frequency range between 20 and 500 kHz was used, with steps of 5 kHz.

The ultrasonic wave propagates through the bone cylinder. To simulate this physical effect, the module ”Pressure Acoustics, Frequency Domain” was used (COMSOL Multiphysics software, version 5.4). Free triangular mesh was used to segment the geometry. An extremely fine mesh was used to solve the physics in the given problem. Computational time among all the frequency range with this geometry was 7 minutes and 4 seconds. For a first approximation, a homogeneous and isotropic cortical bone sample is considered. The material parameters are given in table 1 [15].

TABLE I
ACOUSTIC PARAMETERS USED IN SIMULATIONS

Material	ρ (kg/m^3)	*c (m/s)*	*Z (Pa· s/m)*
Muscle	1090	1588.4	$1.73\text{x}10^6$
Bone	1908	3514.9	$6.70\text{x}10^6$

III. RESULTS AND DISCUSSION

Figure 2 shows the distribution of pressure waves along with the cylinder of cortical bone. The pressure magnitude is indicated by a colored bar and is in the range of -1 to 1 Pa. Increasing the frequency of the transducer reveals clearer acoustic wave patterns. Well-defined waves form around the transducer and fade away when they move far.

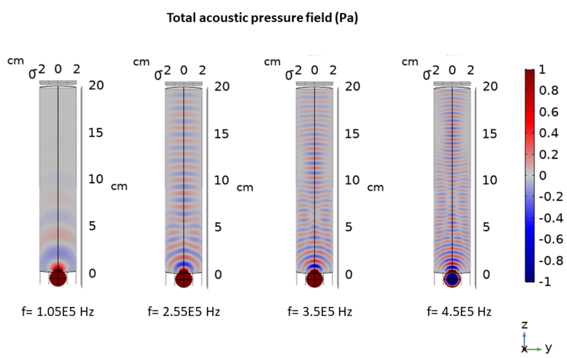

Fig. 2. Pressure waves along cortical bone (YZ plane)

Looking at the figure 3, 10 slices are shown along the cylinder in the XY plane. This graph shows the displacement of the wave throughout the entire cylinder, starting at the tip of the transducer, the colors of the diagram allow to observe the cycles of compression and rarefaction of the wave as well as the attenuation it has with distance. It was considered that acoustic pressure attenuated rapidly in a short distance from the transducer.

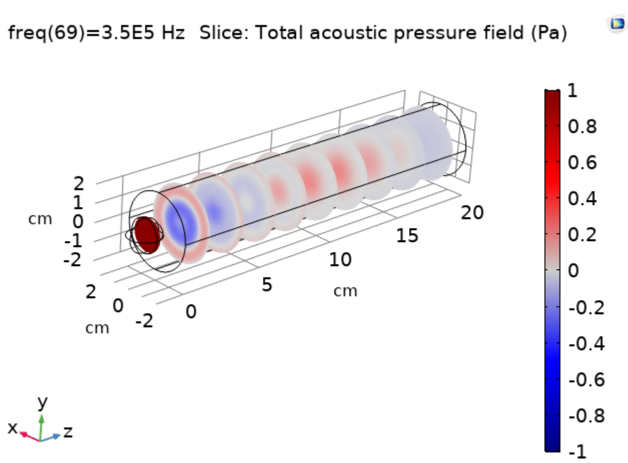

Fig. 3. Pressure waves along cortical bone (XY plane)

A cut line was made in the longitudinal axis of the bone cylinder to analyze the sound pressure passing through the center. The pressure along the cylinder is plotted (see figure 4), studying its behavior at different frequencies. Below 80 kHz, the pressure shows exponential behavior. As the frequency increases, the number of waves and their amplitude increase, which is agree with the analytical solution. Throughout the entire frequency range, the pressure signal attenuates as the length of the sample increases. The region of the sample closest to the tip of the transducer shows the highest increment of acoustic pressure.

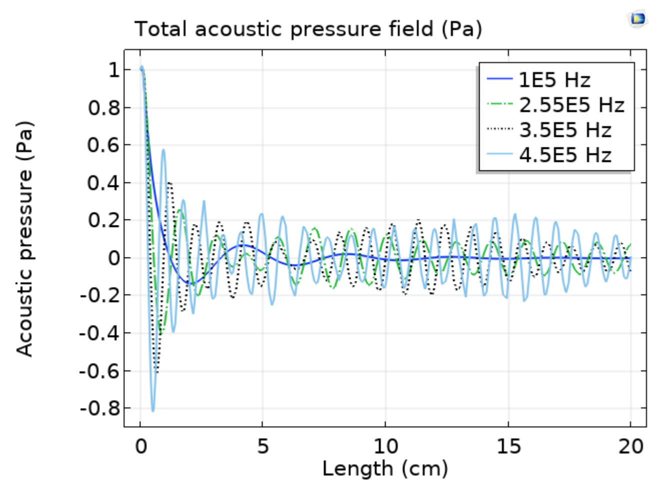

Fig. 4. Pressure in the longitudinal axis of the cortical bone cylinder

Same as radiation, the intensity of ultrasound is attenuated exponentially as it passes through a medium, according to the equation

$$A = A_0 e^{-\alpha x} \tag{8}$$

where A is the amplitude of the sound wave, A_0 is the initial amplitude of the sound wave, α is the attenuation coefficient and x is the distance traveled by the sound wave [16].

The length of the cylindrical sample was adjusted to assess the attenuation behavior, varying it from 5-100 cm. A cut-off point was placed at the end of the cylinder in each length and the absolute pressure was evaluated. All points were plotted and an exponential fit $(y = Be^{-Ax})$ was made out.

For the bone cylinder the exponential fit is

$$y = 0.19e^{-0.12x} \tag{9}$$

with a $R^2 = 0.99$, where y corresponds to the amplitude of the sound wave and x is the distance travelled in the medium. By evaluating each of the points along the cylinder, the exponential decay was verified as mentioned by Hughes [16].

It is possible to modify the sample material to analyze the attenuation coefficient of each material. A second approach was made with a muscle cylinder. The exponential fit for muscle is

$$y = 0.32e^{-0.06x} \tag{10}$$

with a $R^2 = 0.94$, the attenuation coefficient obtained in muscle is half that obtained in bone (muscle behaves as a liquid for ultrasound waves), due to the densities and intrinsic characteristics of each material. In figure 5 acoustic attenuation of two biological tissues (bone and muscle) is observed.

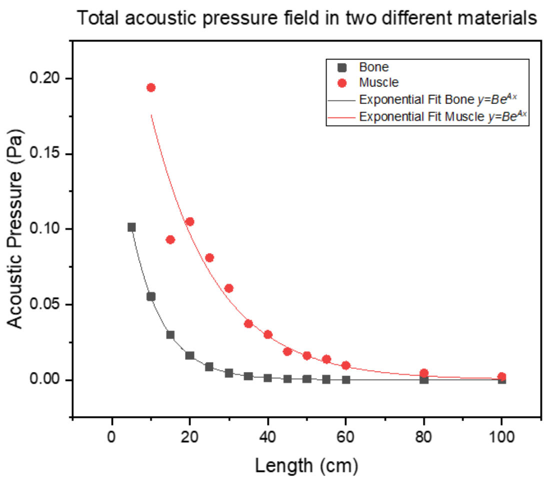

Fig. 5. Final cylinder pressure with increasing length

IV. CONCLUSIONS AND PERSPECTIVES

In this work, the finite element method was applied as a technique to analyze the sound wave propagation trough cortical bone, the numerical results show a great relationship with the behavior of analytical solution of the governing equation, so our model is an excellent starting point to move to an others more realistic models, including porosity in the bone for example. A first simulation approach was made in a 3D geometry, the perspectives of the work are to generalize that initial model into a inhomogeneous system where porosity and other parameters are explicitly included, bringing it closer to real phantoms, as well as to compare the predictions with experimental measurements. All of this is a great approach to initiate models that involve the development of ultrasonic devices with applications in the areas of orthopedics, leaving aside ionizing radiation for diagnosis disease, among others.

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