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The Kapitza's Pendulum as a Concurrent Strategy for Maintaining Upright Posture

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***Abstract*— A Kapitza's pendulum shows that it is possible to stabilize an inverted pendulum by making its base oscillate vertically. This action seems to introduce an inertial effect which will produce an attractor about the upright vertical position. This work shows that the upright posture of the trunk achieved while walking can be explained using a combination of a vertical oscillation and an angular stiffness regulation at the pelvis. This is shown with an estimated oscillation and stiffness obtained from video recordings of an unimpaired and a Parkinsoninan gaits. By simulating the dynamic model of the pendulum for a range of parameters, a series of stability conditions are found. They show that the introduction of the vertical oscillation results in a fast stabilization of the trunk and point to control strategies which rely on the system's dynamics.**

I. INTRODUCTION

Literature regarding posture stabilization mainly falls into two camps. It states that the upright posture is controlled by (i) the stiffness of the joints, specifically of the ankle [1], (ii) and the intermittent control of position [2], [3].These paradigms can sometimes be combined to show intermittent control in the framework of stiffness control [4].

The intermittent control model is often constructed as a continuous-time feedback controller with a delay which incorporates a switching strategy between two controllers in order to reach bounded stability [5].

Intermittent control can be simplified by thinking that the effect of bounded stability could be generated by an imposed oscillation, in a similar mamner to the Kapitza's pendulum. The dynamic model of Kapitza's pendulum describes the stabilizing effect that a vertical oscillation has on the system. It has been shown that the oscillation introduces an inertial effect equivalent to a time varying stiffness which can counteract gravity under certain conditions [6]–[9], and has been dubbed the *"Kapitza stiffness"*. The authors' previous work suggests that this behavior (regarding stiffness regulation of the lower limb) is a generalized strategy employed when walking, and that such an oscillation measured at the pelvis helps the torso to maintain an upright position [10].

To study the effects of the *Kapitza stiffness* in trunk stabilization, an in-depth simulation study is presented here. Using publicly available videos showing an Unimpaired Gait Pattern, and a Parkinsonian Gait Pattern this work attempts to show that introducing the *Kapitza stiffness*, i.e. a vertical oscillation at the pelvis, has the ability to reduce the rise time of the dynamic system. This is true for a wide range of model conditions.

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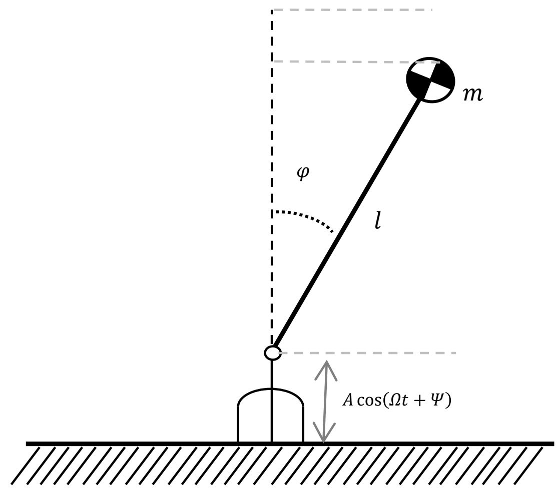

Fig. 1: Inverted pendulum with a vertical oscillation.

II. DYNAMIC MODEL

In this work, a two degrees-of-freedom inverted pendulum is studied. The pendulum in Figure 1 represents the subject's trunk and is considered to have a length l and a point mass m. The degrees-of-freedom are: the angle of rotation of the pendulum(φ) (i.e. the angle at the hip with respect to the vertical) and the vertical oscillation of the base ($A\cos(\Omega t+\psi)$) where A is its amplitude, Ω its frequency, and ψ its phase. Assuming that the revolute joint is also actuated upon with a rotational spring and damper (K and b respectively) we find:

$$ml^2\ddot{\varphi}+b\dot{\varphi}+K\varphi = \left(mlg - mlA\Omega^2\cos(\Omega t+\psi)\right)\sin(\varphi) \quad (1)$$

where mlg can be considered a gravity induced torque. Eq (1) is obtained from the analysis of the dynamic system presented in Figure 1, interested readers may refer to [6], [7], [10].

Note that (1) shows a time-varying, second order system where the effect of gravity on the mass is countered not only by the joint's rotational stiffness and damping, but also by a frequency dependent component referred to as the *Kapitza stiffness*. For ease of notation, we define the *Kapitza stiffness* to be:

$$K_k = mlA\Omega^2\cos(\Omega t + \psi) \quad (2)$$

for a single oscillatory input. When several frequencies of oscillation are introduced to the system, it becomes:

$$K_k = ml\sum_{i=1}^{n} A_i\Omega_i^2\cos(\Omega_i t + \psi_i) \quad (3)$$

where sub-index i refers to the *i-th* frequency.

Finally, for a system where no oscillation is introduced, the steady sate position of the pendulum is given by:

$$\varphi = \frac{mlg}{K}sin(\varphi) \quad (4)$$

which may be found using a suitable iterative method.

In a classical stiffness regulation control framework, the muscles around the joint co-contract creating a position dependent force, which counteracts the effect of gravity and hence maintain balance [7]. The stiffness generated by the muscles can be modulated and modeled by, for example, a non-linear inertial effect such as that provided by the Kapitza's stiffness (K_k). Modulating K_k may decrease the metabolic energy required to maintain the proper joint stiffness (K) trough co-contraction. Additionally, the vertical oscillation of the pendulum could be the result of involuntary reflexive movements, or the self-excitation of the mechanical system.

A. Modeling trunk tilt with an inverted pendulum

A system composed of a single inverted pendulum, modeling the upper part of the body starting at the base of the pelvis, is analyzed. For this, a numerical solution to (1) is determined and requires knowledge of two quantities: the measurable angular deviation of the segment with respect to the vertical (φ), and the joint stiffness (K) which can be determined manually as described in the following section.

Assuming no vertical oscillation of the pendulum, the system will reach an upright position when its joint stiffness is large enough to compensate the gravitational pull. That is $K \leq mlg$. For convenience, this paper will express the value of the joint stiffness to be proportional to the gravitational pull as:

$$K = \alpha mlg \quad (5)$$

This work will focus on cases where the joint stiffness (K) is not enough to bring the system to the upright position on its own. That is, $0 \leq \alpha < 1$. Finally, we shall assume the damping coefficient of the system to be $b = 0.1K_g$.

III. Methods

This work makes use of video recordings of an Unimpaired individual[1] and Person with Parkison's disease[2] Gaits. Measurements of these gaits were performed using the software suite *TRACKER* [11]; specifically, the vertical oscillation of the hip, and the angle of the trunk (φ) were recorded.

[1]https://youtu.be/StwuCOayKBA?t=2
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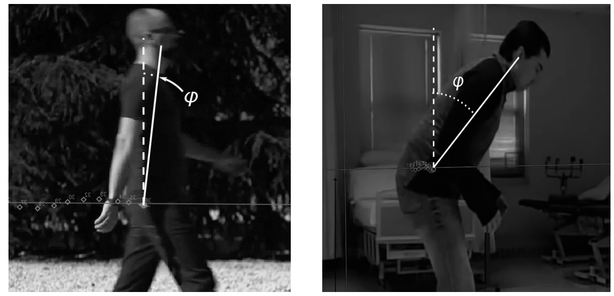

(a) Unimpaired gait (b) Parkinsonian gait

Fig. 2: Trunk tilt angles for distinct walking conditions

The human subjects were assumed to have a height of 1.75 m, and a body mass of 70 kg distributed according to the information published by Winter. This means that the trunk was modeled using an equivalent pendulum with a length of ($l = 0.1802H$) and mass of ($m = 0.678M$) [1].

The vertical oscillation for the unimpaired gait pattern will be henceforth referred to as *UGP*, while that of the Parkinson's subject will the referred to as *PGP*. Both gaits are characterized by a complex hip vertical bobbing which cannot be fully represented by a single pure sine wave. In order to fully describe the hip vertical oscillation we determine their frequency components using an fast Fourier transform (FFT). From (2) we see that the Kapitza's stiffness is a function of the frequency Ω and the amplitude A of the vertical oscillation and can be experimentally measured at the pelvis during walking. Assuming each frequency component as a separate oscillator, the Kapitza's stiffness will be written as in (3).

For *UGP* the maximum oscillation amplitude was under 2 cm and was found at 1.9 Hz while for *PGP* the maximum oscillation amplitude is below 3 mm and was found at 2.1 Hz.

We determined the subject's proper stiffness (K) empirically by simulating the system, with its oscillation component (K_k) until the output angle accurately reproduced the average angular deviation of the torso measured from the videos for the corresponding gait. That is, while including the the Kapitza's stiffness (computed as in (2)) we modulated the proper stiffness to match the equilibrium position in our simulation to the video's. This way, the ratio between the proper stiffness and the gravity induced torque was found to be $\alpha = 0.9785$ for the *UGP*, and $\alpha = 0.89$ for *PGP*. For reference, the steady state value of the tilt angle was found to be $\varphi = 0.06$ radians for the *UGP*, and $\varphi = 0.82$ radians for *PGP*

Previous study of this model suggests that the vertical oscillation of the system helps to stabilize it closer to the vertical, but found that the time response varied substantially between both subjects [10]. Specifically, the time rise for *UGP* was substantially slower after the addition K_k, while the opposite was true for *PGP*. In this work we wish to further study this effect.

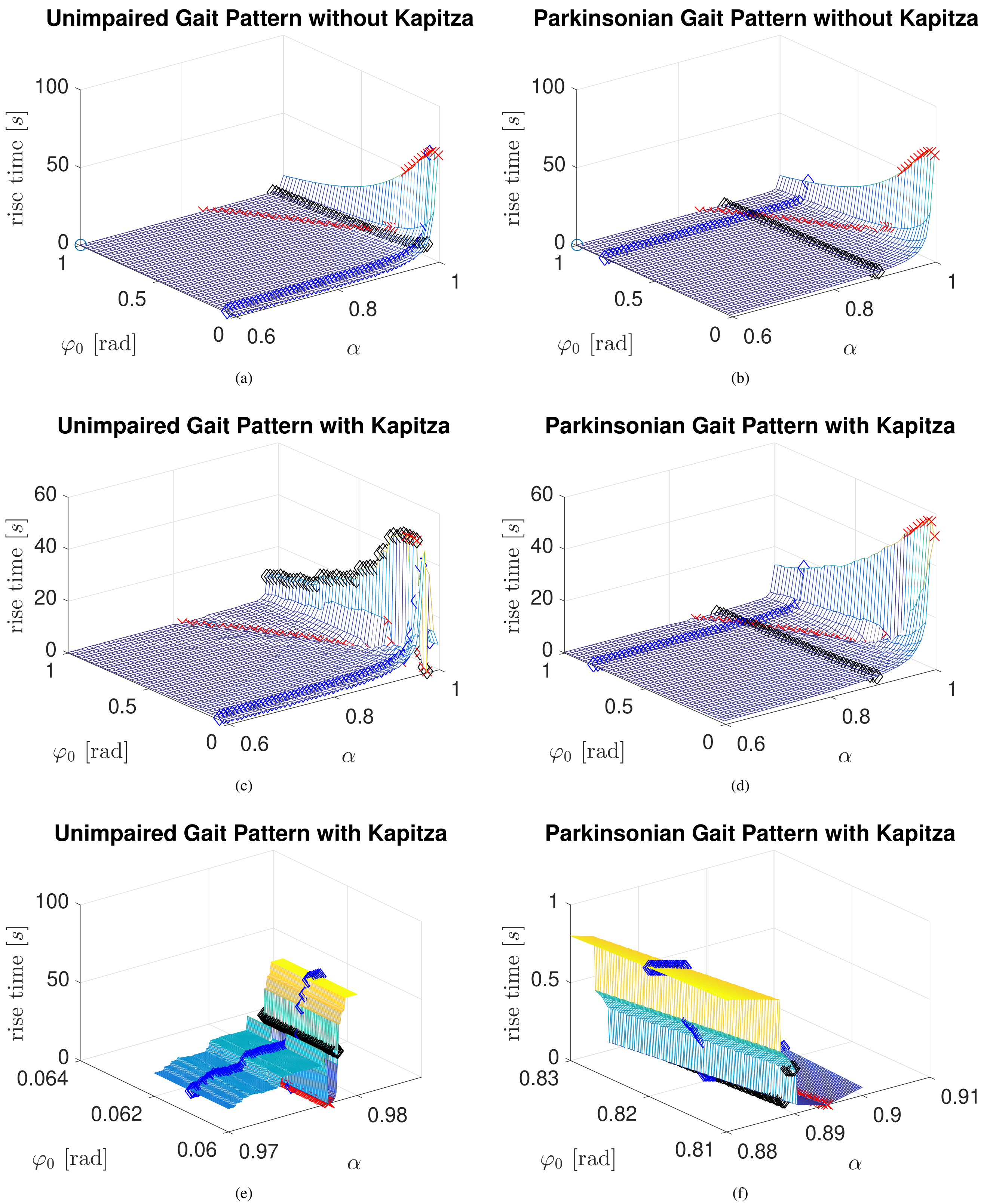

Fig. 3: Simulation results for the studied conditions. The figure shows the dynamic behaviour of the system for a wide range of α and φ_0 values.

To this end, a series of simulations have been performed in which two parameters are varied: (i) the starting angular position of the pendulum (φ_0), and (ii) the ratio of the proper stiffness to that induced by gravity (α). In these simulations we focus on the effect that the parameters have on the rise time of the system, defined as the time required for the response to go from 10% to 90% of its steady state value. To determine the effect of the vertical oscillation on the system, the simulations were performed twice: once with and once without the inclusion of the Kapitza stiffness measured for each gait.

Results are summarized in Figure 3. The surfaces in these figures show the computed rise time for each of the four conditions. In the figures, diamond markers represent the measured α and average trunk angle φ as obtained from the video data respectively. That is to say, the recorded gait corresponds to the intersection of the lines formed by these markers. They are shown in the figure as reference to the known gait patterns. Additionally, red-cross markers were added to represent the steady state equilibrium position and divide the surfaces into two sections: On the one side, the steady state value of the system is closer to the vertical than the initial position used for the simulation ($\varphi_{final} < \varphi_0$). On the other side, the opposite is true. In other words, the curve formed by these red-cross markers, approximates the condition where $\varphi_{final} = \varphi_0$.

IV. Discussion and Conclusions

The dynamics of the Kapitza's pendulum can be reconstructed from Figure 3, which shows the system's time rise given the parameters α and φ_0. Different strategies become apparent for the Parkinsonian Gait (*PGP*) when compared to an Unimpaired Gait (UGP).

Note that any combination of parameters where $\varphi_{final} = \varphi_0$ will not deviate from the starting position. For these values, the system rise time should be equal to zero. The curve shown by the red-cross markers approximates this line.

The introduction of the Kapitza stiffness creates a valley in the time rise surface. The shape of the valley for UGP is narrow and deep as opposed to a shallow valley in *PGP*. In both cases, a flat region on the surface exists for large values of φ_0 and low values of α. While in this region, the subject will appear to be slouched forward.

PGP chose the equilibrium condition in the flat region of the time rise surface. Removing the Kapitza oscillation shows that there is no intersection between the three curves on the surface (Figure 3b). When the Kapitza oscillation is applied, all three lines (measured α, φ, and equilibrium position) intersect at a point showing that the system will quickly converge to its resting state (Figure 3d and Figure 3f).

This contrasts with what is seen for the UGP condition where no intersection exists, even after the addition of the Kapitza oscillation. The three curves never meet at a single point (albeit very close) which is now located in the valley of the time rise surface (Figure 3c and Figure 3e). It could be speculated that UGP operates in an unstable condition which takes a long time to converge to its steady state, but could be quickly stabilized asymptotically via a small change in α.

These results seem to point to a general rule for motion generation: Impaired gait is controlled so as to be close to the steady state all of the time; where the time rise of the system is very small independently of the addition of the oscillation (i.e. located in the flatter region of the time rise surface). When the Kapitza oscillation is present, a wide valley is observed, within which the time rise decreases. Yet, the equilibrium point at which the subject operates is only mildly affected by the effect of the oscillation. This is accomplished by decreasing α and increasing φ_0, keeping the initial position close to the equilibrium. Thus, the subject assumes a slouched posture which position and time to get there is only mildly affected by the effect of the oscillation.

Conversely, unimpaired individuals, can freely take advantage of the dynamic aspect of motion, by modulating both α and φ_0 to position themselves on the steep walls of the gorge generated by the oscillation. In doing so, UGP operates in a meta-stable position that is theoretically unstable but that would take a long time to diverge. In this way, UGP takes advantage of the gravitational effect to move the equilibrium position slightly forward and thus use it as propulsion, knowing that the bottom of the gorge, representing an immediate steady state position, is only a small stiffness adjustment away.

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