Leg-ligament-thigh-trunk Dynamic Model to Describe Posture Recovery after Double-leg Landing Task

Angel Cerda-Lugo¹, E. González-Galván¹, Alejandro González²

Abstract— One of the most common injuries in athletes is that of the Anterior Cruciate Ligament (ACL). This type of injury is commonly analyzed by observing the dynamics of the body in the sagittal plane. ACL injury can be indicated by a the small knee flexion angle and a small angular position of the trunk at start of leg-landing task. In this article a 4 Degrees of Freedom (DOF) dynamic model of the human body restricted to the sagittal plane is presented. The model represents the movement of the legs, an equivalent ligament between the tibia and femur, thighs and trunk. It is used to represent the recovery of vertical posture during a double leg landing task. Initial conditions in velocity are calculated as those resulting from a free fall from a height H. The results obtained from the simulation were satisfactory since the recovery of the vertical posture is achieved and it is possible to approximate the deformation suffered by the equivalent ligament. In conclusion, this model can be very useful in determining the behavior of the ligament and eventually, the possibility of injury after a double-leg landing task.

I. INTRODUCTION

Previous work investigating the most common injuries in the sport community include [1]–[4]. It has been concluded that injuries are most common in the lower limbs especially the knee and ankle. Additionally, these studies point to knee sprain as the most common injury. Within them, the Anterior Cruciate Ligament (ACL) is most often concerned [5]. Seventy percent of ACL injuries are sports related. Women who play sports such as basketball volleyball or soccer have a four times higher incidence of ACL injury than men.

ACL injury does not impact the population's health, but also impacts the economy. In United States, it is estimated that about 38,000 ACL injuries occur in women per year, where their cost is estimated to be \$17,000 dollars per injury, which means that the total cost of injuries could be close to \$646 million dollars annually [6]. Because of this, it is of vital importance to conduct studies that allow the prevention of this injury in female athletes.

ACL injury has been the subject previous studies, where the objective is to find a relationship between kinematic and kinetic parameters of the human body for a given task and the risk of injury to the ligament. It has been observed that injuries due to intrinsic factors are more common than injuries due to extrinsic factors [1], [6], [7].

Hewett *et al.* [6] conducted a study of the kinematics (joint angles) and kinetic loads (joint moments) of the knee during a jump-landing task on 205 female athletes. They found a

Fig. 1: Schematic 4 DOF dynamic model of legs-ligamentthighs-trunk.

correlation between the kinematic and kinetic parameters and injury. Their results show that there is a difference in abduction angle of knee, abduction moment of knee, flexion angle of knee and angular intern displacement of tibia (valgus deformity) during the landing. The values of abduction angle, abduction moment and angular intern displacement of tibia were greater for the injured population than for non-injured athletes. Conversely, the value of flexion angle of knee were smaller in the injured population than non-injured.

II. METHODOLOGY

A. Sagittal Plane Dynamic Model

The present study aims to determine the behavior of the knee ligaments and the articular kinematics of the joints associated to the double-leg landing task. To this end, a 4 Degrees of Freedom (DOF) model in the sagittal plane was developed. It consists of 3 rigid bodies: i) legs, ii) thighs and iii) trunk, and an equivalent ligament between the leg and thigh segments. The equivalent ligament represents the ligaments: (a) ACL, (b) Posterior Cruciate Ligament (PCL), (c) Medial Collateral Ligament (MCL) and (d) Lateral Collateral Ligament (LCL). Additionally, muscles were added to each of the segments of the model in order to recover the upright posture. Both muscles and ligaments are represented as viscoelastic elements in a Kelvin-Voigt model [8]–[10].

¹ACL is with the Faculty of Engineering, Universidad Autónoma de San Luis Potosí, 7800 San Luis Potosí, Mexico A184998@alumnos.uaslp.mx

 2 AG is with CONACyT- Universidad Autónoma de San Luis Potosí, 7800 San Luis Potosí, Mexico alejandro.gonzalez@uaslp.mx

To solve the proposed model, the schematic model shown in Fig. 1 is used. The segment OA represents the legs with mass m_1 and inertia at the center of mass (COM) I_1 , the segment BC represents the thigh segment with mass m_2 and inertia above the center of mass I_2 , \overline{CD} is represents the head-arms-and-trunk with mass m_3 and inertia at COM I_3 and the segment \overline{AB} represents the equivalent ligament of the knee. The angles θ_1 , θ_2 and θ_3 are the orientations of the leg, thigh and trunk segments respectively, while x is the deformation of the equivalent ligament, $\{k_1, c_1\}$, $\{k_2, c_2\}$ and ${k_3, c_3}$ are the stiffness and damping components associated with the muscles of the leg, thigh and trunk segments, respectively, $\{k_L, c_L\}$ are the viscoelastic components of the equivalent ligament.

In order to model the system's dynamics, the Euler-Lagrange equations will be used. For this it is necessary to compute the kinetic T , potential V and dissipated D energies. They can be obtained as:

$$
T = \frac{1}{2} \sum_{i=1}^{p} (m_i v_i^2 + I_i \omega_i^2)
$$
 (1)

$$
V = \sum_{i=1}^{p} \left(m_i g h_i + \frac{1}{2} k_i \theta_i^2 \right) + V_L
$$
 (2)

$$
V_L = \begin{cases} 0 & e < 0 \\ \frac{k_L}{12e_iL_0^2}x^3 & 0 \le e \le 2e_i \\ k_Lx\left(\frac{x - 2e_iL_0}{2L_0}\right) & e > 2e_i \end{cases}
$$
(3)

$$
D = \frac{1}{2}\left(\sum_{i=1}^p c_i\dot{\theta}_i^2 + c_L\dot{x}^2\right)
$$
(4)

where p represents the total number of rigid bodies in the system, $\overrightarrow{m_i}$, v_i , I_i , $\dot{\theta}_i$, h_i , k_i , θ_i , c_i and $\dot{\theta}_i$ are the *ith* rigid body mass, COM velocity, inertia with respect COM, angular velocity, height of COM with respect to O, muscular stiffness, angular position and muscular damping, respectively. In (2) term V_L represents the potential energy in the equivalent ligament. This term can be described as in (3), which is based on Stanev work [9], where k_L represents the stiffness of the equivalent ligament, x is its deformation, e_i is the non-linear strain level parameter, and L_0 the undeformable length of the ligament. The stiffness value k_L was calculated from the sum of the stiffness values of the four ligaments associated with the knee joint and the undeformed length L_0 is obtained from the geometric mean of the undeformed lengths of the knee ligaments. Finally, c_L represents the ligament's damping and \dot{x} is the ligament's deformation velocity.

Applying the Euler-Lagrange methodology to (1) - (4), the equations of motion are obtained as:

$$
\mathbf{M}(\vec{q})\ddot{\vec{q}} + \mathbf{C}(\vec{q}, \dot{\vec{q}})\dot{\vec{q}} + \mathbf{K}(\vec{q})\vec{q} + \vec{G}(\vec{q}) = \vec{\tau}
$$
 (5)

where \vec{q} represents the generalized coordinates vector \vec{q} = $\left[\theta_1 \ \theta_2 \ \theta_3 \ x\right]^T$, M is the inertial matrix, C is the Coriolis Matrix, **K** is the stiffness matrix, \vec{G} is the gravity vector and $\vec{\tau}$ represents articular joint torques vector.

B. Estimating the systems initial condition

As mentioned above, it is desired to know the behavior of the limbs and knee ligaments of the human body in the sagittal plane to the double-leg landing task. Therefore, the aim is to know the behavior of the system after a fall of a height H. It can be assumed that the impact of the feet with the ground is merely plastic, i.e. the coefficient of restitution between the feet and the ground is $e = 0$. To find the solution for (5) it is necessary to first determine the initial conditions for position and velocity of each of the generalized coordinates. The initial conditions in position \vec{q}_0 are assumed constant during the landing task. However, the initial conditions in velocity \vec{q}_0 after impact are not known, therefore an equivalent one degree of freedom θ system will be used, consisting of a single rigid body with mass M, which is calculated how the sum of rigid bodies' masses, inertia with respect to its center of mass I_T calculated from Steiner Theorem and a distance from the pivot point to its center of mass R , which is obtained using the distance between point O and COM. The initial velocity after impact θ of the equivalent system is calculated from the analysis of the amount of motion at the instant of impact with the ground.

Since the original system has 4 degrees of freedom, it is necessary to find 4 equations that relate the original system to the equivalent system. The equations to be used will be: (i) velocity of the center of mass in the direction \hat{i} , (ii) velocity of the center of mass in the direction \hat{j} , (iii) momentum and (iv) total energy.

The equations of the COM velocity and the quantity of motion are linear with respect to the initial velocity vector \dot{q}_0 . However, the equation of the total energy is not linear. Therefore, a Taylor series expansion will be used. From this analysis the following equation was obtained:

$$
\mathbf{J}\dot{\vec{q}}_0 = \vec{f} \tag{6}
$$

where J represents the velocity Jacobian of the equations described above and f represents the vector of the remaining terms of the Taylor series expansion.

III. RESULTS

In this section (5) will be solved in order to determine the position vector \vec{q} . A subject of total mass m and height h was assumed. Winter's anthropometric tables [11] were used to find the mass and geometry values of each of the model's segments. The viscoelastic properties of the ligaments were obtained from the works of [12] and [9]. However, the proposed model considers a single equivalent ligament, because of this the stiffness and damping values are taken as the sum of the stiffness and damping values of the ligaments. Finally, the stiffness and damping values of the muscles were based on our previous work [13]. The articular torques variables were defined using exponential function with form: $\tau_i(t) = (\tau_0)_i \cdot e^{bt}$ where τ_i represents

Fig. 2: Behavior of the generalized coordinates θ_1 , θ_2 , θ_3 and x, during one second after the double-leg landing task.

Parameter	Value		Parameter	Value	
\boldsymbol{m}	70	kg	c ₁	100	N-m-s/rad
h.	1.67	m	c ₂	150	$N-m-s/rad$
k ₁	1800	N-m/rad	c_3	200	N-m-s/rad
k_2	2000	N-m/rad	c_L	4000	N-s/rad
k_3	3300	N-m/rad	τ_1	10	N-m
k_L	48200	N	τ_2	10000	N-m
			τ_3	10000	N-m

TABLE I: Values to perform the calculation of the proposed model.

Parameter	Value		Parameter	Value	
θ_1	0.5236(30)	rad (deg)	θ_1	36.3362	rad/s
θ_2	0.9599(55)	rad (deg)	θ_2	58.3886	rad/s
θ_3	0.5760(33)	rad (deg)	θ_3	29.0784	rad/s
\boldsymbol{x}	0.01	m	\dot{x}	13.3099	m/s

TABLE II: Initial conditions to solve the proposed model.

the *i-th* articular torque, τ_0 is the maximum articular torque value, the constant b was defined as $b = -100$, so that the torques behave as an impulse function and torque function be continuum in order to facilitate the solution in simulation. The values of $(\tau_0)_i$ were chosen arbitrarily, so that the resulting kinematics are as close as possible to the motion of the human body in the sagittal plane observed in the available literature [14], [15]. TABLE I shows the values used to solve the equations of motion.

The initial conditions of the angular positions of the segments of the model were taken from the literature [14]. The initial deformation of the ligament (x) was calculated from data taken from [9], and finally the initial conditions in velocity were calculated utilizing the equation (6) assuming a height in free fall of 0.5 m. TABLE II shows the initial conditions.

In order to solve the equation of motion (5) a 4th order Runge-Kutta method is applied for a simulation time of 1

second. Fig. 2 shows the behavior of generalized coordinates vector.

The Fig. 3 shows the kinematics evolution of human body in the sagittal plane at three different time values: 1) $t = 0$ s, 2) $t \approx 0.5$ s, $t = 1$ s.

IV. DISCUSSION

Since Fig. 3 shows the evolution of human body movement and conclude with angular joints near of equilibrium point $(\theta_i \approx 0)$, the results obtained appear satisfactory. That is, it is possible to know the behavior of the leg-ligament-thightrunk system using a dynamic model in the sagittal plane. It is observed that the behavior of the angular positions are as expected. Therefore, it could be believed that the the values of the viscoelastic components of the muscles were chosen appropriately for the simulation.

However, in Fig. 2 it is observed that the values of the ligament deformation x take negative values, i.e., the thigh and leg segments share the same space. This error is attributed to the failure to consider the impact suffered by the femur with the tibia, as suggested by Abdel-Rahman *et al.* [12].

One of the objectives of the model presented here is to know the deformation of the ligaments, especially the ACL due to is common place in injury. Future work would aim at estimating the deformations of the four ligaments of the knee from the deformation of the equivalent ligament.

Another possible use of this model is the estimation of viscoelastic parameters of muscles and ligaments, as well as inertial parameters of the human body in the sagittal plane.

V. CONCLUSIONS

The model presented here could be very useful for the analysis of ACL deformation and the analysis of upright posture recovery. The main objective of this model is to be used to analyze injuries in ACL ligament and determine

Fig. 3: Evolution of themovement after double-leg landing task. Position at a) $t = 0$ s, b) $t \approx 0.5$ s and c) $t = 1$ s.

a possible correlation with joint kinematics in the sagittal plane.

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